Pipe Flow with Viscosity Gradient and Frictional Resistance*

(1st Report, Flow of a High Viscosity

Fluid in a Heated or Cooled Pipe)

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The flow of a highly viscous fluid in a pipe, of which the wall is heated or cooled at a constant temperature, is studied theoretically and experimentally in order to determine the frictional resistance of the flow with a viscosity gradient. Measurements are performed for the Reynolds numbers $20 < Re < 850$, the Prandtl numbers $500 < Pr < 2400$ and the temperature difference between the fluid and the wall $-40^\circ < \Delta T < 40^\circ$.

The results show that the theoretical analysis, assuming an axisymmetric flow and taking into account the local viscosity gradient, agrees well with the measurements when the buoyancy effect is not large. Even though the buoyancy effect is large, the pressure drop in the flow direction is little influenced by buoyancy and is well predictable by the theory. The pressure drop increases almost linearly in the measured range and the frictional coefficient is expressed by a simple empirical equation. The velocity measurements by Pitot probes in the low Reynolds number range are carefully examined, and the influence of temperature upon the performance of Pitot tubes is also determined.

Key Words : Pipe Flow, Friction Resistance, High Viscosity Fluid, Temperature Gradient, Buoyancy

1. Introduction

The viscosity of a highly viscous fluid is largely dependent upon the fluid temperature. During transportation of such a fluid in a long pipe-line or in a heat exchanger a frictional resistance and a sectional velocity distribution are considered to be much different from those of other fluids, because the viscosity near the wall changes much due to a large temperature gradient depending upon a wall temperature. This is also the case with the flow of melted metal in a molding process.

The flow with a local viscosity gradient has been treated so far in order to determine heat transfer characteristics⁽¹⁾, but the fluid dynamical characteristics such as the change of a sectional velocity profile, pressure drop and a frictional resistance due to a viscosity gradient have not yet been known. It is still difficult to predict the pressure drop and the frictional resistance.

The present study is aimed to determine the frictional resistance of a flow with viscosity gradient, and the flow of a highly viscous fluid in a pipe, of which the wall is heated or cooled at a constant temperature, is studied theoretically and

experimentally. In the first report, a fully developed pipe flow with a rather thin thermal boundary layer (thermal intake region) is studied. The case of constant heat flux is also treated.

2. Theoretical Analysis

The flow of a highly viscous fluid becomes generally a laminar flow and here a fully developed laminar flow in a horizontal circular pipe is treated. The variation of sectional velocity and temperature distributions in the flow direction (x -direction) is analyzed when the wall is heated or cooled stepwise from the temperature T_0 (inlet fluid temperature) to T_w at $x=0$ as shown in Fig. 1 assuming an incompressible axisymmetric flow. When the temperature difference is large between T_0 and T_w , a sectional velocity profile might become non-axisymmetric due to buoyancy effect. But as the main purpose of the present analysis is to determine the in-

Non-dimensional radius: $\eta = r/R$
 Non-dimensional axial distance: $\xi = x/R$
 Pipe diameter: $d=2R$

Fig. 1 Flow model and notations.

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fluence of a viscosity gradient, an axisymmetric flow is assumed in the theory and the buoyancy effect is determined experimentally.

The number of the unknown parameters is four, that is the velocity components (u, v) in x - and r -directions, fluid temperature T and pressure p , and they are determined by the equations of continuity, motion in x - and r -directions and energy balance. The viscosity gradient is to be treated as a function of x and r in the constitutive equations of stress components. The other physical properties such as fluid density ρ , specific heat c_p , thermal conductivity k are treated as constants, because the dependence of these properties upon temperature is much smaller than that of viscosity upon temperature ⁽²⁾.

Assuming that the length of the heated or cooled region is sufficiently large compared with the pipe radius R , and the r -directional gradient $\partial/\partial r$ of flow characteristics is sufficiently steep compared with the x -directional gradient $\partial/\partial x$, the basic equations are simplified as follows using a similar approximation as that used in the boundary layer analysis ⁽³⁾:

$$\frac{\partial V}{\partial \eta} + \frac{V}{\eta} + \frac{\partial U}{\partial \xi} = 0 \quad \dots\dots\dots (1)$$

$$U \frac{\partial U}{\partial \xi} + V \frac{\partial U}{\partial \eta} = -\frac{1}{2} \frac{\partial P}{\partial \xi} + \frac{2}{Re_0} \left(\frac{\partial \mu'}{\partial \eta} \frac{\partial U}{\partial \eta} + \mu' \frac{\partial^2 U}{\partial \eta^2} + \frac{\mu'}{\eta} \frac{\partial U}{\partial \eta} \right) \quad \dots\dots\dots (2)$$

$$\frac{\partial P}{\partial \eta} = 0 \quad \dots\dots\dots (3)$$

$$V \frac{\partial \Theta}{\partial \eta} + U \frac{\partial \Theta}{\partial \xi} = \frac{2}{Re_0 Pr_0} \left(\frac{\partial^2 \Theta}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial \Theta}{\partial \eta} \right) + \mu' \Phi \quad \dots\dots\dots (4)$$

where the suffix 0 denotes the value at $x=0$, and

$$\left. \begin{aligned} \xi &= x/R, \eta = r/R \\ U &= u/u_m, V = v/u_m \\ P &= p/(\rho u_m^2/2) \\ \Theta &= (T - T_w)/(T_0 - T_w) \\ \mu' &= \mu/\mu_0 \\ Re_0 &= u_m d \rho / \mu_0, Pr_0 = \mu_0 c_p / k \end{aligned} \right\} \quad \dots\dots\dots (5)$$

where u_m is a mean velocity of the section. Φ is a dissipation function due to fluid viscosity, and is approximated to

$$\Phi = \frac{2\mu_0 u_m}{\rho c_p d (T_0 - T_w)} \left(\frac{\partial U}{\partial \eta} \right)^2 \quad \dots\dots\dots (6)$$

Equations from (1) to (4) can be numerically solved by an implicit differential method, when the boundary condition is known and the viscosity ratio μ' is expressed as a function of temperature. The boundary condition is given by the parabolic velocity profile of the Poiseuille flow at $x=0$. The fluid viscosity is determined from the measurements and for the other physical properties of fluid the following values are available ⁽²⁾.

$$\begin{aligned} \rho &= 873 \text{ kg/m}^3, c_p = 1.8 \text{ kJ/(kg}\cdot\text{K)} \\ k &= 0.13 \text{ W/(m}\cdot\text{K)} \quad \dots\dots\dots (7) \end{aligned}$$

3. Experimental Apparatus and Procedures

Figure 2 shows the dependence of the kinetic viscosity $\nu = \mu/\rho$ of turbine oil used in the present test upon temperature. It is seen that the kinetic viscosity changes from $300 \times 10^{-6} \text{ m}^2/\text{s}$ at $T=10^\circ\text{C}$ to $20 \times 10^{-6} \text{ m}^2/\text{s}$ at $T=70^\circ\text{C}$.

The test apparatus consists of a horizontally arranged circular pipe circuit of $d=44.5 \text{ mm}$ in diameter and $L=7.5 \text{ m}$ in straight length, two head tanks of 1 m^3 in capacity and 2.6 m in height difference, a circulating pump, heating or cooling equipment and a heat exchanging apparatus. The length of the heated or cooled region is 100 diameters and four velocity measuring apparatus are mounted at $x/d = -11.2, 11.2, 44.8$ and 78.7 . Before the measurements of the heated or cooled region, the inlet velocity profile at $x=0$ was confirmed to agree with that of the Poiseuille flow. During the measurements the temperatures of the wall T_w and the inlet fluid T_0 were kept constant, and the velocity and temperature distributions in one section were measured simultaneously by a Pitot probe coupled with a thermistor. The difference in temperature $\Delta T = T_w - T_0$ between the wall and the fluid was varied in the range of $(-40 \sim 40)^\circ\text{C}$ and the mean velocity u_m in the range $(0.1 \sim 0.6) \text{ m/s}$. The corresponding Reynolds number and Prandtl number ranges are $20 < Re_0 < 850, 500 < Pr_0 < 2400$.

4. Results and Discussions

4.1 Velocity measurements by Pitot probes in the low Reynolds number range

In a low velocity flow it is generally difficult to use Pitot probes, because the velocity factor of a probe varies with the Reynolds number. In the present measurements the velocity factor K_p , defined as the square root of the ratio of the real dynamic pressure to the indicated one by a Pitot probe, was carefully examined in

Fig. 2 Kinematic viscosity of tested oil.

Fig. 3 Velocity factor of a Pitot probe.

order to obtain enough accuracy of the velocity measurements. For many combinations of mean velocity and fluid temperature the following three steps of measurements were performed. In the first step the relation between the flow-rate and the pressure drop in the x-direction was confirmed to satisfy that of the Poiseuille flow, and in the second step the velocity factor was determined from the indicated dynamic pressure by Pitot probes set in the pipe centre and the Poiseuille flow velocity calculated from both the measured flow-rate and the pressure drop in the x-direction. In the last step this velocity factor was confirmed to agree with the ones obtained from the measured dynamic pressure by the Pitot probes set at different radial positions.

Figure 3 shows the dependence of the velocity factor of a Pitot probe upon the Reynolds number Re' defined using the real velocity and the probe diameter. It is seen that the velocity factor is expressed as a function of Re' alone and independent of fluid temperature, which means that it takes different values in the case of the same velocity but different temperatures. In Fig. 3 the scatter of the data becomes larger for higher temperature, which might be because of a natural convection induced

Fig. 4 Comparison of velocity profiles measured and re-calculated.

by the difference in temperature between the fluid and the atmosphere.

In Fig. 4 the velocity data indicated by a Pitot probe is compared with the re-calculated velocity from Fig. 3. It is seen that the indicated values give much larger velocity than the re-calculated ones do and that the latter agrees well with the Poiseuille flow velocity profile. When the wall is heated or cooled, the error in the flow-rate obtained by integrating the re-calculated velocity distribution was within $\pm 4\%$ of the directly measured flow-rate as long as the buoyancy effect was not remarkable.

4.2 Velocity and temperature distributions and buoyancy effect

Figures 5 and 6 show the variation of sectional velocity and temperature distributions and comparison of the theory with the measurements when the wall is heated or cooled. When the wall is heated to a constant temperature, the viscosity of the fluid and the shearing stress near the wall decreases largely and the flow is accelerated, while in the central region the flow

(a) $Re_0=280$, $\Delta T=26.4^\circ$ (b) $Re_0=203$, $\Delta T=39.5^\circ$

Fig. 5 Velocity and temperature distribution in the case that the pipe wall is heated.

Fig. 6 Velocity and temperature distribution in the case that the pipe wall is cooled.

is decelerated to satisfy the continuity equation as shown in Fig.5(a). The theoretical results are seen to give satisfactory agreement with the measurements. But with an increase in temperature difference the buoyancy effect appears, the sectional distribution becomes asymmetrical in the vertical direction and the accuracy of the present theory becomes less satisfactory as shown in Fig.5(b), in which $r/R=+1$ corresponds to the lower wall and -1 to the upper wall. Near the starting point of the heating range the development of a thermal boundary layer is rapid and the change of the velocity profile is also rapid in the range of $x/d \leq 11.2$.

When the wall is cooled, the flow is decelerated near the wall and accelerated in the centre and the velocity profile becomes sharp as shown in Fig. 6(a). When the buoyancy effect appears, the point of peak velocity comes to deviate from the pipe centre toward the upper side. The present theory gives good results for the case of $x/d=78.7$, but near the starting point ($x/d=11.2$) the agreement is not good even in the case that the buoyancy effect is small (Fig.6(a)). This is because of

the difficulty in making a perfect cooling in the experiment to a constant temperature near the starting point.

In order to examine the buoyancy effect, the development of the thermal boundary layer along the pipe axis is shown in Fig.7. The thickness δ_u along the upper wall increases rapidly in the x-direction and its growth becomes more remarkable for a smaller inlet Reynolds number Re_0 and a lower initial temperature T_0 , while the thickness δ_l along the lower wall is almost constant and independent of Re_0 and T_0 . The Grashof number $Gr = gd^3\beta\Delta T/\nu^2$, where β is a thermal expansion coefficient, is usually used to represent the buoyancy effect in a free convection flow. In the present case the flow pattern is determined mainly from the balance between inertia force and viscous force, and the ratio of buoyancy force to inertia force Gr/Re^2 is also considered an important parameter next to the Reynolds number in the case of a large temperature difference ΔT . Adopting the thickness ratio δ_u/δ_l of the upper to the lower thermal boundary layer as the representative parameter of the asymmetry of velocity profile, and plotting the thickness ratio against Gr_0/Re_0^2 gives Fig.8, in which the negative value of the Grashof number corresponds to the cooling ($\Delta T < 0$). It is

Fig. 7 Thickness of thermal boundary layer in the case of heating ($\Delta T=40^\circ$).

Fig. 8 Thickness ratio of thermal boundary layer measured at $x/d=78.7$.

Fig. 9 Pressure drop along the pipe axis and comparison with theory.

recognized that the thickness ratio $\delta u/\delta l$ varies largely in the range of $|Gr_0/Re_0^2| < 0.05$, but varies little for the variation of Gr_0/Re_0^2 in the range of $|Gr_0/Re_0^2| > 0.1$, which indicates that the buoyancy effect is remarkable and is almost saturated.

4.3 Pressure drop in x-direction and frictional resistance

The above-described change in velocity profile makes the pressure variation along the pipe axis much different from that of the steady Poiseuille flow. Figure 9 shows the pressure variation in the x-direction and compares it with the theory for the same condition as in Figs. 5 and 6. Here it is to be noted in Fig. 9 that the pressure drop in the x-direction is still linear in the case of heating and cooling, and also that the theory gives satisfactory results even in the case of $|Gr_0/Re_0^2| > 0.1$, in which the buoyancy effect is remarkable and the velocity profile is largely asymmetrical. But in the case of cooling the agreement between the theory and the measurements is less satisfactory due to the difficulty in cooling the starting region

Fig. 10 Mean friction coefficient and influence of temperature.

in the experiment as is already discussed.

The velocity head of the Poiseuille flow mass-averaged in the section is expressed as $2(u_m^2/2g)$ using the modification constant 2 due to the non-uniformity of the velocity profile. When the velocity profile varies along the pipe axis, the mass-averaged velocity head also changes, which results in energy exchange between velocity head and pressure head. In the case of heating, a part of velocity head is changed to pressure head, and in the case of cooling a part of the latter to the former. Such a change in the velocity profile is caused by the momentum transfer effect of the radial velocity V , of which magnitude is about 1/1000 of the axial velocity U according to the theory.

The mean friction coefficient $\lambda \equiv (\Delta p/\gamma)(1u_m^2/2gd)$ calculated from the measured pressure drop at $x/d=78.7$ is shown in Fig. 10. The friction coefficient in the case of heating or cooling varies nearly parallel with that of the Poiseuille flow, $\lambda=64/Re$, for the variation of the upstream Reynolds number Re_0 . This results in that the product $Re_0\lambda$ is expressed only as a function of temperature difference ΔT .

The influence of temperature difference ΔT appears as a change in the fluid viscosity ν , and then the ratio of the Reynolds number Re_0 based on the upstream temperature T_0 to Re_w based on the wall temperature T_w is considered a representative parameter of the temperature difference. The variation of $Re_0\lambda$ is shown in Fig. 11 for the variation of the Reynolds number ratio. From this figure the following empirical equation is obtained as the mean friction coefficient in the range $0 \leq x/d \leq 78.7$.

$$\lambda \cdot Re_0 = 64 + 28 \log(Re_0/Re_w) \quad (8)$$

This equation gives a mean frictional resistance for the case of a relatively short pipe length and a rather thin thermal boundary layer. In the case of a sufficiently long pipe length or large temperature difference, a thermal boundary layer develops to occupy the whole section of the pipe and the temperature difference between the wall and the fluid approaches zero,

Fig. 11 $Re_0 \lambda$ vs. Re_0/Re_w .

Fig. 12 Non-dimensional pressure drop along the pipe axis and comparison of Eq.(8) with the theoretical results.

which yields again a Poiseuille flow and the local friction coefficient approaches $64/Re_w$ using the viscosity at $T=T_w$. In such cases the friction coefficient should be expressed as a function of Re_0/Re_w and x/d , and this will be discussed in the proceeding report.

The measurements were performed also in the case of constant heat flux. In this case the input heat is relatively small and the temperature rise of the wall is also small near the starting point of the heating region, but at $x/d=78.7$ the wall temperature reaches almost a constant and the product of Re and λ agrees well with the results in Fig.11. It is then concluded that equation (8) is still valid for the case of constant heat flux.

Lastly, in order to examine the validity of the empirical equation (8), comparison with the theoretical results is made in Fig.12 for the case of $Re_0=120$ and $\Delta T=40^\circ$. The theoretical results are expressed as a curve with a small gradient but the agreement with the empirical equation (8) is good in the range $0 < x/d < 200$.

5. Conclusions

(1) The velocity and the temperature distributions in the case that the wall is heated or cooled at a constant temperature are well predicted by the theory assuming an axisymmetric flow, when the buoyancy effect is rather small. But even when the buoyancy effect becomes remarkable and the velocity profile becomes largely asymmetrical, a friction coefficient and pressure

drop along the pipe axis can be well predicted by this theory.

(2) In the range tested ($20 < Re_0 < 850$, $500 < Pr_0 < 2400$, $-40^\circ < \Delta T < 40^\circ$), the pressure decreases almost linearly in the flow direction and the mean friction coefficient is given by Eq.(8).

(3) The buoyancy effect is represented by the parameter Gr_0/Re_0^2 and if $|Gr_0/Re_0^2|$ is larger than 0.1, the buoyancy effect becomes remarkable making the velocity and the temperature profiles in the section largely asymmetrical, but its influence upon the frictional resistance is not large.

(4) The influence of temperature on Pitot probe performances is determined experimentally. The velocity factor of a Pitot probe becomes a function of the Reynolds number alone.

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