Dynamic Behavior of a Liquid in a Cylindrical Container

Subject to Horizontal Vibration*

(Liquid Surface Response in an Elastic Container

Subject to Vibration with Higher Frequencies)

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An experimental study is presented for the dynamic behavior of a liquid in an elastic container subject to horizontal oscillation with high frequencies. The harmonic wave motion, the formation of azimuthal waves, flow pattern on the free surface, frequency characteristics of pressure and the surface disintegration are investigated. The effect of the natural frequency of a liquid-shell system for the formation of azimuthal waves and the surface disintegration worked to decrease the threshold excitation acceleration for formation of the wave and the disintegration. Several circulation flows were formed on the free surface by the lateral vibration. It was observed that the pressure responses had large values in the vicinity of the natural frequency of the system. Many floating liquid drops on the turbulent free surface were observed after the surface disintegration. It was also found that the response of liquid free surface depended on certain combination of excitation frequency and acceleration.

Key Words : Fluid Vibration, Lateral Sloshing, Surface Wave, Resonant Frequency, Surface Disintegration, Drop, Pressure Response

1. Introduction

When a container partially full of a liquid is vibrated, the liquid presents the dynamic behavior responding to the vibrating external force. It is called "liquid sloshing phenomenon". Such phenomenon comes up as an important problem in various fields of technology, and many extensive investigations have been conducted.⁽¹⁾ For example, a severe vibration with higher acceleration occurs during the blast-off and cut-off of the liquid fuel rocket, then the dynamic stability and the control of the liquid fuel surface assume the greatest importance. The frequency range of such a serious problem in space technology extends usually from several Hertz to hundreds Hertz, and acceleration is usually high.⁽²⁾ In seismology, the liquid free surface is resonant to the superior component in the range of displacement waves of earthquake. In this case,

the low frequency range, 0.1 ~ 10 Hz, is of importance.⁽³⁾ Applying the liquid sloshing phenomenon in the low gravity, it is possible to produce a new combined material by mixing gas, liquid and solid particles.⁽⁴⁾ In this case, a relatively higher frequency range from tens to hundreds Hertz with high acceleration is needed. From the circumstance described above, the hydrodynamical clarification of the basic flow mechanism of the liquid sloshing over a wide range of frequencies and accelerations is required.

In general, the liquid with the free surface presents a complicated flow mechanism as the acceleration of an oscillating container is large. For example, the dynamic behavior of the liquid free surface⁽⁵⁾ and the bubble entrainment into the liquid⁽⁶⁾ in a container subject to vertical vibration were reported. In a cylindrical container subject to lateral vibration, the surface response (chiefly with regard to the highest free surface wave with mode 1) to the low frequency excitation was also investigated by many workers.⁽⁷⁾ With regard to the high frequency excitation, the experimental studies on the dynamic stability of an elastic shell-liquid system⁽⁸⁾ and the comparison of the experimental results based on a model with the calculated results⁽⁹⁾ were reported. According to these results, the liquid response in the vicinity of the parametric resonance of the shell-liquid system depends mainly on the vibration of the elastic shell. In

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these investigations, it is not the main purpose to make clear the dynamic behavior of a liquid. So, the essential liquid motion in the frequency range apart from the parametric resonance of a shell and the complicated liquid flow mechanism at the parametric resonance due to the interference between the liquid and the elastic shell motion have been unsolved until now.

We analyzed previously the dynamic stability of a cylindrical elastic container filled with water experimentally.⁽⁸⁾ Based on this result, we have conducted experimental research in order to clarify unresolved problems regarding the liquid vibration from a fluid dynamical point of view. The purpose of the present paper is to make clear the characteristics of the surface wave and the complicated liquid flow in a container subject to horizontal oscillation with high frequencies and high accelerations.

2. Experimental Apparatus and Procedures

A block diagram of a vibration exciter and measuring devices is shown in Fig.1(a), and a coordinate system to a container is shown in Fig.1(b). The electrodynamic shaker was operated at a given frequency, displacement and acceleration. The cylindrical container, of which the natural frequencies were already known in our previous study,⁽⁸⁾ was used here. The container was made of a transparent polyester thin plate, thickness $b = 0.247$ mm, Young's modulus $E =$

5.56 GPa, Poisson's ratio $\nu = 0.3$, density $\rho_c = 1.405 \times 10^3 \text{ kg/m}^3$. The wall plate was bonded to an 18 mm thick duralumin circular disk at the bottom. The cylindrical container, height $L = 160.7$ mm, radius $R = 100$ mm, has a flat bottom and an open top (the coefficient of shape is $L^2\sqrt{1-\nu^2}/(Rb) = 1000$). Water in the temperature range $13^\circ\text{C} \sim 18^\circ\text{C}$ was used as the test liquid.

Experiments were conducted under the horizontal excitation of the container holding water on the table of the electrodynamic shaker. The behavior of liquid free surface in the vibrated container was observed with a 16 mm high speed camera, and the exposed films were analyzed with a film motion analyzer. The displacement of the liquid free surface was also measured with an optical cathetometer and sometimes with an automatic wave height detector. The signals from the wave height detector were analyzed and simultaneously displayed on a FFT analyzer. Further, the liquid flow on the free surface was visualized by a lot of foaming styrol beads adjusting suitably the specific gravity a little smaller than that of water, that is, the motion of the beads on the free surface was photographed with a movie camera. The pressure signals from a pressure transducer probe set at a given position were recorded on the X-Y recorder for a given excitation frequency.

3. Experimental Results and Discussion

3.1 Harmonic standing wave

When a cylindrical container holding a liquid is vibrated laterally, the liquid free surface presents a wave corresponding to the vibration of the container. When the input acceleration increases slowly from zero at a given frequency f_0 , a stable concentric standing wave with small amplitude can be observed initially with the naked eyes. These waves show harmonic response with the same frequency as the input frequency. The shape of the wave is concentric, but their phases differ by π at two symmetric places with respect to

Fig.1 (a) Block diagram of apparatus

Fig.1 (b) Coordinate of cylindrical container

Fig.2 Photographs of harmonic standing waves

the line normal to the excitation direction and through the center. Therefore, the wave amplitudes near the normal line are very small. These waves are generated within a certain range of the excitation amplitudes X_0 . The wave length λ_r does not change with X_0 but changes with f_0 . Fig.2 shows the patterns of these waves for the liquid height ratio (the ratio of liquid height H to the container length L) $H/L = 1.0$. Fig.3 shows the relation between λ_r and f_0 . It is seen from Fig.3 that λ_r decreases as f_0 increases (λ_r does not vary with the excitation acceleration if f_0 is constant). The total amplitude of wave motion, X_L , varies with the excitation acceleration as shown in Fig.4 (the axis of abscissa shows dimensionless acceleration $G_0 = (2\pi f_0)^2 X_0/g$, g is gravitational acceleration). The wave height was measured at a point of 10 mm distance from the wall at $\theta = 0$ with the automatic wave height detector. It is seen from Fig.4 that the wave amplitude increases with the excitation acceleration. This tendency was evident at other frequencies as well, but the rate of increase in the wave amplitude due to the excitation acceleration decreased with an increase in f_0 .

3.2 Azimuthal wave

Further increase in the excitation acceleration from the state of the above harmonic standing wave for a given frequency causes the breakdown of the concentric standing wave in the vicinity of the container wall, and then the azimuthal

Fig.3 Relation of excitation frequency to wave length of harmonic standing wave

Fig.4 Relation of excitation acceleration to amplitude of harmonic standing wave

waves are formed. Fig.5 shows the relation between the threshold excitation acceleration for the azimuthal wave formation, $G_a = (2\pi f_0)^2 X_a/g$, and the excitation frequency f_0 . It is seen from Fig.5 that G_a increases firstly with f_0 , but decreases in excess of a specified value of f_0 . The minimum point of G_a corresponds to the natural frequency of the liquid-shell system ($f_0 = 120\text{Hz}$ for $H/L = 1.0$), and the container wall responds as large-amplitude motions in the vicinity of this natural frequency. It seems that the occurrence of azimuthal waves is promoted by the liquid turbulence due to a large-amplitude wall vibration. It is also seen that the threshold value G_a decreases with an increase in H/L . The occurrence of azimuthal waves is easier as the liquid depth becomes deep.

Fig.6 shows the relation between the wave length and the excitation frequency. It is seen from Fig.6 that the wave length λ_θ (angle in θ direction) decreases with an increase in f_0 . It was also observed that λ_θ did not vary if f_0 was constant. This tendency is evident at other liquid heights as well. The wave amplitude

Fig.5 Relation of excitation acceleration to frequency for threshold of azimuthal wave

Fig.6 Relation of excitation frequency to wave length of azimuthal wave

varied with the excitation acceleration. Fig.7 shows the relation between f_0 and the maximum total amplitude X_L which was measured at the wall of $\theta = 0$. In general, the amplitude of the surface wave increases as the excitation acceleration increases if f_0 is constant, and the rate of increase decreases with an increase in f_0 . The amplitude of the wave also varied along the container wall. This is shown in Fig.8. It seems that the amplitude of the wave varies with the translated liquid velocity induced by the horizontal oscillation of the container. Therefore, the variation of the amplitude depends on $\cos \theta$ as shown in Fig.8.

Fig.9 shows some spectra of wave frequencies analyzed with FFT analyzer. As shown in Fig.9, the response frequency of the wave motion on the free surface varies with f_0 . When the excitation acceleration is lower than G_a [Fig.9(a)], the above harmonic standing wave occupies all the free surface, and when the acceleration exceeds G_a , an other wave with one-half frequency of f_0 emerges [Fig.9(b),(c)]. The latter is an azimuthal wave, and the area occupied by this 1/2-subharmonic waves increases

gradually with f_0 . The photographs in Fig.10 show these two waves (left is the harmonic standing wave and right is the azimuthal wave).

3.3 Liquid flow in container

Liquid flow on the free surface was visualized by a lot of foaming styrol beads to investigate the surface flow when the liquid surface responds as a harmonic standing wave. Regular streak lines appeared clearly on the liquid free surface. Fig.11 shows the results analyzed with the film motion analyzer. The ordinate shows the liquid surface velocity in the direction of the container radius, V , which was measured at the location of $\theta = 0$ and $r = R/2$, and the positive value indicates the velocity toward the wall from the center. For $f_0 = 20$ Hz in Fig.11, it is seen that the velocity toward the center of the container increases with the excitation acceleration. Once the velocity reaches a certain value, it decreases suddenly, and at last the flow reverses. The maximum velocity usually decreases with an increase in f_0 , and at the frequency $f_0 > 30$ Hz the velocity is too small to be

Fig.7 Relation of excitation frequency to amplitude of azimuthal wave

Fig.8 Examples of wave displacement along container wall

Fig.9 Frequency spectrum of wave motions

Fig.10 Photographs of free surface

Fig.11 Relation of excitation acceleration to flow velocity

measured. It can be considered that such a flow (large circulating flow) directed to the container axis is induced by the horizontal oscillation of the container wall. And the flow is reversed with occurrence of an azimuthal wave. So it seems that the reversion of flow is induced as a result of the original flow being disturbed by a small circulating flow⁽¹⁰⁾ generated along the node of the azimuthal wave. Fig.12 shows an overview photograph of the large circulating flow on the free surface. On the other hand, the foaming styrol beads, of which the specific gravity was adjusted suitably, were spread on the bottom of the container, and the movements of these beads were checked during the excitation (the particle pattern induced by the flow on the bottom is shown in Fig.13). The styrol beads gathered around two symmetrical sides ($\theta = 0, \pi$, that is, in the direction of the container excitation) of the container wall at almost all frequencies except in the vicinity of the natural frequency for a lower mode (circumferential mode $N = 1$, vertical mode $m = 1$) in the liquid-shell system. It is confirmed that the two flows toward the opposite corners in the direction of the excitation occur on the container bottom. It was also observed that the velocity of this flow was very small and increased as the excitation frequency was lowered with a constant acceleration, or as the excitation acceleration was raised for a constant frequency. The beads showed similar pattern on the bottom even when the azimuthal wave appearing on the liquid

Fig.12 Flow pattern on free surface

Fig.13 Particle pattern at container bottom

surface had a small wave amplitude. On the other hand, with wall oscillation of the container, the beads showed a more complicated pattern in the vicinity of the natural frequency for the liquid-shell system as shown in Fig.13.

3.4 Pressure response of liquid

The pressure response of the liquid was measured by the probe which was bonded to the pressure transducer at the end of the copper tube. The detection surface of the pressure transducer was kept parallel to the stationary liquid surface during the measurement, and was moved with the container. Fig.14 shows some results of the frequency response of the pressure (the axial location of measurement is $Z/H = 0.88$). The sweep velocity of the frequency was 0.4 Hz/s. In Fig.14, the abscissa shows the excitation frequency f_0 and the ordinate shows the root mean square of pressure variation. It is seen from Fig.14 that a large amplitude response of pressure oscillation occurs when f_0 approaches the natural frequency of the liquid-shell system. The variation of the excitation acceleration and the pressure response at an arbitrary location in a liquid in the vicinity of these frequencies were investigated. Fig.15 shows the radial variation of pressure response at $Z/H = 0.88$. It is seen from Fig.15 that the pressure amplitude increases steeply with the radial distance

Fig.14 Frequency response of pressure

Fig.15 Pressure variation in r-direction

toward the wall from the center of the container. This fact means that the pressure waves induced by the oscillation of the container wall propagate in the liquid with damping from the wall toward the center. Fig.16 shows the pressure response in Z-direction at $r/2R = 0.47$. It is seen from Fig.16 that the pressure amplitude increases with the distance toward the bottom of the container from the free surface. Fig.17 shows the relation between the pressure response and the excitation acceleration for various values of H/L . It is seen from Fig.17 that the pressure amplitude increases with G_0 . It seems to be caused by a large amplitude response of the container wall due to an increase in G_0 . As described above, the liquid pressure amplitudes at the relatively high frequencies depend heavily on the oscillation of the container wall. This means that the dynamic characteristics of the elastic container have important effects upon the pressure

Fig.16 Pressure variation in z-direction

Fig.17 Relation of pressure variation to excitation acceleration G_0

oscillation.

3.5 Surface disintegration

When the container holding water is vibrated above a specific acceleration, the free surface is disintegrated and spray particles are ejected from the surface. Fig.18 shows the relation between f_0 and the threshold acceleration G_d for the surface disintegration. It is seen from Fig.18 that the smaller the dimensionless height H/L is, the larger G_d is in the range lower than $f_0 = 140 \text{ Hz}$. In Fig.18, there is a frequency region where G_d decreases with an increase in f_0 . This corresponds to the natural frequency (mark $\uparrow$ in Fig.18) of the liquid-shell system, that is, the free surface is disintegrated due to a large response of the system even when the excitation acceleration is relatively small. By examining in detail the frequency corresponding to the region of decreasing G_d , it can be comprehended that the decreases in G_d correspond to the natural circumferential modes⁽⁸⁾ of the container wall. Therefore, it is estimated that G_d in the case of a strong stiff container could become very large with f_0 , because the natural frequencies in this case are very high. Conversely it is estimated that G_d of a soft elastic container varies vigorously with an increase in f_0 as shown in Fig.18.

Most of the liquid particles on the free surface were generated in the vicinity of the container wall. This can be due to the fact that the oscillation of the elastic container wall causes surface disintegration in the neighborhood of the wall. Fig.19 shows plots of time versus the number of generated liquid drops which were measured by the high speed movie camera and the film motion analyzer. The ordinate in Fig.19 is the number of liquid drops which are ejected over the container. The number of liquid drops fluctuates with time, and the mean period of the fluctuation is about 0.29 s. Since this period agrees with the period of the swell of the wall vibration, it is inferred that the period has close connec-

Fig.18 Relation of threshold acceleration for surface disintegration to frequency

tion with the swell. The number of liquid drops increased with G_0 . The liquid drops floated on the free surface after the fall and moved toward the center. The flow mechanism on the free surface was complicated; large drops were formed by the collision and coalescence of small drops, and the coalescent waves appeared on the free surface. The photograph in Fig.20 shows a representative pattern of these drops. Such floating phenomenon of liquid drops on the free surface was observed also in the longitudinal liquid sloshing and the basic behavior of these drops was investigated previously by the authors.⁽¹¹⁾

4. Conclusions

A container partially full of liquid was vibrated sinusoidally in the horizontal direction over a wide range of excitation frequencies and accelerations and the characteristics of the liquid flow were investigated. The results obtained are summarized as follows:

(1) The wave length of the concentric standing wave which appears in the range of lower accelerations ($G_0 = 0.01 \sim 0.20$) decreases with an increase in the excitation frequency, and the amplitude of the wave increases with the excitation acceleration if the frequency is kept constant.

(2) Azimuthal waves which respond with the $1/2$ subharmonic of the excitation frequency are generated when the excitation acceleration exceeds the limiting value, that is, the threshold acceleration G_a for the azimuthal wave. The wave length decreases with an increase in the excitation frequency, and the amplitude of

the wave increases with an increase in the excitation acceleration when the frequency is kept constant.

(3) When the free surface presents the harmonic standing waves, the surface liquid at the location $\theta = 0$ and $\theta = \pi$ flows toward the center from the wall, and the surface liquid at $\theta = \pi/2$ and $\theta = 3\pi/2$ flows toward the walls from the center. That is, they form circulation flows, and these flows are turned round by the occurrence of an azimuthal wave. Furthermore, there are two main deflecting flows in the neighborhood of the container bottom, and then the whole liquid flow in the container is complicated.

(4) The liquid pressure amplitude becomes very large in the neighborhood of the parametric resonance of the liquid-shell system, and the pressure amplitude increases with an increase in the excitation acceleration. The pressure amplitude increases with the radial distance toward the wall from the center, and increases with the axial distance toward the bottom from the free surface.

(5) The dimensionless acceleration G_d which is the threshold to the surface disintegration increases with an increase in the excitation acceleration, but decreases in the neighborhood of the parametric resonance of the liquid-shell system. The smaller the liquid depth is, the larger becomes G_d .

(6) A lot of floating drops are formed on the free surface when the free surface disintegrates at the higher acceleration ($G_0 \geq 2$) in the neighborhood of the parametric resonance of the liquid-shell system. These drops move toward the center, coalesce with each other or with the liquid under the free surface, and promote the complicated dynamic phenomena of the gas-liquid interface.

Fig.19 Relation of number of drops to time

Fig.20 Photograph of free surface with liquid drops

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