An Estimation of Power Contribution on Tennis Racket

by Multi-Dimensional Spectral Analysis

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This paper presents a new method to estimate the power contribution on the grip of the tennis racket, in a multiple input system where the input sources may be coherent with one another. Using multi-dimensional spectral analysis, it is found that the biggest part of the vibration source is generated by stress on the grip of tennis racket. This analysis is modeled as multiple input/single output system when the stress generating system of the tennis racket is very complicated. The power contribution analysis of the vibration sources and stress on the grip of the tennis racket is proved to be useful for this purpose. Finally, the overall levels for stress on the grip obtained by multi-dimensional spectral analysis are compared with those measured and calculated by the frequency response function approach method.

1. Introduction

Recently many analysis methods have been studied to identify the vibration sources. Among these methods, up to now, FRF (Frequency Response Function) approach method has been used under the assumption that there is no correlation between inputs. But, in practical structure it was very difficult to identify the vibration sources, because the structure itself was a very complicated system, and there were many vibration sources in the structure. To solve the problem using FRF approach method, we had to identify the vibration sources of system correctly. We applied the multi-dimensional spectral analysis to the tennis racket and classified physical meaning.

In multi-dimensional spectral analysis, we estimated the power contribution of the grip part from each vibration source on frame by using the OCF (Ordinary Coherence Function), PCF (Partial Coherence Function), MCF (Multiple Coherence Function).

2. BASIC THEORY

2.1 The coherence relation in three inputs/single output system

The general theory discussed in references (1,2,3) is applied to the special case of a system with three inputs, which may either be independent or mutually coherent, and a single output. Fig.1 (a) shows a model of inputs which is not correlated with each other. But, in vibrational system, there are many cases where inputs are correlated with each other as shown in Fig.1 (b).

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For example, if we removed the linear effect of $x_1(t)$ from $x_2(t)$ when there was a correlation between $x_1(t)$ and $x_2(t)$, we would get $x_{1.2}(t)$. When the linear effects of $x_1(t)$ and $x_2(t)$ are removed from $x_3(t)$, we get $x_{3.1.2}(t)$.

The output spectrum of the system in Fig.1 (b) can be written as follows

$$S_{yy}(f) = \sum_{i=1}^3 \sum_{j=1}^3 L_{iy}^*(f) L_{jy}(f) S_{ii}(f) + S_{nn}(f) \quad (1)$$

where the asterisk (*) denotes complex conjugate. $L_{iy}(f)$ and $L_{jy}(f)$ are transfer functions of the system in Fig.1 (b), and $S_{ii}(f)$ is the power spectrum of $x_i(t)$. $S_{nn}(f)$ is the power spectrum of independent extraneous noise, $n(t)$. In the case of Fig.1 (a) the output spectrum can be written as follow assuming a correlation between inputs

$$S_{yy}(f) = \sum_{i=1}^3 \sum_{j=1}^3 H_{iy}^*(f) H_{jy}(f) S_{ij}(f) + S_{nn}(f) \quad (2)$$

(a) in the case of incoherent input system

(b) in the case of coherent input system

Fig.1 Model of three input-single output system.

where $S_{ij}(f)$ is the cross spectrum between inputs, $H_{iy}(f)$ and $H_{jy}(f)$ are transfer functions between input and output. But, in the case of eq.(2), the output spectrum can be written as follows if there is no correlation between inputs.

$$S_{yy}(f) = \sum_{i=1}^3 |H_i(f)|^2 S_{ii}(f) + S_{nn}(f) \quad (3)$$

In this study, eq.(3) is called FRF approach.

By using multi-dimensional spectral analysis and removing the correlation element between inputs or each input and output, we find the residual spectrum. For example, the first residual spectrum can be represented by the following equations.

$$S_{ii.1}(f) = S_{ii}(f) - |L_{1i}(f)|^2 S_{11}(f) \quad (4)$$

(i=2,3,y)

$$S_{ij.1}(f) = S_{ij}(f) - L_{iy}(f) S_{1i}(f) \quad (5)$$

(i=2,3 ; i=j)

Similarly, the second residual spectrum can be represented about $x_3(t)$ as follows.

$$S_{33.1,2}(f) = S_{33.1}(f) - |L_{23}(f)|^2 S_{22.1}(f) \quad (6)$$

$$S_{yy.1,2}(f) = S_{yy.1}(f) - |L_{2y}(f)|^2 S_{22.1}(f) \quad (7)$$

$$S_{3y.1,2}(f) = S_{3y.1}(f) - L_{2y}(f) S_{32.1}(f) \quad (8)$$

Next, OCF representing the coherence between inputs or input and output can be written as follows.

$$\gamma_{ij}^2(f) = \frac{|S_{ij}(f)|^2}{S_{ii}(f)S_{jj}(f)} \quad (9)$$

$$\gamma_{iy}^2(f) = \frac{|S_{iy}(f)|^2}{S_{ii}(f)S_{yy}(f)}$$

If we apply the above relation to a multiple inputs system, PCF representing the coherence of each input source in process of the residual spectrum can be written as follows.

$$\gamma_{2y.1}^2(f) = \frac{|S_{2y.1}(f)|^2}{S_{22.1}(f)S_{yy.1}(f)} \quad (10)$$

$$\gamma_{3y.1,2}^2(f) = \frac{|S_{3y.1,2}(f)|^2}{S_{33.1,2}(f)S_{yy.1,2}(f)}$$

The MCF meaning the proportion of the transmitted power from all the of the inputs can be represented as follows.

$$\gamma_{y:x}^2(f) = 1 - \{1 - \gamma_{1y}^2(f)\} \{1 - \gamma_{2y.1}^2(f)\} \{1 - \gamma_{3y.1,2}^2(f)\} \quad (11)$$

In practice, when we search for the vibration sources, we have to measure an exact output. The output spectrum which is multiplied by the measured coherence function is called coherent output spectrum. It corresponds with the power spectrum in input/output system when S/N ratio has a high

value. Using the above relations, the overall level Φ_c of coherent output power spectrum to obtain good information at overall frequency range is defined as follows

$$\Phi_c = \int_0^f \gamma_{iy}^2(f) S_{yy}(f) df \quad (12)$$

Using the above relation, the coherent spectrum to the output of each vibrational source and the overall level as the pure coherence with the correlation elements removed from other inputs can be represented as follows

$$\begin{aligned} \Phi_1 &= \int_0^f \gamma_{1y.2,3}^2(f) S_{yy.2,3}(f) df \\ \Phi_2 &= \int_0^f \gamma_{2y.1,3}^2(f) S_{yy.1,3}(f) df \\ \Phi_3 &= \int_0^f \gamma_{3y.1,2}^2(f) S_{yy.1,2}(f) df \end{aligned} \quad (13)$$

3. EXPERIMENTAL APPARATUS AND THE SIGNAL PROCESSING TECHNIQUE

In this study, we confirmed the basic theory of the multi-dimensional spectral analysis. In this experiment, the tennis racket made of graphite was selected as a subject and the ball was dropped freely from 1.2 meter height. The vibrations caused by collision between tennis racket and ball were assumed as the inputs, and the stress of the grip was assumed as the output, and we analyzed this system modelling three inputs/single output by using multi-dimensional spectral analysis. To measure the vibration of the frame part, we attached accelerometers (RION, PV-90A) at upper(#1), middle(#2), lower part(#3) of the frame. To measure the stress of the grip part, we formed a half bridge circuit with 2 gages and the strain gages (KYOWA, KFC-5-CL-11) were attached symmetrically at upper and lower parts of the grip. As the dropping position was slightly right from the geometrical center of

Fig.2 Schematic diagram of experimental apparatus for multi-dimensional spectral analysis.

tennis racket shape, we could classify the physical meaning about the torsional vibration by multi-dimensional spectral analysis. An experimental schematic diagram for multi dimensional spectral analysis is shown in Fig.2, and the detected signals were analyzed by FFT (Fast Fourier Transform) analyzer and micro-computer.

4. EXPERIMENTAL RESULTS AND CONSIDERATIONS

4.1 The correlation characteristics of the multiple input system (4,5)

Under the assumption of multi-dimensional spectral analysis, we have to examine the correlation between inputs or input and output. Especially, OCF is an important function when we judge the non-correlation or correlation between inputs. The average OCF is represented in Table 1 at overall frequency range. Bendat and Oh reported that a correlation between inputs exists when OCF exceeds 0.5. In this study, multi-dimensional spectral analysis had to be applied because the number of OCFs was larger than 0.6.

The PCF and OCF that remove the correlation elements between inputs are shown in Fig.3. To verify the theoretical assumptions and results in applying multi-dimensional spectral analysis to the tennis racket, MCF estimated from the correlation between inputs and output must have a higher value than 0.5. If the MCF has a low value, the effect of the non-linearity will be ignored. But, in Fig.3 (d), MCF obtained in this study has a very high value at over all frequency range, and we think that the application of multi-dimensional spectral analysis to the tennis racket system is appropriate.

Table 1 Values of total mean ordinary coherence function.

$r_{12}^2(f)$	$r_{13}^2(f)$	$r_{23}^2(f)$	$r_{1y}^2(f)$	$r_{2y}^2(f)$	$r_{3y}^2(f)$
0.629	0.725	0.693	0.675	0.619	0.687

Fig.3 Partial coherence function and multiple coherence function between inputs and output.

- (a) γ_{12}^2
(b) γ_{13}^2
(c) γ_{23}^2
(d) $\gamma_{y,x}^2$

4.2 The estimation of the power contribution of the vibration sources to grip part

The characteristics of frequency response on the frame part and grip of tennis racket are shown in Fig.4. Identifying the vibration sources at the peak value of 31, 73, 152 Hz in Fig.4. (d), the coherent output power spectrum is shown in Fig.5. From the coherent output power spectrum on each vibration source, the quantitative coherent relations between inputs and output at each natural frequency are evaluated by multi-dimensional spectral analysis. For example we know that every input contributes to the output near 32 Hz. When the torsional vibration is about 73 Hz, This is not obtained at the positions of #1 and #3 in impact mechanism, and the vibration at the position of #2 contributes to the output near 73 Hz by multi-dimensional spectral analysis. Near 152 Hz, three inputs contribute to the output in similar value.

As we know the contribution of the frame vibration to the grip part, we can find out the important vibrational modes at frame part of the tennis racket.

Next, to examine the application of the multi-dimensional spectral analysis, the result compared with FRF approach is shown in Fig.6. FRF approach is an analytic method not considering the correlation between inputs, and it has a higher value than the value of the transfer function measured between input and output in Fig.6. On the other hand, the result obtained by multi-dimensional spectral analysis is slightly lower than the measured value. From above results, we know that there are many problems in applying FRF approach method in a system which has a strong correlation between inputs.

We can obtain much information about the power coherence relation and know which vibration of each input source contributes to grip part at each natural frequency. But, to obtain the overall level at overall frequency range, the integrated results of each spectrum are shown in Fig.7. The power contribution of each spectrum to grip part

Fig.4 Typical spectral densities.

- (a) acceleration #1
(b) acceleration #2
(c) acceleration #3
(d) strain of #4

Fig.5 Coherent output power spectrum.
(a) for input #1
(b) for input #2
(c) for input #3

Fig.6 Comparison between predicted spectra by the frequency response function approach and the experimental result by the coherence function approach.

Fig.7 Comparison of integrated energy contributions to the measured coherent output.

has similar value at positions #1 and #2 at overall frequency range. Especially, near 73 Hz, #2 is a more effective vibration source than #1 and #3 in the power contribution. From above results, we can think that it is necessary to attach the damping materials at position #2 or to modify the structure, when we consider the power contribution of grip part.

5. CONCLUSIONS

- (1) The values of input were calculated by OCF in this study, they were higher than 0.6. Because the application of multi-dimensional spectral analysis was able to identify the vibration sources.
- (2) We can verify that the application of multi-dimensional spectral analysis is a useful tool to estimate the power contribution to the grip part of tennis racket compared with FRF approach method when there is a strong correlation between inputs.
- (3) We could determine the natural frequency mode of system that is the important vibration sources of frame part by using multi-dimensional spectral analysis.
- (4) By estimation of the power contribution by overall level, we know that the tennis racket will be improved by reduction of the vibration at the position #2 on the frame.

References

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