Vibration Control for a Beam Structure Using an Electromagnetic Damper with Velocity Feedback
(2nd Report, Shock Response)*

By Hiroyuki KOJIMA**, Kosuke NAGAYA***
Katsumi NAGAI† and Humihiko NIIYAMA†

In this report, the transient response of a beam structure having an electromagnetic damper with velocity feedback introduced in the previous report is investigated theoretically and experimentally. The beam structure is subjected to shock by electromagnetic force generated by a micro computer and an electromagnetic linear actuator, and the transient vibrations are controlled by the electromagnetic damper. In the analysis, the transient response of the vibration displacement of the beam system and the electric current of the control system are derived exactly by use of Laplace transformation and Bernoulli-Euler bending theory. Furthermore, the efficiency and the stability of the vibration control are investigated in detail.

Key words: Vibration Control, Electromagnetic Damper, Velocity Feedback System, Beam Structure, Shock Vibration, Linear Actuator

1. Introduction

Studies of the damper which controls mechanical vibrations actively by providing energy have been reported**-††, and there are ones with an elastic structure. The force acting on a structure is classified into harmonic force and shock. Then, the studies in which the active damper for the elastic structure is investigated analytically and experimentally in detail seem to be few, and may be considered useful in practical application.

In the previous report‡‡, we have made an electromagnetic damper with velocity feedback as a trial, and showed the experimental results about the forced vibration control of a beam structure by using the electromagnetic damper. Furthermore, we have investigated theoretically the control characteristics and the forced vibration response of the system in detail.

In this report, first, the experimental results about the vibration control of a beam structure by use of an electromagnetic damper are shown. The beam structure is subjected to a shock, and the electromagnetic damper consists of a velocity sensor and an electromagnetic linear actuator as well as a control circuit. Then, the vibration control characteristics, the dynamic behavior of the system and the stability are investigated theoretically in consideration of both the beam structure and the control system. In the experiment, the shock is generated by using a micro computer and the electromagnetic linear actuator. As the shock, both a half sinus pulse and a continuous sinus wave are utilized. In the analysis, the transient response of the system is derived exactly by using Laplace transformation and Bernoulli-Euler bending theory. The validity of the expressions derived here is confirmed by comparing the experimental results with the numerical ones. Furthermore, the effects of the control system and the electromagnetic linear actuator upon the vibration control efficiency, the dynamic behavior of the system and the stability are clarified.

2. Summary of the experiment

Figure 1 shows a beam structure subjected to an exciting force by an electromagnetic linear actuator. The beam structure is composed of a beam and two masses \(m_1, m_2\). The exciting force and the control force act on \(m_1\). The mass of the actuators for the control and the exciting actuator as well as the moving part of a velocity sensor is contained in \(m_0\). Origin 0 is set at the left end of the beam. The x axis is located along the neutral axis of the beam, and the flexural displacement of the beam normal to the x axis is y. In the present experiment, the electromotive linear actuators and the velocity sensor which were made in the previous report are utilized.

In general, it is difficult to generate a shock of arbitrary shape mechanically. Therefore, in this experiment, it is tried to generate the shock by using a micro computer (Z80 CPU) and a linear actuator. At first, a half sinus pulse or a continuous...
sinus wave is calculated by the microcomputer and is memorized in RAM with suitable discrete interval. Next, these waves are applied to a summing circuit as voltage by D/A converter. Furthermore, the voltage signal is given to a power amplifier so that the electromotive linear actuator is driven. To execute the D/A conversion rapidly, the machine language is utilized. In the control circuit, the voltage generated in the sensor is given to the control linear actuator through the power amplifier so that the control force is applied to the beam structure. The control electric current is recorded by measuring the voltage of the electric resistance $R_e$. The displacement of right end of the beam structure is detected by a noncontact displacement meter of eddy current type.

The dimensions of the experimental equipment are as follows:

- $\rho = 0.314\, \text{kg}$, $m_1 = 0.38\, \text{kg}$
- $m_2 = 0.084\, \text{kg}$
- $l = 600\, \text{mm}$
- Young's modulus $E = 1.81 \times 10^6\, \text{N/mm}^2$
- Second moment of cross section $I_x = 30.4\, \text{mm}^4$
- Resistance of the control coil $R = 1.4\, \Omega$
- Inductance of the control coil $L = 3.69 \times 10^{-6}\, \text{H}$
- Coefficient of the electromagnetic force of the control linear actuator $\tau = 0.479\, \text{N/A}$
- Coefficient of the electromagnetic force of the exciting linear actuator $\tau' = 0.508\, \text{N/A}$
- Coefficient of the counter electromotive force $\beta = 0.476 \times 10^{-4}\, \text{V.s/mm}$

where $\rho$ is the mass density of the beam, and $A$ is the transverse sectional area.

3. Theoretical analysis

By using Bernoulli-Euler bending theory, the equations of motion of the beam structure and the electric current of the control system in Fig.1 are given as follows:

$$
\begin{align*}
\frac{\partial^2 \psi_y}{\partial x^2} + \rho A \frac{\partial^2 \psi_y}{\partial t^2} &= 0, \quad (n=1, 2) \\
L \frac{d^2 \psi_y}{dt^2} + R \frac{d \psi_y}{dt} &= 0, \quad (1)
\end{align*}
$$

where $i$ is the control electric current, $R$ is the total electric resistance of the control system, $t$ is the time, $\alpha = \frac{\partial^2 \psi_y}{\partial x^2}, \alpha$ is the coefficient of the velocity electromotive force of the sensor coil, and $\alpha'$ is the magnification degree of the power amplifier. The left end of the beam structure is fixed, and the shock $f(t)$ and the control force $F$ are applied to the position of the beam structure where the actuators are attached. Thus, the boundary conditions are obtained as follows:

$$
\begin{align*}
\text{at} \quad x = 0: \psi_y &= 0, \quad \frac{\partial \psi_y}{\partial x} = 0 \\
\text{at} \quad x = L: \frac{\partial \psi_y}{\partial x} &= 0 \\
\text{at} \quad x = h: \frac{\partial \psi_y}{\partial x} &= \frac{m_2 \frac{\partial \psi_y}{\partial t}}{\tau} \\
\text{at} \quad x = h: \psi_y &= \psi_2
\end{align*}
$$

By executing Laplace transformation of Eqs. (1)-(8) under the initial conditions $\gamma_n|_{t=0} = 0, \psi_{n,t} = 0$, the following equations are obtained.

$$
\begin{align*}
\mathcal{L}[\frac{d^2 \psi_y}{dx^2} + \rho A \frac{d^2 \psi_y}{dt^2}] &= 0, \quad (n=1, 2) \\
L \frac{d^2 \psi_y}{dt^2} + R \frac{d \psi_y}{dt} &= 0 \\
x = 0: \psi_y = 0, \quad \frac{d \psi_y}{dt} = 0 \\
x = L: \frac{d \psi_y}{dt} = 0 \\
x = h: \frac{d \psi_y}{dt} = \frac{m_2 \frac{d \psi_y}{dt}}{\tau} \\
x = h: \psi_y = \psi_2
\end{align*}
$$

where $\psi_n(x, S) = \int_0^x \psi_n(x, t)e^{-st} dt$
The general solutions to Eqs. (9) can be written as follows:

\[ Y_1 = A_1 \sin \xi x + A_2 \cos \xi x + A_3 \sinh \xi x + A_4 \cosh \xi x \]  
\[ Y_2 = B_1 \sin \xi (l - x) + B_2 \cos \xi (l - x) + B_3 \sinh \xi (l - x) + B_4 \cosh \xi (l - x) \]  
\[ \left[ \begin{array}{c} A_1 \\ A_2 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{array} \right] = D^{-1} \left[ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right] \]  
(18)

where

\[ S = j\xi \sqrt{\frac{E_l}{\rho A_l l^2}} \]  
(19)

and \( A_1, A_2, B_1, B_2 \) in Eqs. (18)-(20) are unknown coefficients, and \( j^2 = -1 \).

In case that the shock \( f(t) \) is a half sinus pulse, \( f(t) \) is expressed as follows:

\[ f(t) = F_0 \left[ \sin \frac{t}{a} + \sin \frac{t}{a} \right] \]  
(20)

where \( a \) is the unit step function, \( a \) is the pulse width. Applying Laplace transform to Eq. (21) gives

\[ F(S) = F_0 \left[ \frac{\pi a}{S^2 + \left( \frac{\pi a}{S^2 + a^2} \right)} \right] \]  
(21)

By applying Laplace transformation to Eqs. (10)-(19) and substituting Eqs. (18)-(22) into Eqs. (10)-(16), the unknown coefficients in Eqs. (18)-(19) are derived as follows:

\[ A_i, B_i \]  
(22)

where

\[ D = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \]  
(23)

\[ A_1 = \xi \xi^2 - j\xi^2 \]  
(24)

\[ A_2 = -\xi^2 \left( \sin \xi \nu_1 - \sinh \xi \nu_1 \right) \]  
(25)

\[ A_3 = -\xi \left( \cos \xi \nu_1 - \cosh \xi \nu_1 \right) \]  
(26)

\[ a_{11} = 0 \]  
(27)

\[ a_{12} = \sin \xi \nu_1 - \sinh \xi \nu_1 \]  
(28)

\[ a_{13} = \cos \xi \nu_1 - \cosh \xi \nu_1 \]  
(29)

\[ a_{14} = 0 \]  
(30)

In Eq. (30), \( \psi \) and \( \Omega_1 \) are the eigen value and the natural frequency of the first mode of the beam structure, \( \Omega_1 \) is expressed as

\[ \Omega_1 = \sqrt{\frac{E_l}{\rho A_l l^2}} \]  
(31)

Then, the notations in Eqs. (25)-(31) are given as follows:

\[ \xi = \frac{Q_1}{R}, \lambda = \frac{2a_1 \Omega_1}{X_1}, \nu = \frac{I}{I_1}, \]  
(32)

\[ I_1 = \frac{a_1 \Omega_1 X_1}{R}, X_1 = \frac{2I}{3E_l} \]  
(33)

By substituting Eq. (23) into Eqs. (18)-(19), \( \bar{X}_1 \), \( \bar{Y}_1 \), and \( \bar{Y}_2 \) are derived as

\[ \bar{X}_1 = \frac{3 \pi A_1 \Omega_1}{a_1 \Omega_1 \bar{Q}_1}, \bar{Y}_1 = \frac{3 \pi A_1 \Omega_1}{a_1 \Omega_1 \bar{Q}_1} \]  
(34)

where

\[ a_{11} = 0 \]  
(35)

\[ a_{12} = \sin \xi \nu_1 - \sinh \xi \nu_1 \]  
(36)

\[ a_{13} = \cos \xi \nu_1 - \cosh \xi \nu_1 \]  
(37)

\[ a_{14} = 0 \]  
(38)

\[ a_{21} = 0 \]  
(39)

\[ a_{22} = \sin \xi \nu_1 - \sinh \xi \nu_1 \]  
(40)

\[ a_{23} = \cos \xi \nu_1 - \cosh \xi \nu_1 \]  
(41)

\[ a_{24} = 0 \]  
(42)

\[ a_{31} = 0 \]  
(43)

\[ a_{32} = \sin \xi \nu_1 - \sinh \xi \nu_1 \]  
(44)

\[ a_{33} = \cos \xi \nu_1 - \cosh \xi \nu_1 \]  
(45)

\[ a_{34} = 0 \]  
(46)
are obtained. The shock response for the half sinus pulse are expressed as
\[ y_i = \frac{3x Q_i}{\sigma_{Q_i}^2} \exp \left\{ \frac{(\xi_{i1})^2}{\sigma^2} \right\} \]
\[ + u(\tau - a_i) \frac{3x Q_i}{\sigma_{Q_i}^2} \exp \left\{ \frac{(\xi_{i1})^2}{\sigma^2} (\tau - a_i) \right\} \]
\[ \sum \frac{3x Q_i}{\sigma_{Q_i}^2} \exp \left\{ \frac{(\xi_{i1})^2}{\sigma^2} \right\} \]
\[ + u(\tau - a_i) \frac{3x Q_i}{\sigma_{Q_i}^2} \exp \left\{ \frac{(\xi_{i1})^2}{\sigma^2} (\tau - a_i) \right\} \]

where
\[ Q_i = \frac{\rho^2}{2(\xi_{i1})^2} dQ_i \]
\[ Q_i = \frac{3x Q_i}{\sigma_{Q_i}^2} \exp \left\{ \frac{(\xi_{i1})^2}{\sigma^2} \right\} \]
\[ + u(\tau - a_i) \frac{3x Q_i}{\sigma_{Q_i}^2} \exp \left\{ \frac{(\xi_{i1})^2}{\sigma^2} (\tau - a_i) \right\} \]
\[ \sum \frac{3x Q_i}{\sigma_{Q_i}^2} \exp \left\{ \frac{(\xi_{i1})^2}{\sigma^2} \right\} \]
\[ + u(\tau - a_i) \frac{3x Q_i}{\sigma_{Q_i}^2} \exp \left\{ \frac{(\xi_{i1})^2}{\sigma^2} (\tau - a_i) \right\} \]

and, \( a_i \) in Eqs.(47)-(49) means the one in which \( \xi \) in Eqs.(25)-(29) is replaced with \( \xi_{i1} \). Then, \( \xi_{i1} \) is identical with \( \xi_{i2} \). But \( \xi_{i3}, \xi_{i4}, \ldots \) denote the roots of the following frequency equation of the beam structure.

\[ d^2 = 0 \]

The theoretical equation in case of the continuous sinus pulse are obtained by equating the terms belonging to the unit step function to zero.

4. Experimental results and numerical results

In Fig.2- Fig.5, the experimental results are compared with the numerical ones about the vibration control of a beam structure subjected to a shock. In these figures, the shock wave, the experimental vibration wave \( \bar{y} \) in case of no control, the experimental response \( \bar{y} \) in case of control, the theoretical response \( y \), the experimental control current \( i \) and the theoretical control current \( i \) are shown in order, and \( \tau \) denotes the dimensionless time. First, the results in case of the half sinus pulse are shown in Figs.2, 3. In both figures, the vibration amplitudes \( \bar{y} \) and \( \bar{y} \) are damped rapidly so that a remarkable damping effect is recognized. The half sinus wave as the exciting force is approximated by 100 discrete points, and it is seen that sufficient accuracy can be obtained by this number in
Fig. 2 Shock responses by the half sinus pulse

Fig. 3 Shock responses by the half sinus pulse

$k = 0.5, \xi = 0.00114, \omega_n = 1.56
\omega = 1.34$ (rad/sec) $X_s = 0.736$ (mm)
$I_s = 0.0922$ (A)

$k = 0.5, \xi = 0.00114, \omega_n = 3.07
\omega = 1.34$ (rad/sec) $X_s = 0.736$ (mm)
$I_s = 0.0922$ (A)
Fig. 4 Shock responses by the half sinus pulse

Fig. 5 Shock responses by the continuous sinus pulse

\[ H = 0.7 \quad J = 0.00261 \quad \alpha = 3.3 \]
\[ \Omega_1 = 6.3 \text{ (rad/sec)} \quad X_s = 2.07 \text{ (mm)} \]
\[ \Omega_2 = 0.116 \text{ (A)} \]

\[ H = 0.5 \quad J = 0.00114 \quad \alpha = 1.49 \]
\[ \Omega_1 = 19.4 \text{ (rad/sec)} \quad X_s = 0.342 \text{ (mm)} \]
\[ I_s = 0.0494 \text{ (A)} \]
practical use. The control electric current wave is a complicated one containing the components of higher mode vibrations, but the components of higher mode are damped rapidly. On the other hand, the components of higher mode in the beam structure are considerably small. The results in case that the damper is located near the free end are shown in Fig. 4, the efficiency of the damper being sufficient in this case. Then, Fig. 5 shows the results in case that the electromagnetic damper is attached to the center of the beam structure subjected to a continuous sinus exciting force. When the electromagnetic damper is not operated, the vibration wave of higher modes is generated continuously. On the other hand, when the electromagnetic damper is operated, the higher mode components of the beam displacement and the control current are damped rapidly, and only the vibration component corresponding to the exciting force is generated continuously. Furthermore, the vibration modes of the beam structure corresponding to Fig. 3 are shown in Fig. 6. The pulse width of the exciting force is changed in this figure.

Fig. 7 shows the effect of the inductance of the control electromotive linear actuator upon the shock response of the system and the control efficiency. By this figure, it is seen that the efficiency of the vibration control declines and the natural frequency of the system increases, with an increasing value of the inductance. Figure 8 shows the root locus graph in case that the gain of the control system in Fig. 7 is changed, and it is shown quantitatively that the natural frequency and the damping amount change according to the shifting amount of the gain.

5. Conclusions

In the present report, first, the experimental results are shown about the vibration control of a beam structure by utilizing an electromagnetic damper with velocity feedback, made in the previous report as a trial. Then, the vibration control characteristics, the dynamic behavior of the system and the stability are investigated theoretically in consideration of both the beam structure and the control system. The electromagnetic damper is attached to the location on which the shock acts. The results may be summarized as follows:

(1) The shock responses of the displacement of the beam structure as well as

the control electric current are derived by use of Bernoulli-Euler bending theory and Laplace Transformation. The beam structure subjected to the shock is controlled by the electromagnetic damper. As the shock force, a half sinus pulse and a continuous sinus wave are utilized.

(2) It is confirmed that an arbitrary shock force can be generated by using a micro computer and an electromotive linear actuator.

(3) From the experiment of the vibration control for different damper locations, gains and pulse widths, it is seen that the vibration displacement of the beam structure subjected to the control force is damped rapidly. Then, the effects of the location of the electromagnetic damper, the gain of the control circuit, the width of the exciting force and the inductance of
the electromotive linear actuator upon the vibration control efficiency and the shock response of the system are clarified.

(4) The shock experimental results of the vibration response and the control circuit agree well with the numerical ones. Therefore, the validity of theoretical expressions derived here is confirmed.

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