Characteristics of a Slipper Bearing for Swash Plate Type
Axial Piston Pumps and Motors*

(3rd Report, Design Method for a Slipper with a Minimum
Power Loss in Fluid Lubrication)

By Naohiro IBOSHI**

A slipper bearing is one of the machine parts of which careful considerations should be given to realize a good performance, reliability, miniaturization and so on from a standpoint of designing swash plate type axial piston pumps and motors. Optimum slipper sizes, which give a minimum power loss in fluid lubrication, are theoretically analyzed in order to provide a useful guide to the slipper design. First of all, its sizes to give the minimum power loss are obtained on the assumption of uniform clearance. Secondly, considering all the oil-film parameters, a theoretical analysis of its motion is made to examine its lubrication condition. From the relation between a minimum oil film thickness and slipper sizes, it can be said that the operation of the bearing having a minimum oil film thickness larger than the limit determined by the magnitude of surface roughness can be guaranteed in fluid lubrication. As a result, a method for designing a slipper meeting these optimum conditions is shown.

Key words : Fluid Power Systems, Piston Pumps, Slipper Bearing, Theoretical Analysis, Minimum Power Loss, Slipper Design

1. Introduction

A design for a slipper bearing is one of the principal parts of designing swash plate type axial piston pumps and motors. It is very important for providing a useful guide to the slipper design from a standpoint of improving the performance, increasing the reliability and promoting the miniaturization.

A minimum oil film thickness should be larger than surface roughness in order to realize the bearing operation in fluid lubrication. When its power loss due to the viscous drag and the flow rate is investigated, the result of the analysis on the assumption of uniform clearance⁽¹⁾ has not been always guaranteed to be in fluid lubrication, because the minimum oil film thickness is expressed in terms of mean clearance and slipper tilting angle. For that reason, not only the power loss but also the lubrication condition must be investigated to obtain the optimum slipper sizes, which give the minimum power loss in fluid lubrication.

In previous reports^{(2),(3)}, theoretical and experimental analyses on the characteristics of the slipper bearing for rotating a swash plate were made. The availability of theoretical analysis was ascertained by comparing the results of

the theoretical and experimental analyses.

In this report, first of all, a theoretical analysis on the characteristics of the slipper bearing for rotating a cylinder block is made, considering all the oil-film parameters, that is, mean clearance, slipper tilting angle and maximum tilting angle azimuth. The points of analytical difference between two types are the boundary conditions such as its velocity, the magnitude of centrifugal force at high rotational speed, the existence of the frictional moment on spherical bearing due to the revolution of piston. It has been also confirmed that the result of the theoretical analysis can qualitatively agree with that of the experimental one in ref. (4). Secondly, a theoretical expression of its power loss is derived, and the slipper size to minimize the power loss is theoretically analyzed. By means of using the relation between the minimum oil film thickness and the slipper sizes, the operation of the slipper bearing having a minimum oil film thickness larger than the limit determined by the magnitude of surface roughness can be guaranteed in fluid lubrication. And, a slipper design method is shown as follows. The slipper size to give the minimum power loss is obtained on the assumption of uniform clearance so that the calculating time can be shortened. Then, the motion of the slipper is analyzed, considering all the oil-film parameters, and the relation between the minimum oil film thickness and the size is obtained. From the result, the lubrication condition is examined as well as the above procedure.

* Received 24th July, 1984.

** Assistant, National Defense Academy,
1-10-20 Hashirimizu, Yokosuka,
239 Japan.

Finally, this result is investigated and compared with that considering all the oil-film parameters as a function of time.

Nomenclature

- $a = D_1/D_2$
 $A(\omega_0 t), B(\omega_0 t), C(\omega_0 t)$: Velocity coefficients
 D_1 : Recess pressure
 D_2 : Slipper diameter ($= 2 R_2$)
 D_p : Piston diameter
 f_p : Frictional coefficient between piston and cylinder bore
 f_s : Frictional coefficient between slipper cup and ball
 F_1, F_2 : Lateral forces on piston
 h_0 : Mean clearance
 $h_{\min}$: Minimum oil film thickness
 l_0 : Length of restrictor
 l_s : Slipper height
 L : Power loss
 m : Mass
 $P_0 = P_{\text{out}} - P_{\text{sin}}$ (in case of pump operation)
 P_{sin} : Suction pressure
 P_{out} : Delivery pressure
 r_0 : Radius of restrictor
 R : Piston pitch circle radius
 R_b : Ball radius
 α : Slipper tilting angle
 $\beta = D_2/D_p$
 δ_0 : Swash plate tilting angle
 ω_0 : Angular velocity of cylinder block or swash plate

Nondimensional parameters

- $\bar{A}(\omega_0 t) = A(\omega_0 t)/V$
 $\bar{B}(\omega_0 t) = B(\omega_0 t)/V$
 $\bar{C}(\omega_0 t) = C(\omega_0 t)R_2/V$
 $\bar{F} = F H^2/6\mu R_2^3 V$
 $\bar{h} = h/H$
 $\bar{m} = m H^3 V/6\mu R_2^5$
 $\bar{p} = p H^2/6\mu R_2 V$
 $\bar{Q} = Q/\pi H R_2 V$
 $\bar{T} = T H^2/6\mu R_2^4 V$
 $\bar{\alpha} = \alpha R_2/H$
 $\tau = \Omega t$
 $\bar{\omega} = \omega R_2/V$

2. The Forces on a Slipper

2.1 The fluid forces on a slipper

The stationary and moving co-ordinate systems, as in the previous report (2), are defined as shown in Fig. 1. Let us consider a slipper in the following condition for the moving co-ordinate system o-xyz. The rotational angle $\omega_0 t = 0$ is determined as the bottom dead center. In the time t when the cylinder block rotates by $\omega_0 t$, h_0 is the mean clearance, α is a tilting angle made by the rotational axis of the slipper to the z axis and ψ is the maximum tilting angle azimuth. The slipper has a recess and a restrictor for the compensation of pressure.

For convenience, the o-xyz co-ordinate system is represented as cylindrical co-ordinates. The oil film thickness $\bar{h}$ at an arbitrary point $C(\bar{r}, \theta, \bar{z})$ on the slipper bearing surface is given by

$$\bar{h} = \bar{h}_0 + \bar{r}\bar{\alpha} \cos(\psi - \theta) \dots\dots\dots (1)$$

The boundary conditions of the velocity for the motion of the slipper are as follows;

$$\left. \begin{aligned}
 &\text{At } \bar{z} = 0 \text{ (on swash plate surface)} \\
 &\quad \bar{u} = 0, \bar{v} = 0, \bar{w} = 0, \\
 &\text{at } \bar{z} = \bar{h} \text{ (on slipper sliding surface)} \\
 &\quad \bar{u} = \bar{A}(\omega_0 t) \sin\theta + \bar{B}(\omega_0 t) \cos\theta \\
 &\quad \bar{v} = -\bar{B}(\omega_0 t) \sin\theta + \bar{A}(\omega_0 t) \cos\theta \\
 &\quad \quad + \bar{C}(\omega_0 t)\bar{r} + \bar{\omega}_s \bar{r} \\
 &\quad \bar{w} = (H/R_2) \{ \bar{\omega}_s (\partial \bar{h} / \partial \theta) \\
 &\quad \quad + (\partial \bar{h} / \partial \tau) (R_2 \Omega / V) \}
 \end{aligned} \right\} \dots\dots (2)$$

where

$$\bar{A}(\omega_0 t) = \bar{R}\bar{\omega}_0 \left(\frac{\bar{X}_0 \bar{X}_{01}}{\bar{R}_0^2 \cos^2 \delta_0} - \cos \omega_0 t \right)$$

$$\bar{B}(\omega_0 t) = \frac{\bar{R} \bar{\omega}_0}{\cos \delta_0} \left(1 - \frac{\bar{R}}{\bar{R}_0^2} \bar{X}_{01} \right) \sin \omega_0 t$$

$$\bar{C}(\omega_0 t) = \frac{\bar{R} \bar{\omega}_0 \bar{X}_{01}}{\bar{R}_0^2 \cos \delta_0}$$

$$\bar{R}_0 = (\bar{X}_0^2 + \bar{Y}_0^2 + \bar{Z}_0^2)^{1/2},$$

$$\bar{X}_0 = \bar{R} \cos \omega_0 t - \bar{l}_s \sin \delta_0$$

Fig. 1 Diagram showing co-ordinate systems for slipper bearing configuration and friction between piston and cylinder

$$\bar{Y}_0 = \bar{R} \sin \omega_0 t, \quad \bar{Z}_0 = -\bar{X}_0 \tan \delta_0, \quad \bar{X}_{01} = \bar{R} - \bar{I}_S \sin \delta_0 \cos \omega_0 t$$

Neglecting small terms on the assumptions of hydrodynamic lubrication theory, the Navier-Stokes equations of motion in cylindrical co-ordinates reduce to those for the fluid in a slipper bearing. We determine the velocity components between the bearing surfaces, $\bar{u}$ and $\bar{v}$, by integrating those under the boundary conditions of Eq. (2). $\bar{u}$ and $\bar{v}$ are given as follows.

$$\bar{u} = -3 \frac{\partial \bar{p}}{\partial r} \bar{z} (\bar{h} - \bar{z}) + \frac{\bar{z}}{\bar{h}} \bar{U}_r \dots \dots \dots (3)$$

$$\bar{v} = -3 \frac{1}{r} \frac{\partial \bar{p}}{\partial \theta} \bar{z} (\bar{h} - \bar{z}) + \frac{\bar{z}}{\bar{h}} \bar{U}_\theta \dots \dots \dots (4)$$

where

$$\bar{U}_r = \bar{A}(\omega_0 t) \sin \theta + \bar{B}(\omega_0 t) \cos \theta$$

$$\bar{U}_\theta = -\bar{B}(\omega_0 t) \sin \theta + \bar{A}(\omega_0 t) \cos \theta + \bar{C}(\omega_0 t) \bar{r} + \bar{\omega}_s \bar{r}$$

By substituting $\bar{u}$ and $\bar{v}$ into the continuity equation and integrating with respect to $\bar{z}$, the following Reynolds equation is obtained.

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} (r \bar{h}^3 \frac{\partial \bar{p}}{\partial r}) + \frac{1}{r^2} \frac{\partial}{\partial \theta} (\bar{h}^3 \frac{\partial \bar{p}}{\partial \theta}) &= 2 \frac{d \bar{h}_0}{d \tau} \frac{R_2 \Omega}{V} - \bar{\alpha} \{ \bar{A}(\omega_0 t) \sin \psi + \bar{B}(\omega_0 t) \cos \psi \} \\ &+ 2 \bar{\alpha} \frac{\bar{r}}{\bar{\alpha}} \frac{d \bar{\alpha}}{d \tau} \frac{R_2 \Omega}{V} \cos(\psi - \theta) + \bar{\alpha} \bar{r} \{ -2 \frac{d \psi}{d \tau} \frac{R_2 \Omega}{V} + \bar{\omega}_s - \bar{C}(\omega_0 t) \} \sin(\psi - \theta) \dots \dots \dots (5) \end{aligned}$$

Since $\bar{\alpha}$ can be usually considered small enough, the pressure distribution is expressed as a power series in increasing powers of $\bar{\alpha}$. By the perturbation method, equation (5) is reduced to a linear partial differential equation. By integrating Eq. (5), we obtain the distribution of pressures in the slipper bearing. Then, by integrating it over the bearing surface, the fluid force and the moments acting on the slipper are obtained as follows.

$$\bar{P}_z = C_p (\bar{p}_r - \bar{p}_a) + 2 C_w \frac{1}{\bar{h}_0^3} \frac{d \bar{h}_0}{d \tau} \frac{R_2 \Omega}{V} - C_w \frac{1}{\bar{h}_0^3} \{ \bar{A}(\omega_0 t) \bar{\phi}_x + \bar{B}(\omega_0 t) \bar{\phi}_y \} \dots \dots \dots (6)$$

$$\left. \begin{aligned} \bar{M}_x &= -C_1(\tau) \frac{d \bar{\phi}_x}{d \tau} \frac{R_2 \Omega}{V} + C_2(\tau) \bar{\phi}_x + C_3(\tau) \bar{\phi}_y \\ \bar{M}_y &= C_1(\tau) \frac{d \bar{\phi}_y}{d \tau} \frac{R_2 \Omega}{V} - C_2(\tau) \bar{\phi}_y + C_3(\tau) \bar{\phi}_x \end{aligned} \right\} \dots \dots \dots (7)$$

where

$$\bar{\phi}_x = \bar{\alpha} \sin \psi, \quad \bar{\phi}_y = \bar{\alpha} \cos \psi$$

$$C_1(\tau) = 2 C_{rm} \frac{1}{\bar{h}_0^3}, \quad C_3(\tau) = C_{rm} \frac{-\bar{C}(\omega_0 t) + \bar{\omega}_s}{\bar{h}_0^3}$$

$$C_2(\tau) = C_{pm} \frac{\bar{p}_r - \bar{p}_a}{\bar{h}_0} + C_{sm} \frac{1}{\bar{h}_0^4} \frac{d \bar{h}_0}{d \tau} \frac{R_2 \Omega}{V}$$

$$C_p = -\frac{\pi (1 - a^2)}{2 \log a}, \quad C_w = -\frac{\pi}{8} \{ 1 - a^4 + \frac{(1 - a^2)^2}{\log a} \}$$

$$C_{pm} = \frac{3\pi}{32} (4 a^2 + \frac{1 - a^4}{\log a}), \quad C_{rm} = \frac{\pi}{96} (1 - a^2)^3, \quad C_{sm} = \frac{1 - a^2}{a} C_{pm} + 9 C_{rm}$$

2.2 The loads on bearing

The following forces and moments on the slipper can be considered.

- Load due to supply pressure (= cylinder pressure)
- Frictional moment on spherical bearing
- Centrifugal force and moment resulting from motion of slipper about center axis of cylinder block
- Inertia force and moment of inertia resulting from reciprocating motion of slipper in the Z direction
- Inertia force resulting from reciprocating motion of piston

- Spring force in the z direction resulting from one set in cylinder
- The following assumptions can be made in relation to the spherical bearing and the bearing load (b).
- Operation of spherical bearing is in boundary lubrication.
 - Frictional coefficient is constant.
 - Friction is a sliding one.
 - Ball and bearing are perfectly rigid.
 - Displacement between center of ball and that of bearing is negligible.

Based on the assumptions (4) and (5), no moment due to the friction on the spherical bearing acts around the z axis because the point in contact with the spherical bearing and the ball coincides with the center of bearing. And, because $\bar{M}_z$ is small, we obtain $\bar{\omega}_s = 0$.

As the motions of slipper, the translational motion in the x, y and z directions and the rotational motion about the center of the spherical bearing can be considered. Since the translational motions in the x and y directions are determined by the position of ball center, they are not discussed here.

It can be assumed that the following two frictional moments act around the center axis of the spherical bearing in pump (or motor) operation. One of those moments results from the same mechanism as the frictional moment considered in previous report. Another results from the rotation of piston about its own center axis. Frictional moment (b) can be expressed in terms of the sum of two moments. Loads (a) to (f) are expressed as follows.

$$\bar{W} = -\pi \bar{D}_p^2 \bar{p}_s / 4 \quad (8)$$

$$\bar{F}_{cz} = (R_2/H) \bar{m}_s \bar{R}_g \bar{\omega}_0^2 \sin \delta \quad (9)$$

$$\bar{F}_{isz} = -(R_2/H) \bar{m}_s \bar{R} \bar{\omega}_0^2 \sin \delta_0 \cos \omega_0 t \quad (10)$$

$$\bar{F}_{ip} = -(R_2/H) \bar{m}_p \bar{R} \bar{\omega}_0^2 \tan \delta_0 \cos \omega_0 t \quad (11)$$

$$\bar{F}_{sp} = -(\bar{l}_1 - \bar{R} \tan \delta_0 \cos \omega_0 t) \bar{k} \quad (12)$$

$$\bar{T}_{fx} = \bar{R}_b f_s \bar{N} (-1 + \sin \xi) \quad (13)$$

$$\bar{T}_{fy} = \bar{R}_b f_s \bar{N} \cos \xi \quad (14)$$

$$\bar{T}_{cx} = (R_2/H) \bar{m}_s \bar{R}_g \bar{\omega}_0^2 \cos \delta \sin \zeta \bar{l}_g \quad (15)$$

$$\bar{T}_{cy} = -(R_2/H) \bar{m}_s \bar{R}_g \bar{\omega}_0^2 \cos \delta \cos \zeta \bar{l}_g \quad (16)$$

$$\bar{T}_{ix} = 0 \quad (17)$$

$$\bar{T}_{iy} = -(R_2/H) \bar{m}_s \bar{R} \bar{\omega}_0^2 \tan \delta_0 \sin \delta_0 \cos \omega_0 t \bar{l}_g \quad (18)$$

where

$$\bar{N} = -\{\bar{W}_z + \bar{F}_{spz} + \bar{F}_{ipz} - (\bar{F}_{isz} + \bar{F}_{cz})\}$$

$$\sin \xi = \cos \delta_0 \cos \omega_0 t f(\omega_0 t)$$

$$\cos \xi = \sin \omega_0 t f(\omega_0 t)$$

$$f(\omega_0 t) = 1/(\sin^2 \omega_0 t + \cos^2 \delta_0 \cos^2 \omega_0 t)^{1/2}$$

$$\bar{R}_g = \{(\bar{R} \cos \omega_0 t - \bar{l}_g \sin \delta_0)^2 + (\bar{R} \sin \omega_0 t)^2\}^{1/2}$$

$$\sin \delta = \cos \zeta \sin \delta_0$$

$$\cos \delta = \{1 - (\cos \zeta \sin \delta_0)^2\}^{1/2}$$

$$\sin \zeta = Y_0/R_0, \quad \cos \zeta = X_0/(R_0 \cos \delta_0)$$

Loads (a), (e), (f) and the inertia force of (d) are the same as in the rotating swash plate type.

2.3 Friction between piston and cylinder bore

When the effect of friction between the piston and the cylinder bore is considered, the force as in Fig. 1 acts on the piston. It is assumed that the sliding surface between the piston and the cylinder bore is always in boundary lubrication, and the frictional coefficient is constant. In the delivery and suction regions, an equation for the balance of forces in the lateral direction is

$$\bar{P}' \sin \delta_0 + \bar{F}_1 = \bar{F}_2 \quad (19)$$

An equation for the balance of forces in the direction of piston axis is

$$\bar{F}_p = \bar{P}' \cos \delta_0 \mp f_p (\bar{F}_1 + \bar{F}_2) \quad (20)$$

An equation for the balance of moments about the center of spherical bearing is

$$(\bar{s} + \bar{l}_2) \bar{F}_1 \mp (\bar{D}_p/2) f_p \bar{F}_1 = \bar{s} \bar{F}_2 \mp (\bar{D}_p/2) f_p \bar{F}_2 \quad (21)$$

where minus-plus sign follows the fixed order. Minus sign is used in the case of the delivery region, and plus sign in the case of suction region.

$$\bar{F}_p = -(\bar{W} + \bar{F}_{sp} + \bar{F}_{ip})$$

$$\bar{s} = \bar{s}_1 + \bar{R} \tan \delta_0 \cos \omega_0 t$$

$$\bar{l}_2 = \bar{l}_3 - \bar{s}$$

s_1 denotes the distance between the center of spherical bearing and the lower end of cylinder block at $\omega_0 t = \pi/2$. l_2 and l_3 denote the distance as given in Fig. 1. P' denotes the force of the sum of the fluid force due to the fluid pressure and the force in the z direction of the bearing loads (c) and (d) minus the inertia force on the slipper. From the foregoing equations, we can establish the following expression

$$P' = \frac{\bar{F}_p}{\cos \delta_0 \mp \frac{(2\bar{s} + \bar{l}_2 \mp \bar{D}_p f_p)}{\bar{l}_2} f_p \sin \delta_0} \quad (22)$$

3. Motion of a Slipper

The equation of motion for a slipper can be obtained as follows; For translational motion in the z direction

$$\bar{m}_s \frac{d^2 \bar{h}_0}{d\tau^2} \left(\frac{R_2 \Omega}{V}\right)^2 = \bar{P}_z + \bar{W}_z + \bar{F}_{cz} + \bar{F}_{isz} + \bar{F}_{ipz} + \bar{F}_{spz} \quad (23)$$

for rotational motion about the center of the spherical bearing

$$(\bar{I}_d + \bar{I}_g^2 \bar{m}_s) \frac{d^2 \bar{\phi}_x}{d\tau^2} \left(\frac{R_2 \Omega}{V} \right)^2 = \bar{M}_x + \bar{T}_{fx} + \bar{T}_{cx} + \bar{T}_{ix} \quad (24)$$

$$(\bar{I}_d + \bar{I}_g^2 \bar{m}_s) \frac{d^2 \bar{\phi}_y}{d\tau^2} \left(\frac{R_2 \Omega}{V} \right)^2 = -\bar{M}_y - \bar{T}_{fy} - \bar{T}_{cy} - \bar{T}_{iy} \quad (25)$$

Now, it is assumed that the motion of a slipper and the forces and moments acting on the slipper can be expressed in terms of the sum of the position of equilibrium corresponding to time-average value and small deviation from it. When the time-average value of $\bar{h}_0$ is defined as representative clearance H , $\bar{h}_0$ is given by

$$\bar{h}_0 = 1 + \Delta \bar{h}_0, \quad \Delta \bar{h}_0 \ll 1 \quad (26)$$

In addition, $\bar{\phi}_x$ and $\bar{\phi}_y$ as well as $\bar{h}_0$ are given as follows.

$$\bar{\phi}_x = \bar{\phi}_{x0} + \Delta \bar{\phi}_x, \quad \bar{\phi}_y = \bar{\phi}_{y0} + \Delta \bar{\phi}_y \quad (27)$$

In relation to the time-average value, from Eqs. (23) to (25), an equation for the balance of forces is

$$\bar{P}_{z0} + \bar{W}_{z0} + \bar{F}_{cz0} + \bar{F}_{isz0} + \bar{F}_{ipz0} + \bar{F}_{spz0} = 0 \quad (28)$$

and equations for the balance of moments are

$$\left. \begin{aligned} \bar{M}_{x0} + \bar{T}_{fx0} + \bar{T}_{cx0} + \bar{T}_{ix0} &= 0 \\ \bar{M}_{y0} + \bar{T}_{fy0} + \bar{T}_{cy0} + \bar{T}_{iy0} &= 0 \end{aligned} \right\} \quad (29)$$

By integrating the equation of the fluid velocity distribution, the flow rate from the slipper is given as

$$\bar{Q}_{out} = -\frac{\bar{P}_r}{\log a} \bar{h}_0^3 - \left(1 + \frac{1-a^2}{2 \log a}\right) \frac{d\bar{h}_0}{d\tau} \frac{R_2 \Omega}{V} + \left(1 + \frac{1-a^2}{4 \log a}\right) \{\bar{A}(\omega_0 t) \bar{\phi}_x + \bar{B}(\omega_0 t) \bar{\phi}_y\} \quad (30)$$

We assume the mixed restrictor in the same manner as in the previous report. In relation to the time-average, an equation with respect to H is obtained from the relation with the rate of flow into and out of the recess.

$$C_1 H^6 + C_2 H^3 + C_3 = 0 \quad (31)$$

where

$$C_1 = \left(\frac{\pi}{6\mu K_0}\right)^2 p_{s0} \bar{Q}_{out0}^2, \quad C_2 = \frac{4}{3} \frac{\bar{I}_0}{\bar{r}_0^4} \frac{\bar{Q}_{out0}}{R_2^3}, \quad C_3 = -\bar{P}_{s0} + \bar{P}_{r0}$$

$$\bar{Q}_{out0} = -\frac{\bar{P}_{r0}}{\log a} + \left(1 + \frac{1-a^2}{4 \log a}\right) (\bar{A}_0 \bar{\phi}_{x0} + \bar{B}_0 \bar{\phi}_{y0})$$

By solving the simultaneous equations (31), (28) and (29), we can determine each of the time-average values of oil-film parameters. Representative angular velocity Ω is taken as the angular velocity of reciprocating motion of the piston. Representative velocity V is so defined that p_{s0} becomes unity. In this report, therefore, the case of operation with no load ($p_s = 0$ MPa) is not discussed. The results are numerical solutions obtained by Runge-Kutta's method.

Minimum oil film thickness h_{min} is defined as follows.

$$\bar{h}_{min} = \bar{h}_0 - \bar{a} \quad (32)$$

4. Power Loss

The total power loss of a slipper bearing in fluid lubrication is given by the sum of the flow rate and viscous drag losses. An equation of the torque due to the viscous drag is obtained from that of the velocity distribution of the fluid between the bearing surfaces. The power loss due to the viscous drag L_f is given as follows.

$$L_f = \frac{\omega_0}{2\pi} \int_0^{2\pi} \left[\int_0^{2\pi} \int_a^1 dT \right] d(\omega_0 t) \quad (33)$$

where

$$dT = dF_X Y_C - dF_Y X_C$$

$$dF_X = \left(\mu \frac{R \omega_0 \sin \omega_0 t}{h} + \tau_x \cos \delta_0 \right) r d\theta dr, \quad dF_Y = \left(-\mu \frac{R \omega_0 \cos \omega_0 t}{h} + \tau_y \right) r d\theta dr$$

$$\tau_x = \tau_r \cos \theta - \tau_\theta \sin \theta, \quad \tau_y = \tau_r \sin \theta + \tau_\theta \cos \theta$$

$$\tau_r = -\mu \left(\frac{\partial u}{\partial z} \right)_{z=h}, \quad \tau_\theta = -\mu \left(\frac{\partial v}{\partial z} \right)_{z=h}$$

$$X_C = X_0 + r \cos \theta \cos \delta_0, \quad Y_C = Y_0 + r \sin \theta$$

An equation of power loss due to the flow rate, L_q , can be expressed in terms of the flow rate Q_{out} multiplied by the supply pressure P_s .

$$L_q = \frac{1}{2\pi} \int_0^{2\pi} P_s Q_{out} d(\omega_0 t) \dots \dots \dots (34)$$

The total power loss $\hat{L}$ is a nondimensional quantity of the sum of two power losses divided by the theoretical power given by $(1/4) P_0 \omega_0 D_p^2 R \tan \delta_0$.

The approximation to $\hat{L}$ is expressed in terms of $\hat{L}_0$. $\hat{L}_0$ is a quantity given by replacing the oil-film parameter in the equation of $\hat{L}$ by its time-average values. Neglecting small terms, the approximation reduces to

$$\begin{aligned} \hat{L}_0 = & \frac{\mu n}{P_0} \frac{\pi^2}{30 \tan \delta_0} \left[R R_2^2 (1 - a^2) \frac{4}{D_p^2} + \frac{R_2^4 (1 - a^4)}{R} \frac{2 \cos \delta_0}{D_p^2} \frac{C_0 + \omega_{s0}}{\omega_0} \right] \frac{1}{H} \\ & + \frac{P_0}{\mu n \sin \delta_0} \frac{10}{R R_2^2 (1 - a^2)} \frac{P_{s0}}{P_0} H^3 \dots \dots \dots (35) \end{aligned}$$

where the second term of $\hat{L}_{0f}$ in relation with the distance between the origin 0 and a point on the slipper sliding surface is not considered in the efficiency equation derived by Shute and others⁽¹⁾.

The approximation to $\hat{L}_0$ for the rotating swash plate type is given by

$$\begin{aligned} \hat{L}_0 = & \frac{\mu n}{P_0} \frac{\pi^2}{30 \tan \delta_0} \left[\left\{ R \left(1 + \frac{1}{2} \sin^2 \delta_0 \right) + \frac{l_s^2}{R} (1 - \cos \delta_0)^2 \right\} \frac{4 R_2^2 (1 - a^2)}{D_p^2} \right. \\ & \left. + \frac{R_2^4 (1 - a^4)}{R} \frac{2 \cos \delta_0}{D_p^2} \frac{C_0 + \omega_{s0}}{\omega_0} \right] \frac{1}{H} + \frac{P_0}{\mu n \sin \delta_0} \frac{10}{R R_2^2 (1 - a^2)} \frac{P_{s0}}{P_0} H^3 \dots \dots \dots (36) \end{aligned}$$

The behaviour of a slipper will be remarkably dependent on the operating conditions because moments of bearing loads (b) and (c) have great influence on slipper attitude. Then, we investigate the behaviour of the slipper under the following typical operating conditions.

- (1) When the effect of the bearing load (b) on the behaviour of the slipper is predominant, we take $n = 20 \text{ s}^{-1}$, $P_{sout} = 20 \text{ MPa}$,
- (2) when the effect of the bearing load (c) is predominant, we take $n = 25 \text{ s}^{-1}$, $P_{sout} = 5 \text{ MPa}$,
- (3) when the effects of the bearing loads (b) and (c) are comparable, we take $n = 50 \text{ s}^{-1}$, $P_{sout} = 20 \text{ MPa}$.

The three cases are hereinafter called the operating conditions 1, 2 and 3 respectively. In all the numerical examples, viscosity of the fluid μ is $30 \text{ mPa}\cdot\text{s}$, discharge constant of the restrictor K_0 is $0.64 \times 10^{-8} \text{ m}^2 (\text{m}^3/\text{kg})^{1/2}$, suction pressure P_{sin} is 0.5 MPa , frictional coefficient between the piston and the cylinder bore f_p is 0.15 , frictional coefficient between the spherical cup and the ball f_s is 0.01 , and these values are constant.

We take β , a , r_0 as parameters of the slipper sizes which can be selected arbitrarily to some extent for the design condition, and have large effects on the characteristics of the slipper bearing. Where $\beta = D_2/D_p$, $a = D_1/D_2$ and r_0 denotes the radius of the restrictor.

The important values used by numerical calculations are shown in Table 1. In this report, the characteristics of a slipper bearing for the rotating cylinder block type piston pump is mainly discussed. Then, we make comparison between the two types, the rotating cylinder block type, and the rotating swash plate type.

Table 1 Values of numerical examples

$D_p = 25.0 \text{ mm}$	$l_s = 15.0 \text{ mm}$	$m_p = 0.15 \text{ kg}$
$l_0 = 1.5 \text{ mm}$	$R = 40.0 \text{ mm}$	$m_s = 0.05 \text{ kg}$
$l_3 = 105.0 \text{ mm}$	$R_b = 10.0 \text{ mm}$	$\delta_0 = 15.0^\circ$
$l_g = 7.5 \text{ mm}$	$s_1 = 30.0 \text{ mm}$	

The effects of a on the motion of the slipper is shown in Fig. 2, in the case of $\beta = 1.36$, $r_0 = 0.2$ mm. The value of β is so selected that the effect of the operating condition on the power loss appears remarkably. The value of r_0 is given on the condition of maximum bearing stiffness. The motion of slipper is remarkably dependent on the operating condition. When a is small ($a = 0.60$), the deviation of $\bar{h}_0$ in the delivery region increases under all the operating conditions. As a result, the oil film thickness has minimum value $h_{\min}$ in the delivery region. But, when a is large ($a = 0.68$), an increase in the deviation of $\bar{a}$ in the suction region minimizes the oil film thickness. Comparing the operating conditions 1 with 2 and 3, the effect of n appears remarkably in the deviations of $\bar{a}$ and $h_{\min}$ in the suction region.

The effects of a on $\hat{L}$, $\hat{L}_0$ and $h_{\min}$ are shown in Fig. 3. The total power loss $\hat{L}$ remarkably depends on the operating condition. This is the reason why the power loss due to flow rate, $\hat{L}_q$, is predominant under the operating condition 1, and that due to viscous drag $\hat{L}_f$ is predominant under the operating conditions 2 and 3. The difference between $\hat{L}$ and $\hat{L}_0$ originates from that of the deviation of the oil-film parameter. The effects of a on $h_{\min}$ are remarkable as shown in Figs. 2 and 3. It follows therefrom that $h_{\min}$ tends to zero as a increases. This means that the upper limit to a exists in order to realize the bearing operation in fluid lubrication. The value of the limit is dependent on the operating condition. The minimum oil film thickness $h_{\min}$ is composed of $\bar{h}_0$ and $\bar{a}$ as in Eq. (32). The oil film thickness is

minimized at a certain $\omega_0 t$ in a cycle because the deviations of $\bar{h}_0$ and $\bar{a}$ are dependent on the operating condition. Therefore, in the value of a , at which $h_{\min}$ becomes zero, the result considering the deviations differs from one obtained for the steady state (time-average value). The former is entirely different from the result obtained on the assumption of uniform clearance ($\alpha = 0$), too. This means that the behaviour of the slipper must be clarified in order to examine the lubrication condition of the bearing. The minimum oil film thickness must be larger than the surface roughness in order to realize the bearing operation in fluid lubrication. Therefore, we guarantee the bearing operation in fluid lubrication by setting the limit at $h_{\min}$ obtained from the behaviour of the slipper. In this numerical example, 1, 3 and 5 μm are set as the limit. We mark the limits on the curve lines of $\hat{L}$ and $\hat{L}_0$ in Fig. 3.

Equations (35) and (36) for $\hat{L}_0$ can be derived according to the following considerations. In the pump operation, the flow rate in the suction region is negligible as compared with that in the delivery region. Accordingly, when the time-average value of h_0 in a cycle is substituted into Eqs. (35) and (36), the difference between $\hat{L}_q$ and $\hat{L}_{0q}$ becomes large. Thus, we substitute the time-average value of h_0 in the delivery region into $\hat{L}_{0q}$ in Eqs. (35) and (36). Since the effect of the time-average value of $\bar{a}$ on $\hat{L}_0$ is small, we can obtain H by assuming it as zero, that is, $\phi_{x0}, \phi_{y0} = 0$. But, the time-average value of $\bar{h}_0$ in a cycle obtained by assuming $\phi_{x0}, \phi_{y0} = 0$ is substituted into $\hat{L}_{0f}$. It can be said from this standpoint that

Fig. 2 Effects of a on behaviour of slipper

Fig. 3 Effects of a on L , L_0 and $h_{\min}$

Eqs. (35), (36) are for the assumption of uniform clearance. The deviations of $\bar{a}$ are remarkably different in all the regions comparing the operating conditions 1 with 2 and 3. Therefore, the effect of ϕ is considered by using the following equations under the operating condition 1. Eliminating the terms differentiated with respect to time and the terms containing ω_0 , we obtain from Eqs. (24) and (25)

$$\phi_x = C (-1 + \sin \xi) h_0, \quad \phi_y = C \cos \xi h_0$$

$$C = \frac{16 (1 - a^2)}{3 (4a^2 \log a + 1 - a^4)} \frac{f_s R_b}{R_2^2} \dots (37)$$

For the rotating swash plate type

$$\phi_x = C_4 \cos \xi h_0, \quad \phi_y = -C_4 \sin \xi h_0$$

$$\dots (38)$$

where the equations of $\cos \xi$ and $\sin \xi$ have been shown in previous report(2). Hereinafter, we obtain the slipper size minimizing $\hat{L}_0$ on the assumption of uniform clearance, and we guarantee the bearing operation in fluid lubrication by setting the limit at $h_{\min}$ obtained from the behaviour of the slipper having the size.

The values of β and a giving $\hat{L}_{0\min}$ are shown in Fig. 4. The values of β giving $\hat{L}_{0\min}$ under the condition of a constant a are shown in Fig. 4, hereinafter β_0 and a_0 are obtained on the assumption of uniform clearance, and the bearing operation of the slipper in sizes of β_0 and a_0 is in fluid lubrication or metallic contact.

Fig. 4 Optimum slipper size (r_0 is constant)

Then, we examine the lubrication condition and the difference between $\hat{L}$ and $\hat{L}'_{\min}$, $\hat{L}_{0\min}$. The power loss $\hat{L}$ of the slipper in sizes of β_0 and a_0 is expressed in terms of $\hat{L}'_{\min}$ and is shown by a solid line. Setting 1, 3 and 5 μm as the limit, we show the limit of β_0 and a_0 to realize the bearing operation in fluid lubrication. The slipper size giving minimum power loss in fluid lubrication is called the optimum slipper size. The optimum size has such dimensions as to decrease the predominant power loss in each operating condition as in Fig. 3. The difference between $\hat{L}_{0\min}$ and $\hat{L}'_{\min}$ is small and varies with that in the case of constant β (as in Fig. 3). The effect of the deviation of oil film parameter is also small. From Fig. 4, we can have the optimum slipper giving small $\hat{L}'_{\min}$ as we make the surface roughness setting the limit at $h_{\min}$ smaller to a certain degree than its magnitude (for example, at 1 μm). The effect of the frictional coefficient between piston and cylinder bore f_p is also shown in Fig. 4. The values of β_0 , a_0 and $\hat{L}_{0\min}$ vary very much with f_p . This means that it is necessary to take sufficient care in designing the sliding surface between the piston and the cylinder bore.

As β_0 , a_0 and $\hat{L}_{0\min}$ obtained are approximations, we compare these values with one obtained by using the equation of $\hat{L}$. Figures 5 and 6 show $\hat{L}$ and $h_{\min}$, respectively. The operating condition is the same as the condition 3 in Fig. 4.

Fig. 5 $\hat{L}$ (operating condition 3)

Fig. 6 $h_{\min}$ (operating condition 3)

Fig. 7 Effects of r_0

Fig. 8 Comparison of optimum slipper sizes for two types

Comparing the two results, the differences between $\hat{L}_{\min}$ and $\hat{L}'_{\min}$, $\hat{L}_{0\min}$, the optimum size and β_0 , a_0 are less than a few percentage. Reading the value of $h_{\min}$ corresponding to β and a from Fig. 6, we draw a contour line of $h_{\min}$ in Fig. 5. Now, the values of β and a giving $\hat{L}_{\min}$ are expressed in terms of β_{opt} and a_{opt} . The value of a larger than a_{opt} can give a smaller $\hat{L}_{\min}$ than a_{opt} . But, these values are inappropriate from a standpoint of the miniaturization of the systems because the difference between the two is very small and the value of β is considerably larger than β_{opt} . Thus, it seems that Eq. (35) is available as an approximate equation. The errors in the optimum sizes giving $\hat{L}_{\min}$ under the operating conditions 1 and 2 are also in the same degree as under the operating condition 3.

The effect of restrictor radius r_0 on $\hat{L}_{0\min}$ is shown in Fig. 7. We can obtain the optimum value of r_0 by repeating the procedure taken in Fig. 4 for r_0 , but we investigate the power losses $\hat{f}_{0\min}$ only for three values of r_0 . Under the operating conditions 1 and 3, the effect of r_0 on $\hat{L}_{0\min}$ is small, but the values of β_0 and a_0 giving $\hat{L}_{0\min}$ are different. Therefore, the slipper bearings in sizes of β_0 and a_0 vary very much in $h_{\min}$. For a certain β_0 , the value of a_0 becomes large as the restrictor radius r_0 is smaller. When a_0 is large, $h_{\min}$ becomes small because the deviation of $\bar{a}$ is large. On the other hand, when r_0 is large, the value of a_0 becomes small, but the deviations of $\bar{h}_0$ and $\bar{a}$ become large because of no compensation for pressure, and $h_{\min}$ becomes small. From the results, it is known that a value of r_0 maximizing $h_{\min}$ exists. Because p_s is small under the operating condition 2, the value of r_0 maximizing $h_{\min}$ is remarkably different from that under the other operating conditions. But, the existence of the optimum value of r_0 under the operating condition 2 can be explained as under the operating conditions 1 and 3. This means that an optimum value of r_0 under each of the operating conditions exists.

Comparison with the rotating swash plate type is made in Fig. 8. Since the difference of β_0 and a_0 between the two types is small but the bearing loads acting on the slipper is different, the values of β_0 and a_0 to realize the bearing operation in fluid lubrication entirely disagree with respect to the two types. As a result, it is seen that the optimum slipper sizes for the two types are different.

5. Conclusions

In this report, first of all, considering all the oil-film parameters, a theoretical analysis on the characteristics of the slipper bearing for the rotating cylinder block type is made. Secondly, a theoretical expression for the power loss of the slipper bearing is derived, and the slipper size to minimize the power loss and to realize the bearing operation in

fluid lubrication is theoretically analyzed. The following results are obtained:

- (1) The behaviour of a slipper for the rotating cylinder block type can be clarified by a theoretical analysis considering all the oil-film parameters.
- (2) A method for designing a slipper to give the minimum power loss and to realize the bearing operation in fluid lubrication is shown. The slipper size to minimize the power loss is obtained on the assumption of uniform clearance. The bearing operation in fluid lubrication is guaranteed by setting the limit at the minimum oil film thickness obtained from the motion of the slipper.
- (3) The surface roughness of the slipper bearing giving the minimum oil film thickness and the operating condition have great effects on the slipper size giving the minimum power loss in fluid lubrication.

Acknowledgement

The author wishes to express his gratitude to Professor A. Yamaguchi of Yokohama National University for his guidance and encouragement.

References

- (1) Shute, N. A. and Turnbull, D. E., Proc. In. Conv. Lubr. Wear, (1963), p.3.
- (2) Iboshi, N. and Yamaguchi, A., Bull. of the JSME, Vol. 25, No. 210, (1982), p.1921.
- (3) Iboshi, N. and Yamaguchi, A., Bull. of the JSME, Vol. 26, No. 219, (1983), p.1583.
- (4) Hooke, C. J. and Kakoullis, Y.P., proc. Fluid Power Symp., (1981), p.179.