Analysis of Water Hammering in Bubbly Flows*

(1st Report, Two-component Two-phase Flows)

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Water hammer phenomena caused by a sudden valve closure are considered in two-component bubbly flows where the phenomena are more complicated than in single-phase flows because of the presence of a compressible component. Basic partial differential equations based on a one-dimensional homogeneous flow model are solved analytically by linearization and iterated Laplace transformation. As a result, the profiles of the pressure transients, the propagation velocity of a pressure wave with a small amplitude, and the effects of valve closure on transient pressures are found. Further, the effect of friction factor on pressure profiles is shown and approximated by a simple equation. It is also shown that reflection and transmission occur at the interphase where the void fraction changes.

Key Words : Fluid Mechanics, Water Hammering, Analysis of Transient Pressures, Two-component Bubbly Flow, Laplace Transformation

1. Introduction

The water hammer phenomena caused by a sudden valve closure in gas-liquid two-phase flows must be clarified for the safety analysis of LOCA in reactors and further for the safety of boilers, chemical plants, pipeline transport of fluids such as petroleum and natural gas.

The water hammer phenomena in single-phase liquid flows has been investigated for many years and a number of books have been published⁽¹⁾⁻⁽³⁾. However, when gas mixes with liquid, the pressure characteristics change very much owing to the greater compressibility than in a single-phase flow; in particular, the pressure transients in one-component two-phase flows are more complicated than those in two-component two-phase flows because of the occurrence of an interfacial mass transfer between liquid and vapor. In spite of a large number of investigations on shock tubes^{(4),(5)}, only a few investigations have been done on the water hammer phenomena caused by a sudden valve closure. Martin et al.^{(13),(14)} and the authors⁽⁷⁾⁻⁽¹²⁾ have reported on experimental and analytical results for air-water two-component two-phase flows, and about one-component two-phase flows the authors have reported on a series of experiments and numerical results of calculation^{(15),(16)}. These numerical analyses are useful to predict the pressure transients but by such analyses it is difficult to understand the physical meaning of the water hammer phenomenon.

In the present study an analytical approach to the transient profiles of a

pressure wave with a small amplitude is presented using the Laplace transformation method. In this 1st report, a solution for two-component two-phase flows is first shown. The profiles of pressure transients, propagation velocity, effect of valve closing time and effect of distribution of void fractions along the tube on transient pressures can be well expressed in analytical form.

2. Nomenclature

a	: Speed of sound
n	: Polytropic index
P	: Pressure
R	: Reflection rate
s, z	: Laplace transform of time t and distance x
t	: Time
W	: Velocity
α_g	: Void fraction
ρ	: Density
τ	: Valve closing time

Subscripts

g	: Gas phase
l	: Liquid phase
TP	: Two-phase
o	: Steady state

3. Basic Equations

A one-dimensional homogeneous bubbly flow model is shown in Fig. 1. It consists of a stop valve ①, a reservoir tank with an air chamber ②, and a rigid pipe.

Fig. 1 A total system of water hammer

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The following assumptions are made :

- There is no slip between vapor and liquid.
 - The compressibility of pipe wall and liquid is negligibly small compared with that of air.
 - The friction loss is negligible. (In Section 4.2.3 this is taken into account.)
 - The pressure in the bubbles is equal to that in the surrounding liquid.
- Under the above assumptions, the following conservation and momentum equations hold.

$$\frac{\partial}{\partial t}(\alpha_0 \rho_0) + \frac{\partial}{\partial x}(\alpha_0 \rho_0 w) = 0 \quad (1)$$

$$\frac{\partial}{\partial t}[(1-\alpha_0)\rho_l] + \frac{\partial}{\partial x}[(1-\alpha_0)\rho_l w] = 0 \quad (2)$$

$$\frac{\partial}{\partial t}[\alpha_0 \rho_0 w + (1-\alpha_0)\rho_l w] + \frac{\partial}{\partial x}[\alpha_0 \rho_0 w^2 + (1-\alpha_0)\rho_l w^2] + \frac{\partial P}{\partial x} = 0 \quad (3)$$

The gas is assumed to change polytropically, that is,

$$P = c \cdot \rho_0^n \quad (4)$$

where c is a constant and n is a polytropic index which takes the value of unity for the isothermal change and κ for the reversible adiabatic (isentropic) change.

4. Analysis and Results

4.1. Solution by a linearization method

The state quantities in Eqs. (1) to (4) are each considered to consist of a steady-state value (represented by subscript 0) and an infinitesimal fluctuation (represented by Δ), like $P = P_0 + \Delta P$, $\rho_g = \rho_{g0} + \Delta \rho_g$, $\alpha_g = \alpha_{g0} + \Delta \alpha_g$, and $W = W_0 + \Delta W$. By substituting the above equations into Eqs. (1) to (4), neglecting the infinitesimal fluctuation terms of more than the second degree, using non-dimensional forms such as $\tilde{p} = \Delta P/P_0$, $\tilde{w} = \Delta W/W_0$, and performing iterated Laplace transformation of time t and distance x , we obtain the following equations $\tilde{p}(s, z)$, $\tilde{w}(s, z)$.

$$\begin{aligned} &[(a_0^2 - w_0^2)z^2 - 2w_0sz - s^2]\tilde{p}(s, z) \\ &+ \frac{nw_0}{a_0}s\tilde{w}(s, 0) - [(a_0^2 - w_0^2)z - w_0s] \\ &\times \tilde{p}(s, 0) = 0 \quad (5) \end{aligned}$$

$$\begin{aligned} &[(a_0^2 - w_0^2)z^2 - 2w_0sz - s^2]\tilde{w}(s, z) \\ &+ \frac{P_0s}{\rho_{TP0}W_0}\tilde{p}(s, 0) - [(a_0^2 - w_0^2)z - w_0s] \\ &\times \tilde{w}(s, 0) = 0 \quad (6) \end{aligned}$$

where $\rho_{TP0} = \alpha_{g0}\rho_{g0} + (1-\alpha_{g0})\rho_{l0}$ is the mean density of two-phase flow in the initial steady state and $a_0 = \sqrt{nP_0/(\alpha_{g0}\rho_{TP0})}^{*1}$ is the speed of sound in homogeneous flows.

The inverse Laplace transformation of Eqs. (5) and (6) with respect to z gives $\tilde{p}(s, x)$ and $\tilde{w}(s, x)$ at arbitrary location x , that is,

$$\begin{aligned} \tilde{p}(s, x) &= \frac{-a_0\rho_{TP0}W_0}{2P_0}\tilde{w}(s, 0) \\ &\times (e^{-axs} - e^{-\beta xs}) + \tilde{p}(s, 0)\frac{e^{-axs} + e^{-\beta xs}}{2} \quad (7) \end{aligned}$$

$$\begin{aligned} \tilde{w}(s, x) &= \frac{-P_0}{2a_0\rho_{TP0}W_0}\tilde{p}(s, 0) \\ &\times (e^{-axs} - e^{-\beta xs}) + \tilde{w}(s, 0)\frac{e^{-axs} + e^{-\beta xs}}{2} \quad (8) \end{aligned}$$

where $\alpha, \beta = \mp 1/(a_0 \mp w_0)$.

The boundary condition that the pressure in the upstream tank (2) in Fig. 1 is a constant independent of time t holds.

$$x = -L : \tilde{p}(s, -L) = 0 \quad (9)$$

By substituting Eq. (9) into Eqs. (7) and (8), the following solutions for $\tilde{p}(s, x)$ and $\tilde{w}(s, x)$ are given.

$$\tilde{p}(s, x) = E(-a_0\rho_{TP0}W_0/P_0)\tilde{w}(s, 0) \quad (10)$$

$$\tilde{w}(s, x) = F\tilde{w}(s, 0) \quad (11)$$

where $E, F = \{e^{-axs} - e^{-(\beta-a)Ls-axs} \mp e^{-(\beta-a)Ls-\beta xs} \pm e^{-2(\beta-a)Ls-\beta xs}\} / \{1 - e^{-2(\beta-a)Ls}\}$.

The pressure transient at a suddenly closed valve is obtained by substituting $x = 0$ into Eq. (10) as follows.

$$\begin{aligned} \tilde{p}(s, 0) &= (-a_0\rho_{TP0}W_0/P_0)\tilde{w}(s, 0) \\ &\times \{1 - 2e^{-(\beta-a)Ls} + e^{-2(\beta-a)Ls}\} / \{1 - e^{-2(\beta-a)Ls}\} \quad (12) \end{aligned}$$

4.2. Pressure and Velocity Response

The transient responses of the pressure and velocity at arbitrary location and time are expressed by Eqs. (10) and (11). For a sudden valve closure like a step shown by the solid line in Fig. 2, $\tilde{w}(s, 0) = -1/s$ *2

*1 By using the assumption (b) in Eqs. (1) to (4), the propagation velocity of a pressure wave with a small amplitude can be expressed by the following equation instead of $a_0 = \sqrt{nP_0/(\alpha_{g0}\rho_{TP0})}$.

$$a_0 = \{[\tilde{\alpha}_{g0}(1/nP_0 + JD/Eh) + (1-\tilde{\alpha}_{g0})(1/H + JD/Eh)] \times \rho_{TP0}\}^{-1/2}$$

where J is a coefficient that is dependent on the condition of pipe anchoring. Assuming a water-air mixture in a pipe made of steel under the conditions of normal pressure and temperature, the values of $H/nP_0 \approx 10^4$ and of $E/nP_0 \approx 10^6$ are taken and the result is approximated by $a_0 = \sqrt{nP_0/(\alpha_{g0}\rho_{TP0})}$ for $\alpha_{g0} > 10^2$.

*2 Here, the relationship $\Delta W/W_0 = -1$ is taken for the sake of convenience. In the case of $\Delta W/W_0 = -a$ ($a < 1$), where linearization is valid, one ought to consider a factor which is a times the values of $\tilde{p}$ and $\tilde{w}$.

Fig. 2 Initial condition of $\tilde{w}(t,0)$

and the pressure and velocity transient profile at any location is solved against time. The initial maximum pressure rise is expressed by a formula $a_0 \rho \tau P_0 W_0$ ^{*} similar to the well-known Joukowsky's formula $a_0 \rho_0 W_0$ holding for single-phase flows. A compression wave caused by a sudden valve closure propagates upstream with the velocity $(a_0 - W_0)$ and, being reflected at the upstream tank, it propagates downstream as a rarefaction wave of the velocity $(a_0 + W_0)$. On the other hand, the flow velocity becomes W_0 to 0 ($\tilde{w} = -1$) where the first compression wave reaches and begins to flow backwards due to the return of the rarefaction wave (it corresponds to $-W_0$, that is, $\tilde{w} = -2$). This is a fundamental concept of the pressure and velocity responses in the water hammer phenomenon and an analogy of the single-phase flow holds. The pressure transients at the closed valve will be described below.

4.2.1 The effect of closing time τ of the valve

Because in general the valve involves the closing time lag, the initial condition with the time lag τ should be considered as shown by a broken line in Fig. 2. Substituting an initial condition $\tilde{w}(s,0) = -(1-e^{-\tau s})/\tau s^2$ into Eq. (12), the pressure transients $\tilde{p}(t,0)$ at the closed valve are obtained. It denotes a periodic function of the periodic time $2(\beta - \alpha)L = 2L/(a_0 - W_0) + 2L/(a_0 + W_0)$.

$$\begin{aligned} \tilde{p}(t,0) = & \frac{a_0 \rho \tau P_0 W_0}{\tau P_0} \{ t - (t - \tau)U(t - \tau) - 2\{t - (\beta - \alpha) \\ &)L\}U\{t - (\beta - \alpha)L\} + \{t - 2(\beta - \alpha)L\}U\{t - 2 \\ & (\beta - \alpha)L\} + 2\{t - \tau - (\beta - \alpha)L\}U\{t - \tau - (\beta - \alpha)L\} \\ & - \{t - \tau - 2(\beta - \alpha)L\}U\{t - \tau - 2(\beta - \alpha)L\} \} \end{aligned} \quad \dots (13)$$

where $\beta - \alpha = 2a_0/(a_0 - W_0)(a_0 + W_0)$ and U is a unit step function. Figures 3(a) and (b) are given from Eq. (13). Figure 3(a) expresses the region of sudden closure $\tau < (\beta - \alpha)L$. It is shown that the maximum pressure rise $\Delta P_{\max}$ is $a_0 \rho \tau P_0 W_0$, independently of τ . On the other hand, in a slow closing region $\tau > (\beta - \alpha)L$, $\Delta P_{\max}$ decreases since the rarefaction wave reflects at the tank and returns to the closed valve within $t = \tau$, as follows.

^{*}3 In the case of the pressure wave with a finite amplitude, it is expressed as $C_0 \rho \tau P_0 W_0$, where the propagation speed of the shock wave, C_0 , is $\sqrt{n P_2 / (\alpha_0 \rho \tau P_0)}$ and P_2 is the pressure behind the shock wave.

Fig. 3 The effect of τ on pressure transients

Fig. 4 The effect of characteristics of valve closure

$$\begin{aligned} \Delta P_{\max} &= a_0 \rho \tau P_0 W_0 \cdot 2a_0 L / \{(a_0 - W_0)(a_0 + W_0)\tau\} \\ &= \frac{2L}{\tau} \rho \tau P_0 W_0 < a_0 \rho \tau P_0 W_0 \quad \dots (14) \end{aligned}$$

4.2.2 The effect of characteristics of valve closure

The profile of the initial pressure rise changes owing to the valve characteristics. The relationship between the valve flow rate and the pressure difference is expressed as follows by representing the valve characteristics as $\phi = (1 - t/T_c)^m$.

$$\frac{w}{w_0} = \phi \sqrt{\frac{P - P_A}{P_0 - P_A}} \quad \dots (15)$$

where P_A is the pressure at the exit of the valve, T_c is the valve closing time, and m is an index denoting the valve closure characteristics. The coefficient of the valve characteristics ϕ changes as shown by broken lines in Fig. 4 for $m = 0.5, 1$, and 2.5 . Linearizing the above equation against P , the velocity upstream of the valve is expressed as follows.

$$\tilde{w}(t, 0) = \left(1 - \frac{t}{T_c}\right)^m \left\{1 + \frac{P_0 \tilde{p}(t, 0)}{2(P_0 - P_A)}\right\} - 1 \quad \dots\dots (16)$$

In the case of the region of sudden valve closure, one gets

$$\tilde{p}(t, 0) = \left\{1 - \left(1 - \frac{t}{T_c}\right)^m\right\} / P_0 \left\{ \frac{\left(1 - \frac{t}{T_c}\right)^m}{2(P_0 - P_A)} + \frac{1}{a_0 \rho T P_0 W_0} \right\} \quad \dots\dots (17)$$

The non-dimensional pressure transients, which are obtained above, are shown in Fig. 4 by a solid lines for $a_0 \rho T P_0 W_0 / (P_0 - P_A) = 1, 3, 5, \text{ and } 10$ for each value of $m = 0.5, 1, 2.5$. It is evident that the profile of the pressure rise after the valve closure takes a convex shape toward the lower right for $m = 0.5$ and 1 and for $m = 2.5$ it has a concave shape. As the value of the rate of pressure rise $a_0 \rho T P_0 W_0 / (P_0 - P_A)$ is greater, the value $\tilde{p}(t, 0) / (a_0 \rho T P_0 W_0 / P_0)$ becomes smaller for the same value of t/T_c .

4.2.3 The effect of friction loss

By adding the frictional term $\partial P_f / \partial x = AW|W|^{i-1}$ (where i is a constant and A is assumed to be a constant dependent on the physical properties of the fluid, the tube diameter and the void fraction), the pressure profile $\tilde{p}(s, 0)$ is obtained for the region of sudden closure as follows.

$$\begin{aligned} \tilde{p}(s, 0) &= \frac{-2a_0^2 \rho T P_0 W_0 (s+B) \tilde{w}(s, 0)}{P_0 \left\{ (2s+B) a_0 \sqrt{1 - \frac{B^2(a_0^2 - w_0^2)}{(2s+B)^2 a_0^2}} + B w_0 \right\}} \quad \dots\dots (18) \end{aligned}$$

where $B = i A W_0^{i-1} / \rho T P_0$ (in 1/second).

The pressure transient $\tilde{p}(s, 0)$ for the initial condition $\tilde{w}(s, 0) = -1/s$ is, by putting $a_0^2 - W_0^2 = a_0^2$ in Eq. (18),

$$\tilde{p}(s, 0) = \frac{a_0 \rho T P_0 W_0 (s+B)}{P_0 \left\{ \sqrt{s(s+B)} + \frac{B w_0}{2 a_0} \right\} s} \quad \dots\dots (19)$$

A numerical solution is obtained as shown by chain lines in Fig. 5, but if the second term of the denominator is infinitesimal, it is briefly expressed by the inverse Laplace transformation of s as follows.

$$\begin{aligned} \tilde{p}(t, 0) &= \frac{a_0 \rho T P_0 W_0}{P_0} \left\{ e^{-\frac{Bt}{2}} (1+Bt) I_0 \left(\frac{Bt}{2} \right) \right. \\ &\quad \left. + Bt e^{-\frac{Bt}{2}} I_1 \left(\frac{Bt}{2} \right) \right\} \quad \dots\dots (20) \end{aligned}$$

where $I_0(t)$ and $I_1(t)$ are the Bessel functions of the 0th order and first order, that is, $I_0(t) = 1 + t^2/4 + t^4/64 + t^6/2304 + \dots$ and $I_1(t) = t/2 + t^3/16 + t^5/384 + \dots$. In the case of $\beta t \ll 1$, it is approximated by the following equation.

$$\tilde{p}(t, 0) = \frac{a_0 \rho T P_0 W_0}{P_0} \left(1 + \frac{Bt}{2} \right) \quad \dots\dots (21)$$

That is, after the initial pressure rise, a pressure rise of $Bt/2$ can be further recognized. For example, this pressure pattern is shown against the non-dimensional value Bt in Fig. 5. In the figure the solid lines show Eq. (21) and the broken lines show Eq. (20). The result is same as that of $\tilde{w}(s, 0) = -1/s$ for $a_0/W_0 = \infty$ in Eq. (18). A chain line is obtained from Eq. (18) using the numerical calculation method of Gaver⁽¹⁸⁾. It is shown that the subsequent pressure rise after the initial rise is smaller as B and a_0/W_0 become smaller. At the limit $a_0/W_0 = 1$, the subsequent pressure rise does not occur. In the case of smaller Bt and W_0/a_0 , it is shown that it can be approximated by Eq. (21).

Next, the effect of the friction loss on the initial pressure rise is discussed in relation to the valve closing time lag. The initial condition shown by the broken line in Fig. 2 is substituted into $\tilde{w}(s, 0)$ in Eq. (18). The results are shown by solid lines in Fig. 6, but here a_0/W_0 is taken to be infinite since the effect of a_0/W_0 is already shown in Fig. 5. The horizontal axis is for a non-dimensional value t/τ and the parameter is $B\tau$. In the figure the pressure transient for the step change of $\tau = 0$ (the solid line in Fig. 2) is shown by broken lines, and it corresponds to the case of $a_0/W_0 = \infty$ in Fig. 5. That is, the initial pressure rise $\Delta P_{\max}$ becomes greater than $a_0 \rho T P_0 W_0$, and the pressure rise equal to

Fig. 5 The effect of friction loss

Fig. 6 The effect of τ and friction loss

that at $t = \tau/2$ for the broken lines in the figure occurs again $\tau/2$ later for the value of the parameter B equal to B in parameter $B\tau$, as shown by a solid line. The initial pressure rise at $t = \tau$ is expressed as follows.

$$\Delta P(\tau, 0) = a_0 \rho_{TP0} W_0 \left(1 + \frac{B\tau}{4} \right) \dots \dots \dots (22)$$

4.2.4 Transmission and reflection

In a previous report⁽¹⁰⁾, it was shown that a complicated pressure rise occurs further in step form after the first pressure rise from the experiments of the water hammer phenomenon in the test section, where both a bubbly and a single-phase liquid flow exist along the axis of the test section as shown in Fig. 7 (an example of the experiment with an air-water two-phase flow is also shown in Fig. 8). This was verified from both the numerical calculation and the theoretical analysis using a model of reflection and transmission, which were considered to take place at the boundary between two-phase flow regimes. Here, using this analytical method, the characteristics of the pressure rise due to the reflection and transmission at the boundary will be made clear.

For the two-phase section (I) and the single-phase section (II) (the location of the boundary is $x = -l$) in Fig. 7, Eqs. (7) and (8) respectively hold, where for the single-phase flow $\rho_{TP0} \rightarrow \rho_{l0}$, $a_0 \rightarrow a_{l0}$. To distinguish the respective values in the two-phase and the single-phase flow, the subscripts I and II are used. From these four equations and the boundary condition at the upstream end $P_{II}(s, -L) = 0$, the pressure transients $P_I(s, 0)$ at the closed valve are obtained for $L \gg l$.

$$\begin{aligned} \bar{p}_I(s, 0) &= \frac{-(a_0 \rho_0 W_0)_I}{P_0} \\ &\times \frac{1 + \frac{K-1}{K+1} e^{-(\beta-a)ls}}{1 - \frac{K-1}{K+1} e^{-(\beta-a)ls}} \bar{w}(s, 0) \dots \dots \dots (23) \end{aligned}$$

where $K = (a_0 \rho_0 W_0)_{II} / (a_0 \rho_0 W_0)_I$. Substituting $\bar{w}(s, 0) = -1/s$ into Eq. (23) and performing the inverse Laplace transformation, one gets for $j(\beta-\alpha)l < t < (j+1)(\beta-\alpha)l$ ($j=0, 1, 2, \dots$),

$$\bar{p}(t, 0) = \frac{(a_0 \rho_0 W_0)_I}{P_0} \left\{ 1 + 2 \sum_{j=1}^j R^j \right\}$$

Fig. 7 Water hammer for both a bubbly and a liquid flow

$$= \frac{(a_0 \rho_0 W_0)_I}{P_0} \{ 1 + 2R(R'-1)/(R-1) \} \dots (24)$$

where the reflection rate^{*4} $R = (K-1)/(K+1) = [(a_0 \rho_0 W_0)_{II} - (a_0 \rho_0 W_0)_I] / [(a_0 \rho_0 W_0)_I + (a_0 \rho_0 W_0)_{II}]$.

The pressure response of Eq. (24) increases by factors of $2R$, $2R^2$, and $2R^3$ every time the reflected wave returns from the boundary, as shown in Fig. 8. The maximum pressure rise is $\lim_{t \rightarrow \infty} \Delta P = (1+R)(a_0 \rho_0 W_0)_I / (1-R) = (a_0 \rho_0 W_0)_{II}$ from Eq. (24). That is, the pressure rise does not exceed the value of the pressure rise in the single-phase flow $(a_0 \rho_0 W_0)_I$. These phenomena coincide with the results of experiment as well as the previous analytical results, and the phenomena can also be explained physically by making such linearization.

5. Conclusion

In the water hammer phenomenon due to a sudden valve closure in two-component bubbly flows, the pressure and velocity transients are analytically determined. The effects of various factors on the initial pressure rise are discussed. The actual water hammer phenomenon shows a pressure profile with a finite amplitude but it is evident that such linearized analysis expresses the qualitative characteristics of the water hammer phenomenon in two-phase flows. It is concluded that :

- (1) The pressure and velocity responses at arbitrary location and time are expressed by Eqs. (7) and (8).
- (2) The profile of the initial pressure rise differs because of the valve characteristics, but the maximum pressure rise is a constant independent of the closing time and the valve characteristics. It is expressed by (the propagation speed of the pressure wave) $\times \rho_{TP0} W_0$.
- (3) The friction loss leads to a further pressure rise after the initial rise, but in the case of smaller values of $B\tau$

Fig. 8 Pressure response with transmission and reflection

*4 The transmission rate Y is expressed in a similar way as follows.

$$Y = \frac{2(a_0 \rho_0 W_0)_I}{(a_0 \rho_0 W_0)_I + (a_0 \rho_0 W_0)_{II}}$$

- and W_0/a_0 , it is expressed by Eq. (21).
- (4) In the case of a single-phase region existing upstream, reflection and transmission of the pressure wave occur at the boundary between the two-phase and the single-phase flow. The maximum pressure rise in the two-component two-phase flow does not exceed the maximum pressure rise in the single-phase liquid flow.

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