Natural Convective Heat Transfer on a Vertical Plate with Discontinuous Surface Temperature
(Effect of Heat Conduction in the Plate)

By Koki KISHIMARU** and Hakaru SAIITO***

Natural convective heat transfer from a vertical plate having separated heating elements has been studied by numerical computation and experiment. It has been found that convective heat transfer from the plate and the temperature distribution of unheated elements vary depending upon the ratio of thermal conductivity of convective fluid to that of the material of unheated elements, the ratio of thickness to length of unheated elements, and the length and surface temperature of heating elements. The heat transfer data are correlated by using a parameter $S = B_r R_0 / G_{1/4}$, presented in this paper through the vectorial dimensional analysis, and by an observation of convective behavior visualized by Mach-Zehnder interferometer and computational results. An empirical formula has been presented to predict heat transfer coefficients of discontinuously heated plates based on the new parameter $S$.

Key Words: Natural Convection Heat Transfer, Discontinuous Surface-Temperature Wall Temperature Distribution

1. Introduction

Heat transfer from a plate having separated heating elements has become a very important subject for study, relating with natural convective heat exchangers or heat delivering devices which are used in electronic systems[1][2]. In such a case, when a buoyant flow from a heating element passes over an unheated portion on the surface, the temperature distribution in the boundary layer is significantly affected by heat exchange between the flow and the surface. Therefore, the heat transfer behavior is different from an ordinary natural convection along a vertical plate with uniform wall temperature. The heat exchange between the buoyant flow and the unheated surface is to be governed by an energy balancing relation concerning the heat conduction in the wall and the thermal energy flowing into/out from the surface. Accordingly, the thermal properties of the material of the wall have to be taken into account as a fundamental property for describing the convective heat transfer, in addition to the ordinary physical quantities.

For this kind of problem, K-T. Yang[3], N. Hardwick[4], M. Kelleher[5], E. Sparrow[6], T. Jaluria[7], A. Zinnes[8], K. Yamasaki[9] and the authors[10] have published papers. Those works treated the heat transfer by regarding the unheated element as an adiabatic wall, and they did not discuss in detail the effect of heat conduction in an unheated wall. However, it is necessary to study in actual application the heat transfer not only for the case where the whole heat transfer system consists of one set of heat and unheated elements [2] but also for more complicated systems which consist of multi-isolated elements, heating and unheating.

This paper describes experimental and numerical studies of natural convection along a plate which consists of a set of alternating isothermal heating elements and unheated elements. The general tendency of the convective heat transfer was clarified by considering the effect of heat conduction in the unheated wall.

2. Nomenclature

- $b$: thickness of unheated wall
- $L$, $D$: lengths of heating element and unheated element
- $N$: number of set, counted heating and unheated elements as a set
- $Nu$: local Nusselt number

---

* Received 1st April, 1985.
** Associate Professor, Department of Industrial-Mechanical Engineering, Muroran Institute of Technology, 27-1, Misono-cho, Muroran 050, Japan
*** Professor, Muroran Institute of Technology
3. Analyis and Experiment

3.1 Analysis

Fig. 1 shows a model employed in this study. A natural convective heat transfer system with total length \( L_f \) is constructed of heated and unheated elements coupled. The surface temperature of each heating element is \( \theta_w \) and the lengths of heating and unheated elements are equal each other. The unheated elements of \( b \) in thickness are made of a material with thermal conductivity \( \lambda_w \) and heated by conduction from heating elements, being in thermal equilibrium with the buoyant flow flowing over the surfaces.

By taking the length of heating element as a characteristic length, the following dimensionless variables can be obtained [6][7].

- \( X \) : dimensionless length measured along the surface = \( x/L \)
- \( Y \) : dimensionless length normal to the surface = \( (y/L)G_{RL}^{1/4} \)
- \( T \) : dimensionless temperature = \( (\theta - \theta_m) / (\theta_w - \theta_m) \)
- \( G_{RL} \) : Grashof number = \( g\beta L^3(\theta_w - \theta_m)/\nu^2 \)
- \( B \) : dimensionless thickness of unheated element = \( b/L \)
- \( Y \) : dimensionless distance in unheated wall, measured normally from the surface = \( y/L \)

The heat transfer coefficient on the surface can be expressed, by using the dimensionless parameters listed above, as follows:

\[
N_u = \frac{hL}{\lambda} = -G_{RL}^{1/4} \frac{\partial T}{\partial Y} \bigg|_{Y=0} \tag{1}
\]

Since heat flux due to convection on the surface and heat conduction in the wall have to be equal at the unheated plate-fluid interface, the following relation can be given [2][7].

\[
R_t \frac{\partial T}{\partial Y} \bigg|_{Y=0} = -G_{RL}^{1/4} \frac{\partial T}{\partial Y} \bigg|_{Y=\infty} \tag{2}
\]

In the present problem, it is desirable to estimate the total heat delivery from the entire system so that the average Nusselt number for the plate is defined as follows.

\[
\overline{N_u} = \frac{1}{L_f} \int_0^{L_f} N_u \, dx = \frac{1}{L_f} \int_0^{L_f} N_u \, dX \tag{3}
\]

where \( L_f = 2 L \).

The following dimensionless expressions can be derived from the boundary layer and heat conduction equations by applying the dimensionless variables presented above.

![Physical model](image1)

![Experimental apparatus](image2)
Continuity equation:
\[ \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \]  \hspace{1cm} (4)

Momentum and energy equations:
\[ U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = T + \frac{\partial^2 U}{\partial y^2} \]  \hspace{1cm} (5)
\[ U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \frac{1}{\rho} \frac{\partial^2 T}{\partial y^2} \]  \hspace{1cm} (6)

Heat conduction equation:
\[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \]  \hspace{1cm} (7)

where the dimensionless velocities U and V in the X- and Y-directions are defined as follows [6].

\[ U = \frac{UL}{\nu} \cdot G^3L^4, \quad V = \frac{VL}{\nu} \cdot G^3L^4 \]  \hspace{1cm} (8)

In the numerical calculation, first the temperature distribution in the unheated wall was computed, and then the temperature and velocity fields in the fluid region are calculated with finite difference equations derived from the heat conduction equation and the boundary layer equations, using unequal increments both in x and y directions.

3.2 Experimental apparatus
A schematic view of the experimental apparatus is shown in Fig. 2. The apparatus basically consists of a set of isothermal heating elements (aluminum plates with 3 mm in thickness) and unheated elements (phenol resin plates of 10 mm thick) and is located in a large air-conditioned room. The heating elements are heated by hot water from an electric boiler (1) and controlled at a temperature between 35 and 75 °C (within the accuracy 0.1°C). The heat transfer surface is 400 mm in width, insulated on the backside with a foam urethane layer of 100 mm thickness, and side walls of transparent acrylate plates prevent disturbances from the outside. In order to minimize the effect of thermal radiation, a thin aluminum foil with emissivity of 0.07 is attached tightly to the surface of the plate and notched at every 4 to 5 mm to prevent heat conduction through the foil. The heating and unheated elements can be pulled out individually and the number of stages can be altered by changing the combination.

The temperature distribution in the boundary layer and the surface temperature are measured by thermocouples (6); and velocity distribution is measured by the particle trajectory method [2][10]. In addition to those experiments, the authors used a small electrically heated apparatus (Fig.3) for Mach-Zehnder interferometer.

4. Results and Discussions
A vectorial dimensional analysis is made by taking the length L and the wall thickness b, the thermal conductivity of the wall material k as physical properties in addition to those in an ordinary convection problem. When the direction of heat flow in an unheated wall is considered to be parallel to the X direction [11], the following equation can be obtained for the dimensionless temperature distribution:

\[ T_w(X) = (\theta - \theta_w)/(\theta_u - \theta_w) = f(x/L, \nu_e, \nu_e/b, \theta_u/b, \theta_w/b) \]  \hspace{1cm} (9)

From the equation above, the parameter \( \theta_u/b, \theta_w/b \) can be regarded as an important factor describing the distribution of the surface temperatures on a plate. The factor \( \nu_e/b, \theta_u/b \) is also found to be governed by the parameter above, by the same dimensional analysis for the heat transfer coefficient as in Eq.(9). There-

![Fig. 3 Structure of heating surface for electrically heated apparatus](image)

![Fig. 4 Surface temperature distribution on a plate (N = 2)](image)
Therefore, the parameter $S$ describing both the temperature distribution and the heat transfer coefficient is defined as,

$$ S = B \cdot R_s / G_{L}^{1/4} $$

(10)

The results obtained by applying the parameter $S$ to Nusselt number $Nu$ and the temperature distribution on the wall are discussed below.

Fig. 4 shows distributions of the surface temperatures obtained from numerical analysis and experiments for a plate having two sets of heating and unheated elements. The measured data are on the case of $B \cdot R_s = 0.807$ and $G_{L} = 1.85 \times 10^7$ as indicated in the figure. They are in very good agreement with the calculated values for $G_{L} = 2.07 \times 10^7$. It is also found, from a comparison of the results for $B \cdot R_s = 5.39$ and $0.807$, that the dimensionless temperature distribution changes depending more on the parameter $S$ than on the characteristic length $L$ or the heating temperature $\theta_s$. In other words, temperatures on the surface of an unheated element distribute low and do not change with the value of $S$ in the range of low $S$. To the contrary, in the case of large $S$, the temperature distribution is high and is significantly affected by the value of $S$.

According to the computation for the case of any prescribed values of $S$, the temperature distributions in the unheated wall are found to be similar for the value of $R_s > 2$ and temperature gradients in the $Y$-direction cannot be recognized. Therefore, the previous assumption about the direction of heat flow in the dimensional analysis can be justified for $R_s > 2$.

Fig. 5 shows a comparison between the measured and calculated results of the heat transfer characteristics on a plate with two sets of heating/unheated elements. In Fig. 5, one can see the tendency that the local Nusselt number $Nu$ gradually decays downstream, as in an ordinary natural convection, on the heating element of the first stage and falls steeply at the unheated element to the minimum value at the midpoint of the element. On the second stage, a similar distribution of the Nusselt numbers appears again. The local Nusselt number on the heating element increases with an increase in Grashof number $G_{L}$, while it does not significantly change on the unheated element regardless of the values of $S$. The heat transfer rate on the second heating element has the same magnitude as that on the first heating element. It may be seen that the buoyant flow from a preceding stage is cooled on an unheated wall and contributes as a forced convection to the present heating element: the edge effect. The average Nusselt number on unheated elements plays an important role in the average Nusselt number through the whole stages as the parameter $S$ increases. From the discussion above, it is seen that heat transfer from a vertical plate with uniform temperature distribution can be promoted when unheated elements are set on its surface.

From the numerical computations for $0.07 < B < 0.7$, $4 < R_s < 200$, $10^3 < G_{L} < 10^7$, it has been found that the surface temperature distribution $T_s(X)$ and the heat transfer characteristic $Nu/G_{L}^{1/4}$ do not change as long as the parameter $S$ is maintained constant, regardless of the values $B \cdot R_s$ and $G_{L}$. For instance, Fig. 6 shows the result of rearranging the data presented in Fig. 5 by using the parameter $S$. As seen in this figure, the distribution of heat transfer characteristics $Nu/G_{L}^{1/4}$ can be correlated by the parameter $S$.

Fig. 7 shows the measured data on the heat transfer characteristic $Nu/G_{L}^{1/4}$.
as a function of \( Gr_L \), for three-staged plates. Also in this figure, the analytical results for a two-staged plate are plotted for reference, together with the theoretical results[12] which would be obtained if the plate were \( L \) or \( 3L \) in length and heated uniformly. This figure indicates that the experimental results gradually decrease downstream and fall according to the theoretical prediction up to the second stage. An observation shows that natural convection began to fluctuate on the third stage. This resulted in a slight scatter in the data for \( Gr=1.47 \times 10^7 \) at around the third stage in the figure. Except for the case of \( Gr=10^6 \) in which the heat transfer promoting effect decreases slightly, the heat transfer data on the discontinuously heating plates are considerably higher than those of the continuous heating plate with \( 3L \) in length, and this tendency becomes clearer in the case of higher Grashof numbers.

Fig. 8 shows the variation of the local Nusselt number for an eight-staged system as an example of multi-stage heating. The experimental data for four different values of the parameter \( S \) are illustrated in the figure, comparing with the calculations for a corresponding ordinary natural convection along a vertical plate of length \( 8L \). Although the Grashof numbers are in smaller range than in the case of Fig. 7, the heat transfer is still promoted considerably in this case. For example, the Nusselt number at the edge of the eighth stage decreases to 50% of the value at the leading edge in the case of discontinuous heating, while the value at the same position of a continuously heated plate falls to 25% of the value of the leading edge. The interferograms are shown in Fig. 9 for \( Gr=1.4 \times 10^5 \). The stable laminar flow can be observed in the photographs over the entire length of the plate (\( L=500 \) mm).

The Nusselt numbers averaged through the stages, \( \overline{Nu} \), are related with the Grashof number \( Gr \) in Fig. 10. In the range of \( 0.7 < Fr < 2.0 \), the average Nusselt numbers \( \overline{Nu} \) can be assumed to be proportional to the \( 1/4 \)th power of the Grashof number and can be represented by parallel lines, on a double logarithmic sheet, depending on the number of stages. The empirical formulas deduced from these relations are as follows:

\[
N=3: \quad \overline{Nu} = 0.23 (Gr_c \cdot Pr)^{1/4} (10^3 < Gr < 10^6)
\]

\[
N=8: \quad \overline{Nu} = 0.19 (Gr_c \cdot Pr)^{1/4} (10^7 < Gr < 10^9)
\]

\[\text{(11)}\]
5. Conclusions

In this paper, the natural convective heat transfer from a vertical plate with discontinuous isothermal heating elements has been studied by numerical computation and experiment, considering the heat conduction in unheated walls.

It is shown that a considerable heat transfer augmentation can be expected from the edge effect at every heated element. The dimensionless parameter $S$, derived through the dimensional analysis, is found to be the most important factor describing this problem. A formula for prediction of the average Nusselt number $\bar{Nu}$ for whole system is also proposed by using this parameter $S$.

References