Studies on Circular Cascades for Return Channels
of Centrifugal Turbomachinery
(1st Report, Inverse Method and Cascade Design)*

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An inverse method to design a circular cascade for a return channel of a centrifugal turbomachine, whose vane-height varies in the radial direction, was developed using the singularity method. As the vane with a round trailing edge was treated in this study, a reasonable position of the rear stagnation point was decided experimentally when a secondary flow occurs considerably on the end walls and a small separation appears on the vane surface. A desirable circular cascade for the return channel, which has a reasonable pressure distribution on the vane surface and satisfies the radial outflow condition, was designed by the developed inverse method in this report.

Key Words : Fluid Machine Element, Return Channel, Cascade, Centrifugal Turbomachine, Inverse Method, Singularity Method

1. Introduction

To improve the performances of multi-stage centrifugal turbomachines, many studies on impellers and diffusers have been performed actively⁽¹⁾. On the contrary, studies on return channel which leads the flow from the diffuser outlet to the next-stage impeller are limited as follows. The flows and performances of the channel with a rectangular cross-section composed of thin return vanes with constant thickness⁽²⁾⁻⁽⁴⁾ and of the channels without vanes⁽⁵⁾⁽⁶⁾ have been investigated. The flow at the inlet section of the return channel has a considerable whirl velocity component in general. Accordingly, circular cascade is necessary to give a large deflection to the through-flow, so as to get the radial outlet flow. Moreover, the vane thickness of the cascade installed in the channel of a high-pressure large-sized turbomachine becomes thick. The flow through such a cascade induces intense secondary flows on the end walls and flow separations on the vane surfaces. They complicate the internal flow pattern and increase the flow loss. Such problems of circular cascades, however, have not been investigated yet.

This serial study intended to make clear the relation between the flow pattern and performance of the circular cascades for the return channel of a large-sized centrifugal turbomachine with

high pressure ratio. In the 1st Report, the cascade design based on an inverse method was presented. Here, the inverse methods applied to a single aerofoil, straight cascades and circular cascades composed of thin vanes have already been reported⁽⁷⁾⁻⁽¹⁵⁾. An analytical method for the circular cascades composed of thick vanes with large camber, whose height varies in the radial direction, however, has not been established yet.

Nomenclature

- r : radial coordinate (Fig.1).
- θ : counter-clockwise angular coordinate (Fig.1).
- θ^* : clockwise angular coordinate (Fig.1).
- z : distance from the lower end wall (see the 2nd Report).
- b : channel width or vane height.
- v : absolute velocity (Fig.1).
- v_r : velocity component in the radial direction (Fig.1).
- v_θ : velocity component in the counter-clockwise angular direction (Fig.1).
- φ : flow angle measured counter-clockwise from the radial direction (Fig.1).
- β : flow angle $=\varphi-\pi$ (Fig.1).
- i_n : incidence angle decided by the flow angle at the inlet datum section.
- l : length measured on the vane surface from the trailing edge (Fig.1).
- L : total length of vane surface (pressure and suction surfaces).
- p : pressure on the vane surface.
- C_p : pressure coefficient $=(p-p_1)/(1/2\rho\bar{v}_1^2)$.
- ρ : density of air.

Subscripts

- 0 : value at the inlet datum section ($r=390\text{mm}$).
- 1 : value at the cascade inlet section ($r=345.75\text{mm}$, estimated from values at the inlet datum section on the assumption of ideal flow).

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- : mean value weighted by flow rate.

2. Inverse Method

2.1 Singularity method

At first, the pressure distributions on the vane surfaces predicted by the singularity method⁽⁶⁾⁽⁷⁾ considering the variation of the vane-height were compared with experimental ones for the cascades described in Chapter 3. These comparisons show that both distributions coincide pretty well except for the region of separation, in spite of an existence of considerable secondary flow. Therefore, this singularity method was applied to the inverse method.

The channel-width (vane-height) variation can be given nearly by Eq.(1), when its variation is comparatively small in the radial direction.

$$\left. \begin{aligned} b &= (c^* \ln r + c^{**})^{-2} \\ c^* &= \frac{\sqrt{b_1^*} - \sqrt{b_2^*}}{\sqrt{b_1^* b_2^*} \ln(r_1^*/r_2^*)} \\ c^{**} &= \frac{\sqrt{b_1^*} \ln r_1^* - \sqrt{b_2^*} \ln r_2^*}{\sqrt{b_1^* b_2^*} \ln(r_1^*/r_2^*)} \end{aligned} \right\} \dots\dots\dots (1)$$

where, b_1^* and b_2^* are the widths upstream ($r_1^* \geq r_1$) and downstream ($r_2^* \leq r_2$) of the cascade, respectively. Then, the flow on the physical plane considering the width variation (Z'_p -plane, Fig.1) can be predicted by analyzing the flow on the physical plane for the constant width (Z_{pc} -plane), provided the flow pattern does not vary in the width direction. Both flows are in the following relations.

$$v_r = \frac{v_{rc}}{\sqrt{b}}, \quad v_\theta = \frac{v_{\theta c}}{\sqrt{b}} + \frac{c^*}{r} \psi_c \quad \dots\dots\dots (2)$$

where, v_{rc} and $v_{\theta c}$ are the velocity components in the radial and the angular direction and ψ_c is the stream function, on the Z_{pc} -plane. The flow on the Z_{pc} -plane can be induced easily from the flow on the Z_{pc} -plane analyzed by the singularity method that the vortices $\Delta\Gamma_j$ ($j=1, 2, \dots, n$; total number of the vortices) are located on the short segments of the vane contour in the straight cascade on the Z_{pc} -plane transformed conformally by Eq.(3).

$$Z_{pc} = \frac{N}{2\pi} \ln \left(\frac{Z'_p}{r_1} \right) \quad \dots\dots\dots (3)$$

where, Z_{pc} -plane is the mapping plane for the constant width and N is number of vanes in the circular cascade. The flow on the mapping plane considering the width variation (Z_p -plane, Fig.2) can be also obtained easily from one on the Z_{pc} -plane, by using Eq.(2) multiplied by $2\pi r/b$.

Consequently, the velocities at the calculation point $S(x, y)$ on the vane surface in the Z_p -plane, which are obtained from addition of the cascade inlet flow and the flow induced from the vortices located on $S_j(\xi_j, \eta_j)$, are

$$\begin{aligned} u_{s(s)} &= \frac{2\pi r_0}{N} U_0 \left\{ \frac{b_0}{b} \cos \varphi_0 \cos \alpha(s) \right. \\ &\quad \left. + \sin \varphi_0 \sin \alpha(s) \right\} + \frac{1}{2\sqrt{b}} \frac{\Delta\Gamma}{\Delta s} \\ &\quad + \sum_{j=1}^n \left[\frac{1}{\sqrt{b}} \left\{ f_{rs}(s, s_j) - \frac{1}{2} \sin \alpha(s) \right\} \right] \end{aligned}$$

Fig.1 Coordinates and symbols on the physical plane (Z'_p -plane) considering the channel-width variation

Fig.2 Coordinates of the vortex and calculation point on the mapping plane (Z_p -plane) considering the channel-width variation

$$+ \frac{2\pi}{N} c^* \sin \alpha(s) \cdot g_r(s, s_j) \Delta\Gamma_j \quad \dots\dots\dots (4)$$

$$\begin{aligned} u_{n(s)} &= \frac{2\pi r_0}{N} U_0 \left\{ \frac{b_0}{b} \cos \varphi_0 \sin \alpha(s) \right. \\ &\quad \left. - \sin \varphi_0 \cos \alpha(s) \right\} \\ &\quad + \sum_{j=1}^n \left[\frac{1}{\sqrt{b}} \left\{ f_{rn}(s, s_j) + \frac{1}{2} \cos \alpha(s) \right\} \right] \\ &\quad - \frac{2\pi}{N} c^* \cos \alpha(s) \cdot g_r(s, s_j) \Delta\Gamma_j \quad \dots\dots\dots (5) \end{aligned}$$

where, the main symbols are explained in Figs.1 and 2. The velocities $u_{s(s)}$ and $u_{n(s)}$ are the components in the σ -direction and the vertical direction to σ , $\alpha(s)$ is the angle of the tangent to the vane contour at $S(x, y)$ measured from the σ -direction. The vortex is located at the midpoint on the segment of the vane contour. The functions included in the above equations $f_{rs}(s, s_j)$, $f_{rn}(s, s_j)$ and $g_r(s, s_j)$ are given by

$$\begin{aligned} f_{rs}(s, s_j) &= V_{rx} \cos \alpha(s) + V_{ry} \sin \alpha(s), \quad f_{rn}(s, s_j) \\ &= V_{rx} \sin \alpha(s) - V_{ry} \cos \alpha(s) \\ V_{rx} &= -0.5 \sin \{2\pi(y - \eta_j)\} / [\cosh \{2\pi(x - \xi_j)\} \\ &\quad - \cos \{2\pi(y - \eta_j)\}] \\ V_{ry} &= 0.5 \sinh \{2\pi(x - \xi_j)\} / [\cosh \{2\pi(x - \xi_j)\} \\ &\quad - \cos \{2\pi(y - \eta_j)\}] \\ g_r(s, s_j) &= -\frac{1}{4\pi} \ln \frac{\cosh \{2\pi(x - \xi_j)\} - \cos \{2\pi(y - \eta_j)\}}{2\pi^2} \quad \dots\dots\dots (6) \end{aligned}$$

On the Z'_p -plane (Fig.1), the velocity components in the l -direction and in the vertical direction to l are given by

$$v_{s(s)} = \frac{N}{2\pi r} u_{s(s)}, \quad v_{n(s)} = \frac{N}{2\pi r} u_{n(s)} \quad \dots\dots\dots (7)$$

and the velocity components in the radial and angular directions around the cascade are given by

$$\left. \begin{aligned} v_r &= \frac{r_0 b_0}{r b} v_0 \cos \varphi_0 + \frac{1}{\sqrt{b}} \frac{N}{2\pi r} \sum_{j=1}^n V_{rz} \Delta \Gamma_j \\ v_\theta &= \frac{r_0}{r} v_0 \sin \varphi_0 + \frac{1}{r} \sum_{j=1}^n \left\{ \frac{N}{2\pi \sqrt{b}} (V_{r\theta} - \frac{1}{2}) + c^* \cdot g_r(s, s'_j) \right\} \Delta \Gamma_j \end{aligned} \right\} \quad (8)$$

In the inverse method, the strengths of the vortices which show the given velocity distributions are calculated by Eq. (4), and the vane profile which gives $u_{n(s)}=0$ is found by substituting those into Eq.(5), as described in 2.2.

2.2 Analytical method

In the inverse method, the velocity distribution is given on the physical plane (Z_p -plane) and the analysis is performed on the mapping plane (Z'_p -plane). In the analysis, the position of the rear stagnation point of the vane and the x_l -coordinate giving the leading edge position (corresponding to the inlet setting radius of the cascade on the Z'_p -plane) are fixed beforehand, to get one solution. As the leading edge position is movable freely in the y -direction (corresponding to the angular deflection angle), however, the necessary deflection angle and the desirable vane profile may be derived. Besides, the roundness of the leading and trailing edges is also given beforehand.

Now, it is assumed that the velocity distribution is given as a function of non-dimensional length l_{ms}/L_s on the suction side and l_{mp}/L_p on the pressure side, where, l_m and L_s are the local and total lengths measured from the rear stagnation point on the vane contour (subscripts s and p denote the values for suction side and pressure side). At first, assuming an arbitrary vane contour (Contour 0) on the Z_p -plane, the strength of the vortex distributed on its contour is derived by substituting the given velocity into Eqs.(4) and (7). The vertical velocity component $u_{n(s)}$ is not equal to zero, as an arbitrary vane contour is assumed, and its component can be determined by substituting the derived vortices into Eq.(5). In this procedure, the contour assumed on the Z_p -plane must be transformed into the Z'_p -plane, to get the correspondence of the contour length on the Z_p -plane to l_{ms}/L_s and l_{mp}/L_p on the Z'_p -plane.

As the vane contour must be one of the stream lines (i.e. $u_{n(s)}=0$), Contour 0 is modified by turn in the σ -direction from the rear stagnation point so as to make the tangent of the new vane contour coincide with the direction of the resultant velocity $u_{(s)}$ (Fig.3). That is, the j -th calculation point on Contour 0 is moved by Δh_j obtained from Eq.(9) vertically against its contour. Here, the positive value means to move in the outward direction of

Fig.3 Modification of the vane contour on the mapping plane (Z_p -plane)

the contour.

$$\Delta h_j^* = \frac{\sum_{k=2}^j \Delta \sigma_k \left\{ \frac{u_n(s_k)}{u_{s(s_k)}} + \frac{u_n(s_{k-1})}{u_{s(s_{k-1})}} \right\}}{2} \quad \dots\dots\dots (9)$$

where, $\Delta \sigma_k$ is the segment length between $k-1$ -th and k -th calculation points on Contour 0. Then, new coordinates of the modified point are written as follows.

$$x_j^* = x_j + \Delta h_j^* \sin \alpha_{(s_j)}, \quad y_j^* = y_j - \Delta h_j^* \cos \alpha_{(s_j)} \quad \dots (10)$$

The modified n -th calculation point $S_n^*(x_n^*, y_n^*)$, however, does not coincide with the fixed coordinates $S_n(x_n, y_n)$, because the numerical error accumulates in proportion to increase of j . Thus, the correction of its error is carried out in proportion to the length measured from the rear stagnation point on Contour 0. For the x -coordinate, the j -th coordinate x_j^* is remodified by the relation of $x_j^{**} = x_j^* + (x_n - x_n^*) \sigma_j / (\sigma_n - \Delta \sigma_1 / 2)$ ($2 \leq j \leq n$), where, σ_j and σ_n are the local length from the rear stagnation point to the j -th calculation point and the total length of the Contour 0. The symbol $\Delta \sigma_1$ denotes the length measured from the n -th to 1st calculation point including the rear stagnation point. The y -coordinate is also remodified by the same procedure ($y_j^{**} \rightarrow y_j^*$).

The coordinate x_j^{**} is moved still farther to the coordinate given by the relation of $x_j^{***} = \xi_1 + (x_l - \xi_1) / (x_j^{**} - \xi_1) \cdot (x_j^{**} - \xi_1)$, when the x_j^{**} -coordinate giving the leading edge position does not coincide with the fixed x_l -value. The first approximate vane contour is formed by joining smoothly the coordinates $S_j^{***}(x_j^{***}, y_j^{**})$.

The vane profile having the given velocity distribution is designed successfully, when all the calculation points do not move even though the above procedures are repeated.

2.3 Numerical example

Validity of the developed inverse method is checked in this paragraph. The full-line and the dash-line in Fig.4 show the pressure coefficient C_p distributions on the suction and pressure surfaces of Aerofoil B shown in Fig.6. These values were calculated directly by the singularity method presented in 2.1. The vane profile was derived from the velocity distributions given from these C_p values. The deriving process of the vane contour on the Z_p -plane is shown in Fig.5. The final contour (mark $\bullet$) coincides exactly with Aerofoil B (full-line). Accordingly, the

Fig.7 Flow pattern downstream of cascade (Section IV, Aerofoil A, $i_n = -0.8^\circ$, $r/r_0 = 0.41$)

the rear stagnation point nearly equal to the flow angle β . Such relation is shown by the dash-line in this figure. This method seems to be applicable to a circular cascade having a large deflection angle, such as one accompanied with an intense secondary flow and a little flow separation.

When the cascade for the return channel is designed by the above presented inverse method, the velocity distribution ought to be given such that the calculated outlet flow angle β becomes zero (radially flow out) when $\epsilon = 0^\circ$.

4. Cascade Design

4.1 Design conditions

The same design conditions as the preliminary experiments were applied, to facilitate comparison with the performances for Aerofoils A and B. That is, the inlet of the cascade is at $r_1 = 345.75\text{mm}$ and the outlet is at $r_2 = 202.5\text{mm}$. The vane height is varied straight from $b_1 = 20.4\text{mm}$ at r_1 to $b_2 = 28.18\text{mm}$ at r_2 . When the width variation is approximately given by Eq.(1) in the inverse method, we have $b_1' = b_1$ at $r_1' = r_1$ and $b_2' = b_2$ at $r_2' = r_2$. Its error from the actual vane height is smaller than 3.7%. For the flow angle estimation at $r = 160\text{mm}$, we have $b_1' = 29.4\text{mm}$ ($r_1' = 160\text{mm}$) and its error is smaller than 6.5%. The maximum thickness of the vane is about 20mm, and the radii giving roundness to the leading and trailing edges are 4mm and 2mm to get good performance over a wide operating range⁽²⁰⁾. The cascade is also composed of 20 vanes. The flow angle at the datum section is $\beta_0 = 56^\circ$ (nearly mean value for Aerofoils A and B) and the flow angle downstream of the cascade is $\beta = 0^\circ$ (radial direction).

Fig.8 Pressure distributions on the vane surface

Fig.9 Relation between the rear stagnation point of the vane and the downstream flow angle

Though the desirable vane has to suppress the flow separation well and to give the radial flow out condition, it is difficult to obtain the velocity distribution satisfying both demands at the same time. Accordingly, the following procedure is taken in the present paper.

4.2 Design of the cascade

The relation between the velocity distribution and the vane profile was examined using the inverse method (the position of the rear stagnation point is at $\epsilon = 0^\circ$). Where, the velocity on the suction side is expressed as a negative value, from the analytical standpoint. The velocity distribution I showing a straight deceleration along the vane surface was given as the first step [full-line in Fig.10(a)], referred to those of Aerofoils A and B. The obtained vane is thick as shown in Fig.10(b). When abrupt variations are caused to the velocity distribution I near the leading and trailing edges [velocity distribution II, dot and dash-lines in Fig.10(a)], the obtained vane becomes thin thickness (considerably close to the trailing edge) as shown in Fig.10(b). Then, the velocity

(a) Given velocity distribution

(b) Obtained vane profile on the Z_p -plane

Fig.10 Relation between given velocity distributions and obtained vane profiles

distribution III with an abrupt variation near the leading edge and small roundness on the suction side was given [dash-line in Fig.10(a)]. Its obtained vane possesses roughly the desired thickness.

Furthermore, the distribution III was modified, to get the desirable velocity distribution satisfying the above demands for the cascade design. That is, the velocity distribution on the suction side was modified such that the gradient and region of the pressure rise (velocity deceleration) were held as low as possible and the equivalent diffusion ratio became smaller too. In this case, the distribution on the pressure side was also modified slightly, to get the desired maximum thickness of the vane. Such modifications were repeated till the flow angle β' downstream of the cascade composed of the obtained vanes, estimated directly by the singularity method, becomes zero (radial direction).

Two of some velocity distributions determined by the above mentioned modifi-

(a) Given velocity distribution

(b) Obtained vane profile on the Z'_p -plane

(c) Flow angle distribution

Fig.11 Given velocity distributions satisfying the demands for the cascade design, obtained vane profiles and downstream flow angles

cation are illustrated in Fig.11(a). The obtained vane profiles and the outlet flow angles are shown in Figs.11(b) and (c), respectively. Though the profiles differ from each other, both mean outlet flow angles are almost zero (radial direction). The equivalent diffusion ratio of the velocity distribution V is a little lower than that of IV, as the former maximum velocity on the suction side is smaller than the latter one. To evaluate the performance over a wider operating range, the pressure distributions were calculated di-

rectly by the singularity method for three kinds of the incidence angles (Fig.12). Where, marks $\circ$ and $\bullet$ indicate the pressure coefficients estimated from the given velocities shown in Fig.11(a), and their values coincide exactly with those calculated directly for $i_n=0^\circ$. The pressure gradient along the suction surface for the given velocity distribution V is smaller than that for IV when $i_n \geq 0^\circ$, and its region is also narrower when $i_n \leq 0^\circ$.

Judging from these evaluations, the vane profile obtained from the velocity distribution V was adopted for verifying experimentally to obtain the expected performances. The specifications and profile are shown in Fig.13. The maximum camber is 20% of the chord length and the maximum thickness is 20mm (at 55% chord length). The outlet excess angle is 11° and its value is greater than 9.6° estimated from data on the straight cascade⁽¹⁸⁾. The equivalent diffusion ratio is 1.65, the decelerating ratio is 0.69 and the corresponding two-dimensional divergent angle⁽¹⁹⁾ is 9.2° .

5. Conclusions

An inverse method based on the singularity method, considering the vane height variation, was developed. And the cascade for the return channel of a high-pressure large-sized centrifugal turbomachine was designed using this method. The optimum position of the rear stagnation point on the round trailing edge was also derived. The main conclusions are as follows.

- (1) By using the developed inverse method, the vanes of the circular cascade, which have the given velocity distribution on the contour, can be designed successfully.
- (2) For designing the return vanes of the centrifugal turbomachine, the velocity distribution on the vane contour should be given such that the outlet flow runs in the radial direction when the rear stagnation point is located at $\epsilon=0^\circ$ (Fig.9).
- (3) A desirable thick-vane for the return channel, which can suppress well the flow separation on the surface and discharge the flow into the radial direction was designed.

The flow patterns and performances of the cascade composed of a desirable vane designed in this report will be examined in the 2nd Report.

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Fig.12 Pressure distributions on the vane surface for various incidence angles

Fig.13 Aerofoil used for experiments

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