On the Strength of Racks for Jack-up Units
4th Report: Estimation of the Static Strength of the Racks,
by Using Small Scale Specimens, for the Meshings
in the Vicinity of Rack Tooth Tip and Fillet*

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The subject of paper discusses the results of fracture tests for racks of 11.55mm in module (1/7 of that of the full scale rack) and the finite element elastic stress analysis. These results lead to the conclusion that the differential in the ultimate strength between the full scale and 1/7 scale racks is small and that the ultimate strength of the racks is well expressed, for the varying initial meshing points, in terms of the shear strength of the cross-section with which the pinion tooth tip interferes. The authors also elucidate the difference in the pattern of plastic deformation and fracture of the racks resulting from the difference in the initial meshing point.

Key Words : Strength of Materials, Ocean Engineering, Jack-up Unit, Rack, Pinion, Strain, Plastic Deformation, Fracture Behavior, Static Loading Test, Size Effect, Shear Strength

1. Introduction

The previous reports (1)-(3) discussed the fatigue behavior of both torch-cut and machined racks and the results of a static loading test for a full scale rack with a back-plate in service. This report discusses the results of static loading tests for a rack and pinion of the 1/7 scale of the full scale rack and pinion in service. The results of finite element elastic stress analysis and the findings obtained from the comparison with the results of the test for the full scale rack are also discussed.

2. Experimental method

2.1 Material

1/7 scale racks were taken from the full scale rack (2), (3) as shown in Figure 1. It can be seen that the longitudinal direction of the 1/7 scale rack coincides with that of the full scale rack, that five 1/7 scale racks

were taken from the cross-section of the thickness, and that low stresses were generated and no plastic deformation was initiated at the specimen-sample taking location during the static loading test for the full scale rack.

Tables 1 and 2 show the chemical composition and heat treatment condition of the pinion material SNCM630, and its mechanical properties, respectively.

Figure 1 1/7 scale rack specimen sample-taking method

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Table 1 Chemical composition and heat treatment condition of 1/7 scale pinion material

Material	Chemical Composition (%)										Heat Treatment Temperature
	C	Si	Mn	P	S	Cu	Ni	Cr	Mo		
	x100			x1000		x100					
SNCM630	30	29	48	25	20	12	261	288	54	Quenching : 870°C Tempering : 580°C	

Table 2 Mechanical properties of 1/7 scale pinion material

Material	Direction	Yield Strength	Tensile Strength	Elongation	Reduction in Area	Brinell Hardness
SNCM630	Rolling	Yield Point 1080 MPa	1210 MPa	18%	57%	363
	Transverse	0.2% Offset 1070 MPa	1180 MPa	12%	33%	

2.2 Specimen

Table 3 shows the specifications of the 1/7 scale racks and pinions. These racks and pinions are scaled down to 1/7 of the full scale racks and pinions. The face width of the pinion is greater than that of the rack. The racks and pinions were machined by a rack cutter and a formed cutter, respectively. Figure 2 shows the configuration of the 1/7 scale rack. The longitude of approximately 8.5 pitches is long enough to simulate the very long racks in service and three teeth were machined at the center of the longitude. In this experiment, the central tooth was cut away and the lower and upper rack teeth were used for the tests for meshings in the vicinity of the rack tooth tip and the rack tooth fillet, respectively, so that the neighboring tooth did not interfere during the test. The rack

Table 3 Specifications of 1/7 scale racks and pinions

Item	Rack	Pinion
Module	11.55mm	
Pressure Angle	25°	
Number of Teeth	—	7
Reference Pitch Diameter	—	80.85mm
Face Width	18.3mm	—

specimen for the test in the vicinity of the rack tooth tip was taken from the center of the cross-section of the thickness of Rack Tooth B as shown in Figure 1 and that for the test in the vicinity of the rack tooth fillet was taken from the portion nearest to Side (F) of Rack Tooth A. There was no difference in the results of the tension tests resulting from the difference in the specimen sample-taking location; consequently, the difference in the static strength of the rack due to the difference in the specimen-sample taking location is considered to be negligible.

Figure 3 shows the configuration of the 1/7 scale pinion. Two pinion teeth are machined for each specimen and the lower tooth is designed to mesh with the tooth tip of the rack which corresponds to the tip of the full scale rack in the previous reports (2), (3) and the upper tooth is designed to mesh in the vicinity of the rack tooth fillet.

2.3 Test apparatus

Figure 4 shows a schematic diagram of the loading apparatus for the 1/7 scale rack. In the previous reports (2), (3), the rack specimen used

Figure 2 Configuration of 1/7 scale rack specimen

Figure 3 Configuration of 1/7 scale pinion specimen

was of a type with rack teeth on both sides of a cylinder (a chord) as shown in Figure 1; however, the rack specimen with rack teeth on one side only and without a chord was adopted in this study. The part corresponding to the back-plate is machined to 3.57mm thickness and the right end of the 3.57mm thickness corresponds to the center line of the back-plate for the full scale rack. A component with pinion teeth which is to mesh with a rack is inserted into the apparatus and the testing for any meshing condition can be conducted by changing the phase of the pinion teeth. The right hand side and the lower part of the left hand side of the rack are supported by roller slides through the contact plate. The friction coefficient of the roller slide is an extremely small value of 0.003 through 0.004 according to the catalogue. The main body of the apparatus consists of two plates fixed by reamer and ordinary bolts with a bed plate at its bottom. The rack specimen is guided by a slot machined on the main body and the clearance between the rack specimen and the slot width can be adjusted to any given value.

2.4 Experimental method

A multi-purpose static loading test machine, manufactured by Shimazu Manufacturing Company, was used for the testing. A spherical bearing, a load cell, and a plate supporting the load cell were placed between the loading head of the test machine and the upper part of the rack specimen shown in Figure 4. The location, shown by an arrow, was set at the center of the load application. It is therefore considered that the load distribution is uniform across the face width. The loading head was stopped at 1 ton intervals for the range that the rack specimen was in elastic condition except at the contact region with the pinion. The displacement measurement of the loading head by using a dial gage, the photographing of the meshing condition, and the strain measurement for the designated location of the rack and pinion specimens were conducted for each stop position of the loading head. After the rack specimen was in the elastic-plastic range, the loading head was stopped under the displacement control, initially at the interval of 0.3mm and then gradually increased to a final interval of 1mm for the previously mentioned measurements.

The biaxial and uniaxial gages used are SFC-2bL6x6 and N11-FA-1-126-11 of 1mm in gage length, respectively, made by Nihondenki-Sanei Company. The biaxial and uniaxial gages were applied to the most critically strained fillet surface locations of the pinion, determined by Hofer's method,⁽⁴⁾ at the center of the face width, in the vicinity of the tooth end, and midway between the center of the face width and the tooth end. The biaxial gages were also applied to the rack fillet surfaces of approximately 25° in tangential angle

Figure 4 Loading apparatus for 1/7 scale rack specimen

(A) Meshing in the Vicinity of the Rack Tooth Tip (B) Meshing in the Vicinity of the Rack Tooth Fillet

Figure 5 Strain gage locations for 1/7 scale racks

in the vicinity of tooth ends, the side surfaces of the rack tooth, and the back-plate surfaces, as shown in Figure 5.

3. Method of analysis

Elastic stress analysis was conducted for the 1/7 scale rack specimen under plane stress formulation by using MISA,⁽⁵⁾ a large scale finite element structural analysis program. Figures 6 and 7 show the meshing, load, and boundary conditions of the finite element model of the 1/7 scale racks with the loads being applied in the

Constraint (1) : All the specified constraints are effective.
 Constraint (2) : Shaded constraints are removed.

Figure 6 Finite element model of 1/7 scale rack with load applied in the vicinity of tooth tip

Constraint(1) : All the specified constraints are effective.
 Constraint(2) : Shaded constraints are removed.

Figure 7 Finite element model of 1/7 scale rack with load applied in the vicinity of tooth fillet

vicinity of the tooth tip and fillet, respectively. The dimensions shown in these figures indicate 7 times those of the 1/7 scale rack, thus corresponding to the dimensions of the full scale rack. The meshing pattern of the fillets corresponds to that shown in the 2nd Report of this research.⁽²⁾ Young's modulus E and Poisson's ratio ν were set at 2.06×10^5 MPa and 0.3, respectively.

The analysis was first conducted for Constraint (1), as shown in Figures 6 and 7, for the meshings in the vicinity of the rack tooth tip and fillet. The constraints with the tensile reaction force at the bottom of the back-plate, which are shown as shaded triangles in Figures 6 and 7, were then released and the analysis was conducted for this condition which is denoted as Constraint (2).

4. Experimental and analytical results

4.1 Elastic behavior

Figures 8 and 9 show the tangential strain values obtained by using FEM and those obtained by using strain gages, at $\theta = 25^\circ$ for tensile and compressive fillets, respectively. The strain values, which are normalized by Young's modulus E , the jacking load P , the face width b , and module m , were obtained by applying the least square method to the measured strains in the range that all parts of the rack specimen are in the elastic range. The FEM tangential strain values on the fillet surfaces were obtained by using an extrapolation method as stated in the previous report.⁽²⁾ Almost no difference can be seen in the computed strain values between Constraints (1) and (2). The results obtained for these boundary conditions with the load being applied

Figure 8 Relation between tangential angle and normalized strain on the tensile fillet surface of 1/7 scale rack

at the rack tooth tip also agreed with the results of the analysis of the full scale rack.⁽²⁾ It can be seen that the strain gage values are in very good agreement with the FEM values, especially at compressive fillets. With regard to the FEM values only, the most critically strained locations for the load applied in the vicinity of the rack tooth fillet are at $\theta = 62^\circ$ for both tensile and compressive fillets and these locations are on the bottom land side compared with the case for the load applied in the vicinity of the rack tooth tip. This result is considered to be due to the fact that the ratio of the bending of the rim and back-plate to that of the rack tooth increases as the point of load application is shifted toward the tooth fillet.

Table 4 shows the values of k , the ratio of the strain in the face width direction to the tangential strain in the elastic range, for each given case. The values of k for the rack were obtained for the vicinity of the tooth end, and those for the pinion were obtained for the center of the face width, respectively. It can be seen from Table 4 that the value of k is small and positive, which indicates nearly plane strain condition, for the tensile fillet of the rack tooth for the meshing in the vicinity of its fillet. This result may be obtained because the contact area between the rack and pinion is adjacent to the fillet. On the other hand, k values are approximately -0.3 for other fillet locations of the rack and thus indicate plane stress

Figure 9 Relation between tangential angle and normalized strain on the compressive fillet of 1/7 scale rack

conditions. Since the constraint is large at the center of the pinion face width, it can be seen that the plane strain condition prevails there.

Figures 10 and 11 show the measured and FEM values of the stresses σ_x and σ_y in the longitudinal and transverse directions, respectively, of the rack on the lines of the strain gage locations of the back-plate for the meshing in the vicinity of the rack tooth tip. Figures 12 and 13 show those for the meshing in the vicinity of the rack tooth fillet. Measurement was conducted by using strain gages and the strain per unit load was obtained by using the least square method for the range that the rack specimen is in elastic condition. From these strains, the above mentioned stresses were obtained by using the following formulae:

$$\sigma_x = E \cdot (\epsilon_x + \nu \epsilon_y) / (1 - \nu^2), \text{ and}$$

$$\sigma_y = E \cdot (\epsilon_y + \nu \epsilon_x) / (1 - \nu^2).$$

The difference in FEM values between Constraints (1) and (2) is negligibly small and these values almost coincide with those computed for the boundary condition used for the full scale rack. A slight difference can be observed between FEM and strain gage values in Figure 10, while these values coincide with each other in Figures 11 through 13. In any case, the strain distributions in the longitudinal direction of the racks form high gradient curves due to the bending of the rim and back-plate, and this

Table 4 k values at various locations on the fillet surfaces of 1/7 scale racks and pinions

Meshing Location	Rack Tooth Tip		In the Vicinity of Rack Tooth Fillet	
	Pinion	Rack	Pinion	Rack
Face Width Location	Center	Near End	Center	Near End
Tensile Fillet	-	-0.285	-0.088	0.101
Compressive Fillet	-0.079	-0.273	-0.110	-0.354

Figure 10 Stresses σ_x in the longitudinal direction of the back-plate of 1/7 scale rack with load applied in the vicinity of tooth tip

Figure 11 Stresses σ_y in the transverse direction of the back-plate of 1/7 scale rack with load applied in the vicinity of tooth tip

tendency is clearer in the case of the meshing point in the vicinity of the rack tooth tip than in the vicinity of the rack tooth fillet and in the case of σ_y than in the case of σ_x .

These results lead to the conclusion that the elastic stress conditions at the strain gage locations do not vary with the difference in the constraint conditions in the case that the chord does not exist. In addition, if the rack is in the plastic range, the influence of the constraint is considered to decrease due to the greater load component in the downward tooth height direction resulting from the bending and contact plastic deformation of the rack.

4.2 Relation between load and displacement

Figure 14 shows the relation between the displacement of the loading head and the jacking load, both of which are nondimensionalized by module of the rack m , face width of the rack b , and Young's modulus E , for the meshings in the vicinity of the tooth tip and tooth fillet of the 1/7 scale rack, and the meshing in the vicinity of the tooth tip of the full scale rack, respectively. The load-displacement curves for the meshings in the vicinity of the tips of the 1/7 and full scale racks are close

Figure 12 Stresses σ_x in the longitudinal direction of the back-plate of 1/7 scale rack with load applied in the vicinity of tooth fillet

Figure 13 Stresses σ_y in the transverse direction of the back-plate of 1/7 scale rack with load applied in the vicinity of tooth fillet

Figure 14 Relation between nondimensionalized jacking load and stroke displacement

to each other until the jacking load reaches its maximum; therefore, the size effect on the static strength is considered to be small. The relation between the nondimensionalized module $m_n = (m/80.851)$ and the nondimensionalized jacking load P_n (the maximum jacking load for the given value of m_n divided by that for $m_n=1$) can be expressed in the conventional form for the size effect

for the static strength (6) as $P = m_n^{-0.0251}$ by connecting two data points as shown in Figure 15. The size effect is, therefore, concluded to be small for the range of $m_n = 1/7 - 1$ and its vicinity.

It is observed from the relation between the jacking load and the displacement of the loading head for the meshing in the vicinity of the tooth fillet of the 1/7 scale rack that the maximum value of (P/Ebm) is 4.27 and thus is greater than 3.19 and 3.03 which are those for the meshing in the vicinity of tips of the 1/7 scale and full scale racks. It can also be seen that the displacement $(\delta/\pi m)$ for the maximum jacking load is 0.138 and is consequently smaller than the value of approximately 0.4 for the meshings in the vicinity of the rack tooth tip. These features may result from the difference in the deformation pattern of the rack tooth as will be stated in 4.3.

4.3 Plastic deformation and fracture behavior of rack teeth

Figure 16 shows the deformation pattern of the 1/7 scale racks for the meshings in the vicinity of the rack tooth tip and fillet, respectively. The deformation pattern for the former meshing appears to be identical to that for the full scale rack stated in the previous report. (3) It can be seen that the contact region between the pinion and rack expands toward the rack tooth fillet as the rack tooth is plastically deformed and then the rack is sharply bent on the compressive tooth surface. However, contrary to the full scale rack, a crack is not observed in the case of the 1/7 scale rack. This difference is considered due to the fact that the full scale rack is torch-cut and thus has high hardness and brittleness with defects on its surface

Figure 15 Relation between nondimensionalized module and nondimensionalized maximum jacking load

while the 1/7 scale rack is machined and thus has a better surface finish.

With regard to the results of the meshing in the vicinity of the tooth fillet of the 1/7 scale rack, the rack tooth is fractured as if it were cut by the pinion tooth tip instead of the plastic bending deformation which is seen for the meshing in the vicinity of the rack tooth tip. The contact region between the rack and pinion expands from the rack tooth fillet side to the rack tooth tip side and a crack is observed on the tensile tooth surface at Measurement Number 35 contrary to the fact that a crack is initiated on the compressive tooth surface in the case of the testing for the full scale rack subject to the meshing in the vicinity of the rack tooth tip.

From these observations, it is concluded that the plastic deformation pattern of the rack tooth varies considerably depending on the initial meshing condition of the rack and pinion. However, the observation of the rack tooth deformation pattern for the maximum jacking load reveals a common

(A) For the initial meshing in the vicinity of rack tooth tip

Figure 16 Plastic deformation pattern of 1/7 scale rack

(B) For the initial meshing in the vicinity of rack tooth fillet

Figure 16 Plastic deformation pattern of 1/7 scale rack (continued)

phenomenon that the rack tooth mainly suffers shearing deformation by the pinion tooth; consequently, the ultimate strength of the rack tooth is deduced to be well expressed by the shear strength against the pinion tooth tip. Division of the maximum jacking load by the initial cross-sectional area for the location defined by Line of Interference, as shown in Figure 16, results in the almost constant values of the shear strength τ_B of 410MPa, 420MPa, and 430MPa for the meshings in the vicinity of the full scale rack tooth tip, the 1/7 scale rack tooth tip, and the 1/7 scale rack tooth fillet, respectively. The ratio of τ_B to the tensile strength σ_B , therefore, ranges from 58% to 61% and is in excellent agreement with the ratio of the yield stress for the pure shear to that for the tension (τ_N/σ_N) = 57.7% determined by the criterion of von Mises. (7)

5. Conclusions

The findings obtained may be summarized as follows:

- (1) The finite element elastic stress analysis resulted in the conclusion that the stresses (strains) at the center of the back-plate and on the rack tooth fillet surface are not significantly influenced by the difference in constraint shown in this report. The finite element stress values and the measured stress values by using strain gages are found to be relatively in good agreement except for the limited cases.
- (2) In the elastic region, the rack tooth fillet stress condition in the vicinity of the tooth end was found to be subject to plane stress except at the tensile fillet of the rack tooth for the meshing in the vicinity of the rack tooth fillet, which is adjacent to the contact area between the rack and pinion teeth. The stress condition of the pinion fillet at the center of the face width was found to be subject to a nearly plane strain state due to the large constraint in the face width direction.
- (3) The jacking load - displacement curves, which are nondimensionalized by module m , Young's modulus E , and the face width b , for the meshings in the vicinity of tips of the 1/7 and full scale racks were found to be in excellent agreement until the jacking load reached its maximum. The maximum jacking load for the meshing in the vicinity of the rack tooth fillet was found to be greater than that for the meshing in the vicinity of the rack tooth tip and the displacement for the maximum jacking load was smaller in the former case than in the latter case.
- (4) The ratio of the ultimate strength for the full scale rack to that for the 1/7 scale rack (size effect) was found to be small and the relation between the nondimensionalized ultimate strength, P_n , and the nondimensionalized module, m_n , was

found to be expressed as $P_n = -0.0251$.

- (5) The ultimate strength of the rack tooth was found to be expressed by the shear strength τ_B of the cross-section which is determined by Line of Interference as defined in Figure 16. It was also shown that the ratio of τ_B to σ_B was in good agreement with the ratio of the yield stress for the pure shear to that for the tension (τ_y/σ_y) determined by the criterion of von Mises.
- (6) The deformation patterns for the meshings in the vicinity of tooth tips of the 1/7 and full scale racks were found to be similar to each other; however, cracks were not initiated in the case of the 1/7 scale rack, due presumably to difference in the surface finish.
- (7) The contact area between the rack and pinion expanded from the rack tooth fillet side to the rack tooth tip side for the meshing in the vicinity of the 1/7 scale rack tooth fillet and a crack was initiated at the location on the tensile rack tooth surface which was adjacent to the pinion tooth tip. This is considered to be due to the high stiffness of the rack tooth surface region with which the pinion tooth tip interferes.

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