A Transversely Isotropic Elastic Plate Pressed between Two Rigid Cylindrical Surfaces*

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This paper considers a fundamental contact problem of a transversely isotropic elastic plate pressed between two identical rigid cylindrical surfaces under concentrated loads. Two harmonic type stress functions are introduced under plane strain conditions and the method of Fourier transforms leads to a pair of dual integral equations which are reduced to Fredholm integral equations of the 2nd kind through the method of Abel transforms. The effects of anisotropy and plate thickness on the stresses and deformations as well as the contact pressure and the relations between load and contact area are examined.

Key Words: Contact Problem, Transversely Isotropic Elastic Plate, Plane Strain, Cylindrical Surfaces, Fredholm Integral Equation

1. Introduction

The problem of an isotropic or anisotropic elastic half plane with its free edge pressed by a rigid stamp with flat or curved base is one of the most fundamental contact problem in the two dimensional elasticity theory and a solution to this problem is shown together with its analytical method by Muskhelishvili⁽¹⁾ and Galin⁽²⁾ using a complex stress function approach. In this case, if the contact area is assumed to be sufficiently smooth, it is well known that the value of the contact pressure is independent of the elastic constant and so both results for isotropic and anisotropic half planes are the same⁽³⁾⁻⁽⁵⁾. On the contrary it is very complicated to analyze the similar problem of a strip or elastic layer with finite thickness, and so various methods of solution are investigated. For example, the numerical method based on the point-matching technique⁽⁶⁾, the asymptotic method by Wiener-Hoff technique^(7,8), the integral equation method of Fredholm type⁽⁹⁾ and the series expansion method using the orthogonal function are proposed for the contact problem between a stamp with cylindrical surface base and an elastic layer or thick plate. Most of these investigations, however, are concerned with the isotropic mediums and very few of them with the anisotropic ones⁽⁵⁾. It is now expected for the anisotropic elastic medium that the stresses and displacements as well as the contact pressure are remarkably affected by the elastic anisotropy unlike the isotropic one. Therefore this paper considers a fundamental

contact problem of a transversely isotropic elastic plate pressed between two identical rigid cylindrical surfaces, and show a solution and the effects of the anisotropy on the stresses and deformations such as the contact pressure. Two harmonic type stress functions are introduced under plane strain conditions and the method of Fourier transforms leads to a pair of dual integral equations which are reduced to Fredholm integral equations of the 2nd kind by introducing an auxiliary function and using the method of Abel transforms. Although this is a non-linear mixed boundary value problem⁽¹⁰⁾ and the contact zone varies with the magnitude of the compressive load, we assume for the sake of analytical and numerical conveniences that the contact zone is prescribed in advance and the compressive load is unknown.

2. Method of Solution

Consider a contact problem as shown in Fig.1 where two identical rigid cylinders of infinitely long in the z direction are pressed into both sides of a transversely isotropic elastic plate whose material direction coincides with x axis and plane of isotropy with yz plane. Then the elastic plate is in plane strain state and the stresses and displacements can be expressed in terms of two harmonic type stress functions as follows:

$$\left. \begin{aligned} u &= \frac{\partial \varphi_1}{\partial x} + \frac{\partial \varphi_2}{\partial x}, \quad v = k_1 \frac{\partial \varphi_1}{\partial y} + k_2 \frac{\partial \varphi_2}{\partial y} \\ -\frac{\sigma_x}{C_{66}} &= (1+k_1) \frac{\partial^2 \varphi_1}{\partial y^2} + (1+k_2) \frac{\partial^2 \varphi_2}{\partial y^2} \\ -\frac{\sigma_y}{C_{66}} &= (1+k_1) \frac{\partial^2 \varphi_1}{\partial x^2} + (1+k_2) \frac{\partial^2 \varphi_2}{\partial x^2} \\ \frac{\tau_{xy}}{C_{66}} &= (1+k_1) \frac{\partial^2 \varphi_1}{\partial x \partial y} + (1+k_2) \frac{\partial^2 \varphi_2}{\partial x \partial y} \end{aligned} \right\} \quad \dots (1)$$

where $(\partial^2/\partial x^2 + \mu_i \partial^2/\partial y^2) \varphi_i(x, y) = 0$, $k_i = (C_{11}\mu_i - C_{66})/(C_{12} + C_{66})$ and $\mu_i (i=1, 2)$ are the roots of

$$C_{11}C_{66}\mu^2 + \{C_{12}(2C_{66} + C_{12}) - C_{11}C_{22}\}\mu + C_{22}C_{66} = 0 \quad \dots (2)$$

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Fig.1 Transversely isotropic elastic plate pressed between two identical rigid cylinders

In addition, c_{mn} 's are the reduced elastic constants given by 5 independent elastic constants of a transversely isotropic elastic medium. Now press the rigid cylinders of radius R into the elastic plate by the compressive load P and then, by assuming the contact area to be sufficiently smooth and the contact length c to be sufficiently small compared to the cylinder radius R , the boundary conditions can be stated as follows:

$$(v)_{y=\pm b} = \pm(v_0 - x^2/2R), \quad |x| \leq c \quad (3.a)$$

$$(\sigma_y)_{y=\pm b} = 0, \quad |x| > c \quad (3.b)$$

$$(\tau_{xy})_{y=\pm b} = 0, \quad -\infty < x < \infty \quad (3.c)$$

where v_0 is the normal displacement at the middle of the contact area and it is an unknown constant. In order to seek for a solution satisfying the boundary condition given by Eq.(3) we take the stress functions in the following form by taking account of the symmetry of stress state and the conditions at infinity:

$$\varphi_i = \int_0^\infty f_i(\lambda) \cosh \lambda \eta / \sqrt{\mu_i} \cos \lambda \xi d\lambda \quad (i=1, 2) \quad (4)$$

where $\text{Re} \sqrt{\mu_i} > 0$, $\xi = x/b$, $\eta = y/b$ and $f_i(\lambda)$'s are unknown functions which are to be determined from the boundary conditions. By substituting Eq.(4) into the expression for τ_{xy} in Eq.(1), putting $\eta = \pm 1$ ($y = \pm b$), by equating it with the boundary conditions given by Eq.(3.c) and using the method of Fourier transforms we have

$$f_i(\lambda) = -\frac{1+k_1}{1+k_2} \sqrt{\frac{\mu_2}{\mu_1}} \frac{\sinh \lambda / \sqrt{\mu_1}}{\sinh \lambda / \sqrt{\mu_2}} f_1(\lambda) \quad (5)$$

Substitution of Eq.(4) into Eq.(1) together with Eq.(5) leads to the following expressions for the stresses and displacements:

$$\begin{aligned} u &= -\frac{1}{b} \int_0^\infty \lambda \frac{1+k_1}{\sqrt{\mu_1}} \\ &\quad \times \sinh \lambda / \sqrt{\mu_1} \left\{ \frac{\sqrt{\mu_1}}{(1+k_1)} \frac{\cosh \lambda \eta / \sqrt{\mu_1}}{\sinh \lambda / \sqrt{\mu_1}} \right. \\ &\quad \left. - \frac{\sqrt{\mu_2}}{(1+k_2)} \frac{\cosh \lambda \eta / \sqrt{\mu_2}}{\sinh \lambda / \sqrt{\mu_2}} \right\} f_1(\lambda) \sin \lambda \xi d\lambda \\ v &= \frac{1}{b} \int_0^\infty \lambda \frac{1+k_1}{\sqrt{\mu_1}} \\ &\quad \times \sinh \lambda / \sqrt{\mu_1} \left\{ \frac{k_1}{(1+k_1)} \frac{\sinh \lambda \eta / \sqrt{\mu_1}}{\sinh \lambda / \sqrt{\mu_1}} \right. \\ &\quad \left. - \frac{k_2}{(1+k_2)} \frac{\sinh \lambda \eta / \sqrt{\mu_2}}{\sinh \lambda / \sqrt{\mu_2}} \right\} f_1(\lambda) \cos \lambda \xi d\lambda \\ \sigma_x &= -\frac{1}{b^2} \int_0^\infty \lambda^2 \frac{1+k_1}{\sqrt{\mu_1}} \\ &\quad \times \sinh \lambda / \sqrt{\mu_1} \left\{ \frac{\cosh \lambda \eta / \sqrt{\mu_1}}{\sinh \lambda / \sqrt{\mu_1}} \right. \end{aligned}$$

$$\begin{aligned} &\quad \left. - \frac{\cosh \lambda \eta / \sqrt{\mu_2}}{\sinh \lambda / \sqrt{\mu_2}} \right\} f_1(\lambda) \cos \lambda \xi d\lambda \\ \frac{\sigma_y}{C_{66}} &= \frac{1}{b^2} \int_0^\infty \lambda^2 \frac{1+k_1}{\sqrt{\mu_1}} \\ &\quad \times \sinh \lambda / \sqrt{\mu_1} \left\{ \frac{\sqrt{\mu_1} \cosh \lambda \eta / \sqrt{\mu_1}}{\sinh \lambda / \sqrt{\mu_1}} \right. \\ &\quad \left. - \frac{\sqrt{\mu_2} \cosh \lambda \eta / \sqrt{\mu_2}}{\sinh \lambda / \sqrt{\mu_2}} \right\} f_1(\lambda) \cos \lambda \xi d\lambda \\ \frac{\tau_{xy}}{C_{66}} &= -\frac{1}{b^2} \int_0^\infty \lambda^2 \frac{1+k_1}{\sqrt{\mu_1}} \\ &\quad \times \sinh \lambda / \sqrt{\mu_1} \left\{ \frac{\sinh \lambda \eta / \sqrt{\mu_1}}{\sinh \lambda / \sqrt{\mu_1}} \right. \\ &\quad \left. - \frac{\sinh \lambda \eta / \sqrt{\mu_2}}{\sinh \lambda / \sqrt{\mu_2}} \right\} f_1(\lambda) \sin \lambda \xi d\lambda \end{aligned} \quad (6)$$

From Eq.(6) the normal displacement v on the plate surfaces ($\eta = \pm 1$) and the stress σ_y are obtained as follows:

$$\frac{(v)_{\eta=\pm 1}}{b} = \mp \int_0^\infty \lambda^{-1} \{ \alpha + \psi(\lambda) \} S(\lambda) \cos \lambda \xi d\lambda \quad (7)$$

$$\frac{(\sigma_y)_{\eta=\pm 1}}{C_{66}} = \int_0^\infty S(\lambda) \cos \lambda \xi d\lambda \quad (8)$$

where

$$\begin{aligned} S(\lambda) &= \frac{1+k_1}{b^2 \sqrt{\mu_1}} \lambda^2 \sinh \lambda / \sqrt{\mu_1} \{ \sqrt{\mu_1} \coth \lambda / \sqrt{\mu_1} \\ &\quad - \sqrt{\mu_2} \coth \lambda / \sqrt{\mu_2} \} f_1(\lambda) \end{aligned} \quad (9)$$

and α , $\psi(\lambda)$ are a constant and a known function, respectively, which are given

$$\begin{aligned} \alpha &= -\frac{k_1 - k_2}{(1+k_1)(1+k_2)(\sqrt{\mu_1} - \sqrt{\mu_2})} \\ \psi(\lambda) &= -\frac{k_1 - k_2}{(1+k_1)(1+k_2)(\sqrt{\mu_1} \coth \lambda / \sqrt{\mu_1} - \sqrt{\mu_2} \coth \lambda / \sqrt{\mu_2})} \\ -\alpha &= O(e^{-2\lambda \max(\sqrt{\mu_1}, \sqrt{\mu_2})}) \end{aligned} \quad (10)$$

Then by equating Eqs.(7) and (8) with the boundary conditions (3.a) and (3.b) we have

$$\begin{aligned} \int_0^\infty \lambda^{-1} \{ \alpha + \psi(\lambda) \} S(\lambda) \cos \lambda \xi d\lambda \\ = \frac{v_0}{b} - \frac{b \xi^2}{2R}, \quad |\xi| \leq \beta \end{aligned} \quad (11.a)$$

$$\int_0^\infty S(\lambda) \cos \lambda \xi d\lambda = 0, \quad |\xi| > \beta \quad (11.b)$$

in which $\beta = c/b$.

Now differentiating Eq.(11.a) with respect to ξ yields

$$\int_0^\infty \{ \alpha + \psi(\lambda) \} S(\lambda) \sin \lambda \xi d\lambda = \frac{b}{R} \xi, \quad |\xi| \leq \beta \quad (11.a)'$$

Therefore by solving a pair of dual integral equations (11.a)' and (11.b) the unknown function $S(\lambda)$ is determined, from which the unknown constant v_0 is also determined. So we express the unknown function $S(\lambda)$ with a newly introduced auxiliary function $\phi(t)$ as follows:

$$\begin{aligned} (R/b) S(\lambda) &= \lambda \int_0^\beta \phi(t) J_1(\lambda t) dt - \phi(0) \\ &= \int_0^\beta \phi'(t) J_0(\lambda t) dt - \phi(\beta) J_0(\lambda \beta) \end{aligned} \quad (12)$$

where $J_0(\cdot)$, $J_1(\cdot)$ are Bessel functions of the 1st kind and $\phi'(t)$ represents the first derivatives of $\phi(t)$ with respect to t , i.e., $d\phi(t)/dt$.

Here, by substituting Eq.(12) into Eq.(8), we have

$$\begin{aligned} \frac{(\sigma_y)_{\eta=\pm 1}}{C_{66}} &= \frac{b}{R} \left\{ \int_0^\beta \frac{\phi'(t)}{\sqrt{t^2 - \xi^2}} dt - \frac{\phi(\beta)}{\sqrt{\beta^2 - \xi^2}} \right\} \\ &\quad \begin{cases} 0 \leq \xi \leq \beta \\ 0, \xi > \beta \end{cases} \end{aligned} \quad (13)$$

and therefore we find that Eq.(11.b) is automatically satisfied. Noting that the contact pressure $(\sigma_y)_{y=0}$ is finite at the edge of the contact area ($\xi = \beta$) we have

$$\phi(\beta) = 0 \quad (14)$$

Eq.(12) and the first of Eq.(13) become as follows when taking account of Eq.(14):

$$S(\lambda) = (b/R) \int_0^\beta \phi'(t) f_0(\lambda t) dt \quad (15)$$

$$\frac{(\sigma_y)_{y=0}}{C_{66}} = (b/R) \int_0^\beta \frac{\phi'(t)}{\sqrt{t^2 - \xi^2}} dt \quad (16)$$

Then substituting Eq.(15) into the integral equation (11.a)', exchanging the order of integration⁽¹²⁾ and further using the mathematical formulae we have

$$\alpha \int_0^\beta \frac{\phi'(t)}{\sqrt{t^2 - \xi^2}} dt = F(\xi), \quad 0 \leq \xi \leq \beta \quad (17)$$

where $F(\xi)$ is given by

$$F(\xi) = - \int_0^\beta \phi'(t) \int_0^\infty \phi(\lambda) f_0(\lambda t) \sin \lambda \xi d\lambda dt + \xi \quad (18)$$

Application of Abel transforms⁽¹²⁾ to Eq.(17) yields

$$\phi'(s) = \frac{2}{\alpha \pi} \frac{d}{ds} \int_0^\beta \frac{\xi F(\xi)}{\sqrt{s^2 - \xi^2}} d\xi \quad (19)$$

and noting that

$$\left. \begin{aligned} \frac{d}{ds} \int_0^\beta \frac{\xi \sin \lambda \xi}{\sqrt{s^2 - \xi^2}} d\xi &= \frac{\pi}{2} \lambda s f_0(\lambda s) \\ \frac{d}{ds} \int_0^\beta \frac{\xi^2}{\sqrt{s^2 - \xi^2}} d\xi &= \frac{\pi}{2} s \end{aligned} \right\} \quad (20)$$

we reduce Eq.(19) to the following Fredholm integral equation of the 2nd kind:

$$\phi'(s) + \int_0^\beta K(s, t) \phi'(t) dt = \frac{s}{\alpha} \quad 0 \leq s \leq \beta \quad (21)$$

where $K(s, t)$ is an integral kernel given by

$$K(s, t) = \frac{1}{\alpha} \int_0^\infty \lambda s \phi(\lambda) f_0(\lambda s) f_0(\lambda t) d\lambda \quad (22)$$

Accordingly by using $\phi'(s)$ determined from Eq.(21), $S(\lambda)$ and $f_0(\lambda)$ are determined from Eqs.(15) and (9), respectively, and all the stresses and displacements are obtained.

On the other hand, by substituting Eq.(15) into Eq.(11.a) and putting $\xi = 0$, the normal displacement v_0 at the middle of the contact area is obtained as follows:

$$\frac{R}{b^2} v_0 = \int_0^\beta \lambda^{-1} \{ \alpha + \phi(\lambda) \} \int_0^\beta \phi'(t) f_0(\lambda t) dt d\lambda \quad (23)$$

and then the distance between the two cylinders is given by $\delta = 2v_0$.

The compressive load P is obtained as follows from its relation to the contact pressure:

$$P = - \int_{-c}^c (\sigma_y)_{y=0} dx = -2b \int_0^\beta (\sigma_y)_{y=0} d\xi \quad (24)$$

and substituting Eq.(16) into the above equation and changing the order of integration⁽¹²⁾ we have

$$P = \frac{\pi b^2 C_{66}}{R} \phi(0) = \frac{\pi C^2 C_{66}}{R \beta^2} \phi(0) \quad (25)$$

in which $\phi(0) = - \int_0^\beta \phi'(t) dt$

Furthermore the contact pressure given by Eq.(16) can be expressed as follows by taking account of Eq.(25):

$$\frac{(\sigma_y)_{y=0}}{\sigma_m} = - \frac{2\beta}{\phi(0)\pi} \int_0^\beta \frac{\phi'(t)}{\sqrt{t^2 - \xi^2}} dt \quad (26)$$

where σ_m is the average value of the contact pressure and it is given by $\sigma_m = P/2c$.

3. Numerical Calculations

For the contact problem treated in this paper the compressive load P is prescribed as usual and then the contact length β , the stresses and displacements are obtained by solving Eqs.(21) and (25) simultaneously, which is not easy to do. On the other hand, by assuming the contact length β as known it is also possible to get the corresponding compressive load P from Eq.(25) where the auxiliary function $\phi'(t)$ is obtained from Eq.(21). The latter is employed here for numerical convenience. Now it seems difficult to solve Eq.(21) analytically and so it is solved numerically by applying Simpson's integration rule to the regular part of it, when the integral kernel given by Eq.(22) is also numerically evaluated by use of Simpson's rule. For the evaluation of the contact pressure, the stresses and displacements it is convenient to use an approximate expression for the auxiliary function which is derived from the numerical values of $\phi'(t)$ thus obtained. That is, noting that the auxiliary function $\phi'(t)$ is an odd function, we assume

$$\phi'(t) = a_1 t + a_3 t^3 + \dots + a_{2N-1} t^{2N-1} \quad (27)$$

where Q_{2k-1} ($k=1, \dots, N$)'s are unknown coefficients and they are determined by using the numerical values of $\phi'(t)$ and the least square method. Then substituting Eq.(27) into Eq.(26) yields the contact pressure as follows:

$$\frac{(\sigma_y)_{y=0}}{\sigma_m} = - \frac{2\beta}{\phi(0)\pi} \sum_{k=1}^N a_{2k-1} A_{2k-1}(\xi) \quad (28)$$

where

$$A_{2k-1}(\xi) = \sum_{r=0}^{k-1} \frac{A_r C_r}{(2k-2r-1)!} (\beta^2 - \xi^2)^{k-r-1/2} \xi^{2r} \quad (29)$$

in which A_r, C_r are binomial coefficients. Similarly the normal displacement v_0 at the middle of the contact area is expressed as follows:

$$\frac{C_{66} v_0}{P} = \frac{1}{\phi(0)\pi} \sum_{k=1}^N a_{2k-1} D_{2k-1} \quad (30)$$

where

$$D_{2k-1} = \int_0^\beta \lambda^{-1} \{ \alpha + \phi(\lambda) \} H_{2k-1}(\lambda) d\lambda \quad (31)$$

$$H_{2k-1}(\lambda) = \frac{\beta^{2k}}{2} \sum_{r=1}^k (-1)^{r-1} \frac{(k-1)!}{(k-r)!} \left(\frac{2}{\lambda \beta} \right)^r J_r(\lambda \beta) \quad (32)$$

and $k! = k \cdot (k-1) \cdot (k-2) \cdot \dots \cdot 1$.

In like manner the other stresses and displacements are given in the form of the summation of the series with the unknown coefficients. Finally, in order to examine the effects of anisotropy on the stresses, displacements and so on, we express the two roots of Eq.(2) with the engineering elastic constants:

$$\frac{\mu_1}{\mu_2} = A \pm \sqrt{A^2 - B} \quad (33)$$

in which

$$\begin{aligned} A &= \left\{ -\frac{\nu'}{1-\nu'} + \frac{1}{1-\nu'} \frac{(E'/E)}{(G'/G)} \right\} / (E'/E) \\ B &= \frac{1-\nu'^2}{(1-\nu'^2)} \frac{(E'/E)}{(E'/E)} \end{aligned} \quad (34)$$

where E' , G' and ν' are Young's modulus, shear modulus and Poisson's ratio, respectively, in the material direction and also E , G , and ν are the corresponding coefficients in the plane of isotropy. It is seen from these equations that the stresses and displacements are dependent on E'/E , G'/G , ν' and ν .

4. Numerical Results

Numerical calculations are carried out for comparison with glass fiber- and graphite fiber reinforced epoxy resin composites^{(13),(14)} and an isotropic material given in Table 1. In solving Eq.(21), Simpson's integration rule is used by dividing the integral interval into 20, 20, 40, and 80 sub-domains for different values of the contact length β of 0.1, 0.5, 1 and 2, respectively. The infinite integral given by Eq.(22) is evaluated in the interval of 0 ~ 20 considering that the integrand $\psi(\lambda)$ decreases exponentially as shown in Eq.(10). The approximate expression given by Eq.(27) with $N=1 \sim 4$ suffices to get good accuracy.

Fig.2 shows the outlines of the auxiliary function $\phi'(t)$ for the contact length $\beta=2$. Fig.3 shows the contact pressure distributions where the ordinate indicates the non-dimensional quantity by $\sigma_m (= -P/2c)$ and the abscissa the distance from the middle of the contact area. The contact pressure is the highest at the middle of the contact area in all materials and it decreases monotonously as it tends to the edge of the contact area where it vanishes.

The contact pressure makes a difference in quantity at the middle of the contact area, and it is higher for E-glass / Epoxy and lower for Graphite / Epoxy than for the isotropic material. However, as β is decreased, i.e., the plate thickness becomes larger compared to the contact area, the difference diminishes and for sufficiently large plate-thickness we find that the value of the contact pressure is the same for these three materials.

Fig.4 shows the distributions of stresses $(\sigma_x)_{y=\pm 1}$ in the material direction. Although σ_x yields the maximum compressive stress at the middle of the contact area ($\xi=0$) it decreases gradually toward the edge of the contact area and turns out into a tensile stress near the edge ($\xi=2$) where it becomes the largest. Furthermore, away from the edge, the stress decreases again to vanish. The maximum values of the compressive and the tensile stresses are

Fig.2 Outlines of auxiliary function

Fig.3 Contact pressure distributions

Table 1 Elastic constants of materials used in calculations

	E'/E	G'/G	ν	ν'
E-glass/Epoxy ($V_f=63\%$)	3.44	1.014	0.37	0.24
Graphite/Epoxy ($V_f=50\%$)	11.5	1.456	0.32	0.30
Isotropy	1.0	1.0	0.30	0.30

Fig. 4 Distributions of surface stresses , $(\sigma_x)_{y=\pm 1}$

Fig. 5 Ratio of maximum to average contact pressure as a function of contact length

Fig. 6 Compressive load as a function of contact length

the largest for Graphite/Epoxy followed by E-glass/Epoxy being several times as large as that of the isotropic material.

Fig. 5 shows the ratio of the maximum to the average contact pressure as a function of the contact length β where it in-

Fig. 7 Distance between the two cylinders as a function of contact length

Fig. 8 Effect of elastic constants on maximum contact pressure

creases monotonously with an increased value of β and the quantitative differences between these materials are distinct.

Fig. 6 shows the compressive load P as a function of the contact length β . When the compressive load is small the contact length increases rapidly with an increased value of P , whereas the increasing rate reduces for the larger compressive load. From this figure we can evaluate the contact length for a given compressive load.

Fig. 7 shows the distance between the two cylinders, δ , as a function of the contact length β , from which together with Fig. 6 we can estimate the distance for a given compressive load.

Fig.8 shows the effect of the elastic constants on the maximum contact pressure for the shear modulus ratios $G'/G=1$ and 2 with a fixed value of Poisson's ratio $\nu = \nu' = 0.3$. Here the ordinate represents the ratio of the maximum to the average contact pressure and the abscissa Young's modulus ratios E'/E of 1~50. It is seen from this figure that the ratio of the maximum to the average contact pressure is almost independent of E'/E and G'/G and so it is constant.

5. Conclusions

In order to show a solution to the two dimensional anisotropic elastic contact problem and examine the effects of anisotropy on the contact pressure etc., a fundamental problem of a transversely isotropic elastic plate with its both surfaces pressed between two identical rigid cylinders is considered by using two harmonic type stress functions and reducing it to a Fredholm integral equation of the 2nd kind. Numerical calculations are carried out for comparison with E-glass/Epoxy and Graphite/Epoxy composites and further an isotropic material to show the contact pressure and the surface stress distribution.

The relations of the compressive load and the contact area, and the distance between the two cylinders are graphically

illustrated, from which the contact area and the distance between the two cylinders are found to be estimated for a given compressive load.

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