A Study on the Dynamic Characteristics of Tennis Racket by Modal Analysis

By Jae Eung Oh* and Sung Ha Yum**

A study of spectral analysis as a field of applied engineering has been recently advanced for the identification of structural dynamic characteristics, and many analysis methods to detect vibration sources are under study. In this paper, an experimental modal analysis of tennis racket was carried out at the University of Han Yang in order to obtain the modal parameter.

Air type and epoxy type rackets were compared with each other to investigate the damping effect due to the existence and/or nonexistence of damping material, by examining its maximum bending moment and logarithmic decrement after impact between racket and ball. In order to clarify the dynamic behaviour of tennis racket, mode shapes corresponding to each natural frequency were obtained, and its sweet spot was determined by considering the node line at each natural frequency. The final modification was applied in a new prototype and its dynamic behaviour was checked by experimental modal analysis.

Nomenclature

M	;mass matrix
C	;damping matrix
K	;stiffness matrix
ω	;frequency (rad/s)
ω_r	;natural frequency of rth order mode (rad/s)
$\{\psi_r\}$	;normal modal matrix of rth order mode
$\{\psi_r\}^T$	;transpose matrix of $\{\psi_r\}$
$\{F\}$	;exciting force matrix
m_r	;equivalent mass of rth order mode
c_r	;equivalent damping of rth order mode
ζ_r	;damping factor of rth order mode
k_r	;equivalent stiffness of rth order mode
ψ_{ir}, ψ_{jr}	;shape coefficients of rth order complex mode at points i and j respectively
X_i	;displacement response value at point i
F_j	;the magnitude of the exciting force at point j

1. Introduction

In designing and making the sports mechanism, the strength has, up to now, been the most important problem. Thus, the main task was that the mechanism should not be destroyed in use. There were many cases that the mechanism was, because of safety, too strongly designed for the impact to the

tennis racket. Now, the dynamic characteristics, as well as static strength, must be considered to control the ball easily, to enlarge the sweet spot, and so on. To make the body condition more adequate, it is weight must be reduced. But when weight is too much reduced the "elbow" phenomenon will occur by vibrations.

Under these conditions, the methods to analyze dynamic characteristics in step of design were developed. In this study, it is the main task to develop an effective method to reduce the vibrations of the present or new tennis racket by the experimental modal analysis. To identify the dynamic characteristics, modal parameters are obtained by the transfer function that manifests the characteristics of system. To know the damping effect such as damping materials, we compared the dynamic characteristics of tennis racket whose frame is filled with air or epoxy, and decided enlarged sweet spot that has few vibrations.

2. Theoretical Basis of Modal Analysis

If the tennis racket has a damping proportional to velocity, the dynamic motion can be written in the traditional configuration on space form;

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{f(t)\} \quad (1)$$

The damping matrix C of eq.(1) is represented by linear sum of mass and stiffness matrix. If the vibration of each measuring point is represented by linear combination of the vibrational mode, eq.(1) will be transformed as follows;

$$\{X\} = \sum_{r=1}^n \frac{\{\psi_r\}^T \{F\} \{\psi_r\}}{-\omega^2 m_r + j\omega c_r + k_r}$$

* Assistant Professor, Dept. of Mechanical Engineering, Han Yang University
17 Haeng Dang-dong, Seong Dong-ku,
Seoul Korea

** Professor, Dept. of Mechanical Engineering, Han Yang University
17 Haeng Dang-dong, Seong Dong-ku,
Seoul Korea

$$= \sum_{r=1}^n \frac{\{\phi_r\}^T \{F\} \{\phi_r\}}{k_r \left\{ 1 - \left(\frac{\omega}{\omega_r} \right)^2 + j 2 \zeta_r \left(\frac{\omega}{\omega_r} \right) \right\}} \quad (2)$$

$$= \sum_{r=1}^n \frac{\{\phi_r\}^T \{F\} \{\phi_r\}}{m_r (-\omega^2 + \omega_r^2 + j 2 \zeta_r \omega \omega_r)}$$

From eq.(2), we know that the transfer function is represented by superposition of each one-degree of freedom system. In practice, it can be measured in a specific range of frequencies because of the function of analyzer. The relation between i and j components of matrix in the transfer function, that is, the relation between the force at j point and the response of i point is

$$H_{ij} = \frac{X_i}{F_j} = \sum_{r=1}^n \frac{\phi_{ir} \phi_{jr}}{-\omega^2 m_r + j \omega c_r + k_r} \quad (3)$$

$$= \sum_{r=1}^n \frac{\frac{\phi_{ir} \phi_{jr}}{m_r}}{-\omega^2 + \omega_r^2 + j 2 \zeta_r \omega \omega_r}$$

Here, we know an important thing that the denominator of eq.(3) depends only upon natural frequency and damping ratio regardless of exciting point i and measuring point j . The measured transfer functions are same wherever they are taken in the system. From eq.(3), the vibrational mode existing lower and higher than measuring frequency range have an effect on the transfer function of measuring range. In practice, if we take the measuring frequency range. 1-m, eq.(3) will be transformed as follows;

$$\frac{X_i}{F_j} = \sum_{r=1}^{l-1} \frac{\phi_{ir} \phi_{jr}}{-\omega^2 m_r} \cdot \frac{1}{\left\{ 1 - \left(\frac{\omega_r}{\omega} \right)^2 - j 2 \zeta_r \left(\frac{\omega_r}{\omega} \right) \right\}} \quad (4)$$

$$+ \sum_{r=l}^m \frac{\phi_{ir} \phi_{jr}}{-\omega^2 m_r + j \omega c_r + k_r}$$

$$+ \sum_{r=m+1}^n \frac{\phi_{ir} \phi_{jr}}{k_r} \cdot \frac{1}{\left\{ 1 - \left(\frac{\omega}{\omega_r} \right)^2 + j 2 \zeta_r \left(\frac{\omega}{\omega_r} \right) \right\}}$$

At lower frequency range than the measuring frequency range, ω_r/ω term will be zero and if vice versa ω_r/ω will be zero. And eq.(4) can be written approximately as

$$\frac{X_i}{F_j} = \sum_{r=1}^{l-1} \frac{\phi_{ir} \phi_{jr}}{-\omega^2 m_r} + \sum_{r=l}^m \frac{\phi_{ir} \phi_{jr}}{-\omega^2 m_r + j \omega c_r + k_r} \quad (5)$$

$$+ \sum_{r=m+1}^n \frac{\phi_{ir} \phi_{jr}}{k_r}$$

The first and third terms of eq.(5) are called inertia restraint and residual flexibility.

Modal parameters for the transfer function will be obtained by selecting an adequate curvefitting method depending on characteristics of transfer function.

3. Experiment

3.1 Experimental apparatus and method

Experimental apparatus to identify the dynamic characteristics of tennis racket is shown in Fig.1. Tennis racket was mounted on a frame like a racket-hand system, and it was excited by impact hammer and the vibration of tennis racket by impacting was measured by accelerometer. Tennis racket was midsize, made of graphite, its mass being 450 g and tension of gut being 30.5 kgf.

3.2 Measuring system and signal processing technique

Measuring and analyzing systems used in this study are shown in Fig.2. Each signal measured by force transducer and accelerometer was converted from analog to digital by the dynamic spectral analyzer, and transmitted to the micro-computer. Modal analysis was carried out by micro-computer using the input data. The relation between input and output is modelled as single input/single output that is assumed to be a linear system. Average number of data was 32.

4. Experimental Results and Considerations

4.1 The effect of damping materials on tennis racket

Fig.1 Impact excitation for modal analysis.

Fig.2 Schematic diagram of experimental apparatus.

Typical response characteristics of tennis racket generated by impact is shown in Fig.3. Table 1 gives comparisons of bending moment and logarithmic decrement between air and epoxy type. The logarithmic decrement between air and epoxy type. The logarithmic decrement of epoxy type racket is about 3.9% larger than that of air type racket. This phenomenon represents the effect of damping material well. It is clearly shown that the logarithmic decrement is minimum at the geometrical center of tennis racket's gut part. We think that this phenomenon is caused because the vibration is transmitted slowly to the racket's frame by the effect of damping.

The influence of the impact point on the transfer function of tennis racket system is shown in Fig.4. From Fig.4 we know that the natural frequency of epoxy type is higher than that of air type at higher frequency than the third mode. To know these results more in detail, we tabulate the modal parameters such as natural frequency and damping ratio in Table 2 and show the curve-fitting results in Fig.5.

4.2 Mode shapes by the transfer function

Mode shapes from the first mode to the sixth mode are shown in Fig.6 within 1000 Hz. The first mode is bending vibration without

node line, the second mode is torsional vibration, the third is second bending vibration with one node line, the 4th is the third bending vibration with two node lines, the 5th is combination of these vibrations, that is the first bending vibration at frame and the first membrane vibration, and the 6th is combination of these vibrations too, that is the second bending vibration and the second membrane vibration. From the 4th mode, we know that the vibration of grip part intensifies and this will give good data to modify the structure for reduction of the vibration.

4.3 Determination of sweet spot considering the mode shapes

Generally, the sweet spot is a safety impact point area formed in gut when the ball hits the racket; it is a closed space where the vibration is very small, an area where spring constant is not changed and the displacement is constant when the ball hit the racket. And we consider the node lines where the displacement becomes zero for each mode and determine the sweet spot by the second, the third and the 6th mode shapes. In Fig.7 (a),(b),(c), dashed-line represents the node line and from these lines we determine the sweet spot. A dot-line is the sweet spot of epoxy type, a

Fig.3 Typical response of tennis racket.
(a) force (b) acceleration

Table 1 Comparison of air type and epoxy type.

Impact position	Air type		Epoxy type		Ratio (Epoxy/Air)	
	Max.B.M.	Log. decrement	Max.B.M.	Log. decrement	Max.B.M.	Log. decrement
#1	209.6	0.00623	203.6	0.00635	0.969	1.019
#2	195.5	0.00582	189.6	0.00611	0.970	1.050
#3	180.2	0.00573	174.3	0.00591	0.967	1.031
#4	169.1	0.00579	166.5	0.00599	0.983	1.035
#5	152.7	0.00581	148.2	0.00617	0.971	1.062
Max.B.M.: Maximum bending moment				avg. ratio	0.972	1.039

Fig.4 Comparison of air type and epoxy type
(a) impact position #1
(b) impact position #2
(—; air type, ----; epoxy type)

Table 2 Comparison of air type and epoxy type for modal resonance.

Mode No.	Air type		Epoxy type	
	D.N.F (Hz)	D.R (%)	D.N.F (Hz)	D.R (%)
first	31.45	2.97	31.55	3.82
second	72.95	1.65	74.75	2.21
third	152.27	1.83	162.81	1.79
fourth	445.10	1.16	456.39	1.22
fifth	552.53	0.18	614.72	0.17
sixth	851.67	0.25	958.84	0.31

D.N.F : Damped natural frequency

D.R : Damping ratio

solid line is that of air type in Fig.7 (d)

We know that the sweet spot of the epoxy type tennis racket is larger than that of the air type due to the damping material used. And the sweet spot obtained in this study is located slightly lower than the geometrical center of elliptic frame.

Fig.5 Result of curvefitting.
(---;measured, —;curvefitting)

Fig.6 Mode shapes of tennis racket within the frequency range of 1000 Hz.

Fig.7 Comparison of sweet spot considering each mode.

- (a) 2nd mode (b) 3rd mode
(c) 6th mode
(--- ;node lines,
----- ;stable criterion)
(d) sweet spot
(--- ;epoxy type, — ;air type)

5. Conclusions

- (1) In case of epoxy type tennis racket, bending moment is reduced about 2.8% and logarithmic decrement is increased about 3.9% compared with air type.
- (2) By modal parameters obtained by impact method, we confirmed that the epoxy type tennis racket has the damping ratio increased at the natural frequency is shifted to higher frequency than the the third mode compared with air type.
- (3) The sweet spot of the epoxy type tennis racket is larger than the air type due to the damping material used.
- (4) The sweet spot obtained by modal analysis is located slightly lower than the geometrical center of elliptic frame.

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