Study of a Bounded Jet Flow Considering the Initial Turbulence*
(2nd Report, In the Case of Relatively Large Nozzle Aspect Ratio)

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For an approximate calculation of the bounded jet flow, a velocity distribution function on the bounded jet center-plane is proposed, considering the effects of the wall turbulence and the free turbulence. Also, in order to determine the empirical parameter contained in this function, experiments were carried out using a nozzle having an aspect ratio of 16 with the initial turbulence intensity prescribed. As a result, the velocity distributions of the bounded jet flow are well expressed by the proposed function, regardless of the initial turbulence intensity. Furthermore, the variation of the flow patterns towards downstream can be shown by means of the parameter contained in the function.

Key Words : Boundary Layer, Turbulence, Bounded Jet, Nozzle Aspect Ratio, Secondary Flow

1. Introduction

Studies of a bounded jet flow issuing into the space between two parallel flat plates have been done by Foss and Jones™, McCabe™, and Holeman and Foss™ so far, and the secondary flow which occurs in the bounded jet is qualitatively explained from a point of view of deformation and stretching of the vortex filament. The deformation of this vortex filament, as reported by the authors™, comes from the interaction between the boundary layer effect in the vicinity of the bounding plates and the free turbulent diffusion as a jet. In order to calculate such a flow, it is possible to use the integral method assuming a probable approximate equation of the velocity distribution, but the study of the bounded jet flow by means of such a method has not been done yet, so far as the authors are aware.

In order to carry out the approximate calculation of the bounded jet flow by the integral method, it is necessary to assume the velocity distribution function in the diffusion and the depth directions considering the flow structure. It has been reported in the case of a nozzle aspect ratio of 3™ that the velocity distribution in the diffusion direction is well expressed by that of the two-dimensional free jet™ experimentally. To obtain the velocity distribution function in the depth direction, a velocity distribution function on the center-plane which is the typical plane of the bounded jet is newly proposed.

The velocity distribution on the jet center-plane changes depending upon the initial turbulence intensity and the nozzle shape, and a peculiar distribution appears, that is, the velocities in the vicinity of the bounding flat plate are larger than those in the middle part. For the wall jet similar to such a flow, Spalding™ and Escoodier and Nicoll™ proposed a velocity distribution equation using the wake function and the shape parameter by Coles™. This wake function has little physical basis and is not satisfactory. In this report, a modified seventh power law by Latzko™ is used for the boundary layer, and the term which corresponds to the wake function and expresses the effect of the free turbulence is expressed by polynomials considering the boundary conditions. Then the velocity distribution on the jet center-plane is given by superposition of those two terms.

To determine the empirical parameter contained in the velocity distribution function, experiments were done using a nozzle having an aspect ratio of 16, and prescribing the initial turbulence intensity which affects the bounded jet flow. By using this parameter, it is attempted to explain the aspect of the flow varying with the downstream location.

2. Approximate Calculation of Boundary Layer on Flat Plate

In order to determine the velocity distribution in the depth-direction, the boundary layer thickness is to be calculated in the case of a two-dimensional jet flow along the flat plate. Schematic models of the nozzle part and the velocity distribution of the flow for the nozzle having an aspect ratio of 16 are shown in Fig.1. In this figure, the origin O is taken at the center of
the nozzle exit, the axis of in the streamwise direction at the origin 0 and the axes of and perpendicular to the axis of . Let be the distance from the side plate. The -plane is defined as the mid-plane of the bounded jet, and the -plane is the center-plane. Let be the velocity along the center line of a two-dimensional jet, the velocity on the mid-plane and the velocity on the center-plane. Denoting by , and the axis velocity components, the equation of motion and the continuity equation are given by

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_{x}}{\partial x} \\
\end{align*}
\]  

and

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0,
\]

using the conventional symbols and applying the boundary layer approximation. At the outer edge of the boundary layer, there exists a two-dimensional free jet flow. Let and be the velocity components of and directions and the shearing stress at the outer edge. Then Eq.(1) becomes

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} &= \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau_{x}}{\partial x} \\
\end{align*}
\]

After integrating Eq.(1) with respect to , the following momentum integral equation is derived from Eqs.(2) and (3):

\[
\begin{align*}
\frac{\partial}{\partial x} \int_{y} u(y) dx + \frac{\partial}{\partial y} \int_{z} u(y) dx + \frac{\partial}{\partial z} \int_{x} u(y) dz + \frac{\partial}{\partial y} \int_{z} (v - u) dz + \frac{\partial}{\partial z} \int_{x} (v - u) dz \\
= \frac{1}{\rho} \frac{\partial}{\partial x} \int_{z} (r_{x} - r_{y}) dz + \frac{1}{\rho} \frac{\partial}{\partial x} \int_{z} (r_{x} - r_{y}) dz \\
= \frac{1}{\rho} \frac{\partial}{\partial x} \int_{z} (r_{x} - r_{y}) dz + \frac{1}{\rho} \frac{\partial}{\partial x} \int_{z} (r_{x} - r_{y}) dz \\
\end{align*}
\]

The boundary layer developed on the jet center-plane is to be discussed here. Let be the velocity component of the direction and be the one at the outer edge of the boundary layer. In the jet center-plane, from the symmetry of the jet, we obtain

\[
\begin{align*}
\frac{\partial u}{\partial y} &= 0, \\
\frac{\partial v}{\partial y} &= 0, \\
\end{align*}
\]

Let be the boundary layer thickness. Then Eq.(4) becomes

\[
\begin{align*}
\frac{\partial}{\partial x} \int_{y} u(y) dx + \frac{\partial}{\partial y} \int_{z} u(y) dz + \frac{\partial}{\partial z} \int_{x} u(y) dz \\
= \frac{1}{\rho} \frac{\partial}{\partial x} \int_{z} (r_{x} - r_{y}) dz + \frac{1}{\rho} \frac{\partial}{\partial x} \int_{z} (r_{x} - r_{y}) dz \\
\end{align*}
\]

The second term of the left hand side and the first term of the right hand side of Eq.(5) become

\[
\begin{align*}
\int_{y} u(y) dy &= \int_{y} u(y) dy \\
\end{align*}
\]

and

\[
\begin{align*}
\frac{1}{\rho} \int_{z} (r_{x} - r_{y}) dz \\
= \frac{1}{\rho} \int_{z} (r_{x} - r_{y}) dz \\
\end{align*}
\]

respectively.

The term \(\frac{\partial}{\partial y} u\) in Eq.(6) is assumed as follows. It is shown by the experiment that the velocity distribution of the bounded jet in the diffusion direction is similar to that of a two-dimensional free jet at the outer edge of the boundary layer regardless of the depth . Therefore, using the velocity on the center-plane as a reference velocity, it is assumed that the relation

\[
\frac{u}{U_{1}} = \frac{\delta_{i}}{\delta_{i}}
\]

is valid between the velocity components. Supposing that there exists a two-dimensional flow with the centerline velocity of the two-dimensional free jet along the wall, the velocity distribution of the boundary layer on the jet center-plane is assumed as

\[
U_{i} = U_{0} \delta_{i}
\]

where \(\delta_{i} = \delta_{i} \). Then Eq.(8) yields

\[
\frac{\partial}{\partial y} u_{i} = \frac{\partial}{\partial y} u_{i} = \frac{U_{i}}{U_{1}}
\]

Therefore, using Eq.(9), the last expression becomes

\[
\frac{\partial}{\partial y} u_{i} = \frac{\partial}{\partial y} u_{i} = \delta_{i}
\]

Using the continuity equation at the outer edge of the boundary layer, Eq.(11) becomes
\[
\left( \frac{\partial \phi}{\partial y} \right)_{y=0} = -\frac{dU_2}{dz} \phi_0(z) \quad \cdots \cdots \cdot (12)
\]

Concerning the term \(1/\rho(\partial \tau_{\text{yy}}/\partial y)_{y=0}\) in Eq.(7) using its value \(1/\rho(\partial \tau_{\text{yy}}/\partial y)_{y=0}\) at the outer edge of the boundary layer and the velocity distribution function \(\phi_0(z)\), it is assumed as follows:

\[
\frac{1}{\rho} \left( \frac{\partial \tau_{\text{yy}}}{\partial y} \right)_{y=0} = \frac{1}{\rho} \left( \frac{\partial \tau_{\text{yy}}}{\partial y} \right)_{y=\infty} \phi_0(z) \quad \cdots \cdots \cdot (13)
\]

On the jet center-plane, Eq.(3) reduces to

\[
U_2 \frac{dU_2}{dz} = -\frac{1}{\rho} \left( \frac{\partial \tau_{\text{yy}}}{\partial y} \right)_{y=0} \quad \cdots \cdots \cdot (14)
\]

Then Eq.(13) becomes

\[
\frac{1}{\rho} \left( \frac{\partial \tau_{\text{yy}}}{\partial y} \right)_{y=0} = U_2 \frac{dU_2}{dz} \phi_0(z) \quad \cdots \cdots \cdot (15)
\]

According to the above assumptions, the momentum integral equation (9) becomes

\[
-\int d\left[ U_2 \frac{dU_2}{dz} \phi_0(z) \right] \int_{z=0}^{z} \phi_0(z) dz = 0 \quad \cdots \cdots \cdot (16)
\]

Using the modified seventh power law by Latzko\textsuperscript{a} for the velocity distribution function, we put

\[
\phi_0(z) = \zeta^m \left( \frac{8}{7} - \frac{8}{7} \zeta \right) \quad \cdots \cdots \cdot (17)
\]

It is assumed that the velocity distribution on the jet center-plane is similar at various distances from the origin. In this boundary layer, by using Eq.(17), the second term of the right hand side of Eq.(16) is given as

\[
\left( \frac{1}{\rho} \frac{\partial \tau_{\text{yy}}}{\partial y} \right)_{y=0} = aU_2 \left( \frac{U_2}{\nu} \right)^{10} \quad \cdots \cdots \cdot (18)
\]

Then the momentum integral equation (16) becomes

\[
\left( \frac{U_2}{\nu} \right)^{11} \frac{dU_2}{dz} + \int \left( \frac{8}{7} - \frac{8}{7} \zeta \right) d\zeta = aU_2 \left( \frac{U_2}{\nu} \right)^{10} \quad \cdots \cdots \cdot (19)
\]

Performing the integrations, we obtain

\[
\frac{56}{1035} U_2 \frac{dU_2}{dz} + \frac{56}{1035} \frac{dU_2}{dz} = aU_2 \left( \frac{U_2}{\nu} \right)^{10} \quad \cdots \cdots \cdot (20)
\]

which is solved as

\[
\frac{\delta_1}{\delta_0} = \left( \frac{5175}{224} \left( \frac{U_2}{U_0} \right)^{10} \left( \frac{U_0}{U_2} \right)^{10} \int \left( \frac{U_2}{U_0} \right) d\zeta \right) \quad \cdots \cdots \cdot (21)
\]

where \(C\) is an integral constant and the value of \(a\) is 0.0225. Let \(U_0\) be the velocity at the nozzle exit, \(b_0\) half the width of the nozzle and \(\nu\) the kinematic viscosity of the fluid. Then the Reynolds number is defined as

\[
R_s = \frac{2U_0 b_0}{\nu} \quad \cdots \cdots \cdot (22)
\]

Actually in the jet center-plane, there exists not only the effect of the wall turbulence but also the effect of the free turbulence as a jet, then the velocity distributions on the jet center-plane are not always similar in the flow direction as shown by Eq.(17). Therefore, the term expressing the influence of the free turbulence on the flow is taken into consideration as well as the wall turbulence. Putting \(z' = \delta_1\), the velocity distribution on the jet center-plane is expressed by

\[
\frac{U_2}{U_1} = g_1(z') + g_2(z') \quad \cdots \cdots \cdot (23)
\]

where \(\delta_1\) is the distance from the side plate inside which the interaction of the wall turbulence and the free turbulence exists. The second term of the right hand side of Eq.(23) corresponds to the wake function proposed by Spalding\textsuperscript{a} and others, is called the secondary component.

The following boundary conditions are assumed for \(g_1(z')\):

\[
\begin{align*}
\zeta_0 &= 0 ; \quad g_1(0) &= 0, \\
\zeta_1 &= 1 ; \quad g_1(1) &= 0, \\
\zeta_1 &= 1 ; \quad \frac{d\phi_1}{d\zeta} &= 0, \\
\zeta_1 &= 1 ; \quad \frac{d\phi_2}{d\zeta} &= 0.
\end{align*}
\]

The polynomial of \(\zeta\) satisfying these boundary conditions is obtained as follows:

\[
\phi_1 = k_0(1-\zeta)^{10} \quad \cdots \cdots \cdot (24)
\]

leaving one parameter unknown. In Eq.(24), \(k_0\) is the shape parameter of the velocity distribution expressing the extent of the secondary flow which differs at each downstream location and is only determined by the experiment. The schematic models of the velocity profile are shown in Fig.2 for the positive and negative values of \(k_0\).

As mentioned above, the velocity distribution function on the center-plane is expressed by

\[
\frac{U_2}{U_1} = \zeta^{10} \left( \frac{8}{7} - \frac{8}{7} \zeta \right) + k_0(1-\zeta)^{10} \quad \cdots \cdots \cdot (25)
\]

Fig.2 Models of the velocity distribution
3. Experiments

The velocity distribution function on the jet center-plane Eq. (25) contains the parameter $k$, which is an empirical one. Therefore, in order to determine its value, measurements of the velocity distribution on the jet center-plane were done. In order that the effects of the wall turbulence from only one side plate and of the free turbulence may appear, a relatively large nozzle aspect ratio of 16 is used, where the aspect ratio of the nozzle is defined by $H/b_0$, using the nozzle width $b_0$ and a distance between the side plates $H$. It was reported from the experiment using the nozzle aspect ratio of $3^{10}$ that

\[ T_i = \frac{\sqrt{u'}}{U_i} \]  

(26)

In this study, the experiments were done for $T_i = 0.008$, $0.030$ and $0.080$. The controlling method of the initial turbulence intensity is the same as shown in the previous paper [18].

4. Experimental Results and Discussions

4.1 Measurement of velocity distribution on the center-plane

The experimental results of the velocity distribution on the jet center-plane are shown in Fig. 3. This figure shows the aspect of the variation of the velocity in the range of $0 \leq z'/b_0 \leq 8$, that is, half the distance from the side plate to the mid-plane. Since the velocity does not change in the range from $z'/b_0 = 8$ to the mid-plane in spite of downstream location, the results are omitted here. Comparing with Figs. 3(a), (b) and (c), the aspect of the flow changes largely with the initial turbulence intensity $T_i$. Especially in the case of $T_i = 0.008$, the velocity excess, which means that the velocities near the side plate are larger than those in the middle part of the flow, does not appear unlike the case of the other initial turbulence intensities.

At each downstream location, the ve-
Locality on the jet center-plane approaches a constant value with an increased distance from the side plate, and becomes equal to the center axis velocity. The center axis velocity \( U_c \) is equal to the center-plane velocity of the two-dimensional free jet \( U_0 \), as mentioned later.

4.2 Jet center axis velocity

For each initial turbulence intensity, Fig. 4 shows the center axis velocity decay in the streamwise direction. In this figure, the solid lines represent the center-line velocity of the two-dimensional jet:

\[
\frac{U_0}{U_c} = (203.35 \times \frac{x}{b_0} + 1)^{-\frac{1}{8}}
\]

in which \( x \) is the length of the zone of flow establishment where the velocity at the nozzle exit \( U_n \) is kept, and \( x \) is an empirical constant contained in the eddy kinematic viscosity. Both \( x \) and \( x \) are functions of the initial turbulence intensity \( T_0 \).

The jet center axis velocity \( U_c \) is equal to the center-line velocity of the two-dimensional jet in the upstream location, but, independent of the initial turbulence intensity. Then \( U_c \) becomes smaller than \( U_0 \) in the range larger than \( x/b_0 = 35 \). This means that the influence of the side plates extends to the mid-plane.

Using the center-line velocity of the two-dimensional jet \( U_c \), the boundary layer thickness \( \delta \) is calculated from Eq. (21). According to the structure of the bounded jet, the calculation is carried out separately for the zone of flow establishment where the center-axis velocity is kept constant, for the transitional zone where the center-axis velocity decays but does not obey Eq. (27), and for the zone of established flow where the center-axis velocity varies with Eq. (27).

Since in the zone of flow establishment the velocity at the outer edge of the boundary layer is equal to the velocity at the nozzle exit \( U_n \), the boundary layer in this region is so calculated as to develop along a flat plate placed in a uniform stream. The equation (17) is used as the velocity distribution function in the boundary layer. The initial condition is \( \delta = 0 \) at \( x/b_0 = 0 \). The length of the zone of flow establishment is \( x/b_0 = 8 \) for \( T_0 = 0.008 \), \( x/b_0 = 6 \) for \( T_0 = 0.030 \), and \( x/b_0 = 2 \) for \( T_0 = 0.080 \).

In the transitional zone, the center axis velocity cannot be expressed by Eq. (27), and the empirical equations for the velocity

\[
T_0 = 0.008: \quad \frac{U_c}{U_0} = 1 - 1.25 \times 10^2 \left( \frac{x}{b_0} \right) - 3.49 \times 10^{-3} \left( \frac{x}{b_0} \right)^3
\]

\[
T_0 = 0.030: \quad \frac{U_c}{U_0} = 1 - 2.79 \times 10^{-2} \left( \frac{x}{b_0} \right) + 4.53 \times 10^{-3} \left( \frac{x}{b_0} \right)^3
\]

\[
T_0 = 0.080: \quad \frac{U_c}{U_0} = 1 - 1.03 \times 10^{-2} \left( \frac{x}{b_0} \right) - 1.67 \times 10^{-3} \left( \frac{x}{b_0} \right)^3
\]

are derived by the interpolation of the experimental results of the center axis velocity for each initial turbulence intensity. The results are substituted in Eq. (21) and the boundary layer thickness \( \delta \) is calculated. For this zone, the value of \( \delta \) at the end of the zone of flow establishment for each initial turbulence intensity is used as the initial condition. Furthermore, the transitional zone extends from \( x/b_0 = 8 \) to 18 for \( T_0 = 0.008 \), \( x/b_0 = 6 \) to 15 for \( T_0 = 0.030 \) and \( x/b_0 = 2 \) to 10 for \( T_0 = 0.080 \) respectively.

In the zone of established flow, \( \delta \) is calculated by substituting Eq. (27) into Eq. (21). The value of \( \delta \) at the end of the transitional zone for each initial turbulence intensity is used as the initial condition for this zone.

The results of calculation of \( \delta \), obtained by the method mentioned above, are shown in Fig. 5. In this figure, the result in the case of the boundary layer, for which the velocity at the outer edge is

**Fig. 4 Jet center axis velocity decays**

**Fig. 5 Calculated results of \( \delta \)**

**Fig. 6 Variation of \( \delta \)**

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kept constant is also shown for comparison. As the streamwise velocity decays in the jet, $\delta$ becomes larger than that for the uniform stream.

Now, $\delta$ is the thickness which shows the domain, where the free turbulence diffusion is affected by the side plate, i.e., the distance to a point where the velocity on the jet center-plane $U_e$ becomes equal to the center axis velocity $U_c$. Experimentally $\delta$ is determined as the distance to a point where $U_e$ is equal to $1.01U_c$ in the case of the so called velocity excess, which means $U_e$ is larger than $U_c$ somewhere, and $U_e$ is equal to $0.99U_c$ in the case of no excess, taking the experimental error into account. Moreover, at some downstream location, $\delta$ is determined by the trial and error method. Figure 6 shows the values of $\delta$ determined by the above method. In the case of $T_e=0.008$, $\delta$ does not show such a large value in the upstream location, but shows a large value in the region $x/b_o>30$ and is the same as that in the case of a larger initial turbulence intensity. Namely, a region where the two-dimensional flow is not kept is developed far from the side plate.

Using the values of $\delta$, shown in Fig. 5, $\delta$ in Fig. 6 and the velocity on the jet center-plane $U_e$ in Fig. 3, the shape parameter $k$ contained in Eq. (25) is obtained by the method of least squares. Figure 7 shows the aspect of the variation of $k$ in the streamwise direction. In the case of $T_e=0.008$, the values of $k$ are negative from the nozzle exit to $x/b_o=30$. This means that the flow pattern becomes as shown in Fig. 2(b), and that, on account of the interaction between the wall turbulence and the free turbulence, the displacement effect from the side plate is amplified. Farther downstream, the values of $k$ become positive. It is considered that the flow being directed to the side plate, which is called the effect of marching, is produced, changing to a flow pattern as shown in Fig. 2(a). The change of these flow patterns corresponds to the change of the turbulence intensity in the streamwise direction.

Namely, it is reported by Nozaki et al. that experimentally that, even if the initial turbulence intensity is small, the turbulence intensity in the jet becomes large in the downstream region far from the nozzle exit. Therefore, in this region, the effect of displacement from the side plate weakens and the flow pattern becomes as shown in Fig. 2(a). In the case of $T_e=0.008$, the values of $k$ are positive from the upstream location, and the flow pattern is as shown in Fig. 2(a). In the case of $T_e=0.030$, the flow pattern is an intermediate one between $T_e=0.008$ and $0.080$.

Using the velocity function, Eq. (25), with the values of $\delta$, $\delta$ and $k$ just obtained above, the velocity distribution on the jet center-plane for each turbulence intensity is calculated, and the results are shown in Fig. 3 with solid lines.

The velocity on the jet center-plane can be expressed by Eq. (25) in a wide range of flows, regardless of the initial turbulence intensity and the existence of a velocity excess.

In the region far from the nozzle exit, as shown in Fig. 4, the velocity on the jet center axis $U_e$ becomes smaller than the center-line velocity of the two-dimensional jet $U_c$. In order to discuss such phenomenon, using X-type hot-wire probe, the distribution of the velocity components $W$ in the $z$-direction was measured.
in the jet center-plane. The results are shown in Fig. 8. In the case of \( T_s = 0.008 \), at the locations \( x/b = 10 \) and 20, \( W \) shows a positive value, and the effect of displacement is the same as that of the boundary layer on a side plate. At \( x/b = 40 \), however, \( W \) shows a negative value over a wide range, whereas the effect of displacement appears very close to the side plate. It is considered that this means the existence of a marching effect by which the flow is directed to the side plate. As a result, the flow deflects to the side plates as a whole, and the velocity on the center axis is considered to be smaller than the center-line velocity of the two-dimensional jet. On the other hand, in the case of \( T_s = 0.080 \), the marching effect begins to appear at relatively upstream location, so that the velocity distribution on the jet center-plane shows an excess as shown in Fig. 3(c).

3. Conclusions

In order to carry out an approximate calculation by the integral method for a bounded jet which issues into the space between two parallel flat plates, a velocity distribution function on the center-plane was proposed from a point of view that the flow consists of the interaction of the wall turbulence and the free turbulence. The velocity distribution function proposed by Latsko, a modified seventh power law, which is used for the approximate calculation of the turbulent inlet length, has been adopted for the wall turbulence. To represent the interaction effect of the wall and the free turbulence, and an additional polynomial expression is superposed on the above velocity distribution function, and the resultant function is used as the velocity distribution function on the jet center-plane.

Further, in order to determine the shape parameter contained in this function, an experiment was done for a relatively large aspect ratio of 16, prescribing the initial turbulence intensity which affects the bounded jet. As a result, the proposed velocity distribution function was found to well express the bounded jet flow in a wide range from the nozzle exit, regardless of the initial turbulence intensity. Even for an aspect ratio of 16, downstream far from the nozzle exit, it was found that the aspect of the bounded jet flow varies compared with the two-dimensional one, for example, the velocity on the center axis decays somewhat rapidly. To discuss about this, the velocity distribution in the depth direction was measured by using an X-type hot-wire probe. As a result, in the downstream location where the aspect of the flow is different from that of the two-dimensional jet, it was found that there exists the effect of the side plates and the flow is directed to the side plate.

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