Studies on Circular Cascades for Return Channels of Centrifugal Turbomachinery
(1st Report, Inverse Method and Cascade Design)∗

By Tomitaro TOYOKURA∗∗, Toshiaki KANEMOTO∗∗∗ and Morisaki HATTAA∗∗∗∗

An inverse method to design a circular cascade for a return channel of a centrifugal turbomachine, whose vane-height varies in the radial direction, was developed using the singularity method. As the vane with a round trailing edge was treated in this study, a reasonable position of the rear stagnation point was decided experimentally when a secondary flow occurs considerably on the end wall and a small separation appears on the vane surface. A desirable circular cascade for the return channel, which has a reasonable pressure distribution on the vane surface and satisfies the radial outflow condition, was designed by the developed inverse method in this report.

Key Words : Fluid Machine Element, Return Channel, Cascade, Centrifugal Turbomachine, Inverse Method, Singularity Method

1. Introduction

To improve the performances of multi-stage centrifugal turbomachines, many studies on impellers and diffusers have been performed actively. On the contrary, studies on return channel which leads the flow from the diffuser outlet to the next stage impeller are limited as follows. The flows and performances of the channel with a rectangular cross-section composed of thin return vanes with constant thickness and of the channels without vanes have already been investigated. The flow at the inlet section of the return channel has a considerable swirl velocity component in general. Accordingly, circular cascade is necessary to give a large deflection to the through-flow, so as to get the radial outflow. Moreover, the vane thickness of the cascade installed in the channel of a high-pressure large-sized turbomachine becomes thick. The flow through such a cascade induces intense secondary flows on the end walls and flow separations on the vane surfaces. They complicate the internal flow pattern and increase the flow loss. Such problems of circular cascades, however, have not been investigated yet.

This serial study intended to make clear the relation between the flow pattern and performance of the circular cascades for the return channel of a large-sized centrifugal turbomachine with high pressure ratio. In the 1st Report, the cascade design based on an inverse method was presented. Here, the inverse methods applied to a single aerofoil, straight cascades and circular cascades composed of thin vanes have already been reported. An analytical method for the circular cascades composed of thick vanes with large camber, whose height varies in the radial direction, however, has not been established yet.

Nomenclature

r : radial coordinate (Fig.1).
θ : counter-clockwise angular coordinate (Fig.1).
θ' : clockwise angular coordinate (Fig.1).
: distance from the lower end wall (see the 2nd Report).
b : channel width or vane height.
 : absolute velocity (Fig.1).
v : velocity component in the radial direction (Fig.1).
,vw : velocity component in the counter-clockwise angular direction (Fig.1).
p : flow angle measured counter-clockwise from the radial direction (Fig.1).
r : flow angle =θ-θ (Fig.1).
i : incidence angle decided by the flow angle at the inlet datum section.
l : length measured on the vane surface from the trailing edge (Fig.1).
L : total length of vane surface (pressure and suction surfaces).
P : pressure on the vane surface.
Cp : pressure coefficient \(=\frac{\rho - \rho_{0}}{\rho_{0}}\).
s : density of air.

Subscripts

0 : value at the inlet datum section (r = 300mm).
1 : value at the cascade inlet section (r = 345.75mm, estimated from values at the inlet datum section on the assumption of ideal flow).
- \( \hat{S} \): mean value weighted by flow rate.

2. Inverse Method

2.1 Singularity method

At first, the pressure distributions on the vane surfaces predicted by the singularity method were compared with experimental ones for the cascades described in Chapter 3. These comparisons show that both distributions coincide pretty well except for the region of separation, in spite of an existence of considerable secondary flow. Therefore, this singularity method was applied to the inverse method.

The channel-width (vane-height) variation can be given nearly by Eq.(1), when its variation is comparatively small in the radial direction.

\[
\hat{r} = \left( c' \ln \frac{r}{r_c} + c'' \right)^{\frac{1}{2}}
\]

where, \( c' \) and \( c'' \) are the widths upstream (\( r > r_c \)) and downstream (\( r < r_c \)) of the cascade, respectively. Then, the flow on the physical plane (\( Z_r \)-plane, Fig.1) can be predicted by analyzing the flow on the physical plane for the constant width (\( Z_r \)-plane), provided the flow pattern does not vary in the width direction. Both flows are in the following relations.

\[
u = \frac{\nu_c}{z} \quad \alpha = \frac{\alpha_c}{z} + \frac{c'}{r}
\]

where, \( \nu_c \) and \( \alpha_c \) are the velocity components in the radial and the angular direction and \( Z_r \) is the stream function, on the \( Z_r \)-plane. The flow on the \( Z_r \)-plane can be induced easily from the flow on the \( Z_r \)-plane analyzed by the singularity method that the vorticities \( \Gamma_j (j = 1, 2, \ldots, n) \) (total number of the vortices) are located on the short segments of the vane contour in the straight cascade on the \( Z_r \)-plane transformed conformally by Eq.(3).

\[
Z_r = \frac{N}{2\pi} \ln \left( \frac{Z_r}{r_c} \right)
\]

where, \( Z_r \)-plane is the mapping plane for the constant width and \( N \) is number of vanes in the circular cascade. The flow on the mapping plane considering the width variation (\( Z_r \)-plane, Fig.2) can be also obtained easily from one on the \( Z_r \)-plane, by using Eq.(2) multiplied by \( 2\pi N \).

Consequently, the velocities at the calculation point \( S(x, y, z) \) on the vane surface in the \( Z_r \)-plane, which are obtained from addition of the cascade inflow and the flow induced from the vortices located on \( S(x, y, Z_r) \), are

\[
\begin{align*}
\nu(x, y, z, \phi) &= \frac{\nu_c}{z} \cos \alpha_c \cos \phi_c + \frac{1}{2} \frac{c'}{r} \cos \alpha_c \sin \phi_c \\
\alpha(x, y, z, \phi) &= \frac{\alpha_c}{z} + \frac{c'}{r} \sin \alpha_c \sin \phi_c \\
\end{align*}
\]

\[
\begin{align*}
\nu(x, y, z, \phi) &= -8.5 \sin \left[ 2\pi (z - y) - \frac{1}{2} \cos \phi \right] \\
\alpha(x, y, z, \phi) &= -8.5 \sin \left[ 2\pi (z - y) - \frac{1}{2} \cos \phi \right] \\
\end{align*}
\]

\[
\begin{align*}
\gamma(x, y, z, \phi) &= -1 \frac{1}{4\pi} \cos \left[ 2\pi (z - y) - \frac{1}{2} \cos \phi \right] \\
\end{align*}
\]

Fig.1 Coordinates and symbols on the physical plane (\( Z_r \)-plane) considering the channel-width variation.

Fig.2 Coordinates of the vortex and calculation point on the mapping plane (\( Z_r \)-plane) considering the channel-width variation.
On the Z̄ plane (Fig.1), the velocity components in the i-direction and in the vertical direction to i are given by

\[ \nu_i = \frac{N}{2\pi r} \left( \frac{\partial u}{\partial i} \right) \]  

and the velocity components in the radial and angular directions around the cascade are given by

\[ v_r = \frac{\tau}{r} \sin \theta \]  

\[ v_\theta = \frac{\tau}{r} \cos \theta \]  

\[-\frac{1}{2} \frac{\partial^2 u}{\partial i^2} \]  

In the inverse method, the strengths of the vortices which show the given velocity distributions are calculated by Eq. (4), and the vane profile which gives \[ \omega = 0 \] is found by substituting those into Eq.(5), as described in 2.2.

2.2 Analytical method

In the inverse method, the velocity distribution is given on the physical plane (Z̄ plane) and the analysis is performed on the mapping plane (Z̄ plane). In the analysis, the position of the rear stagnation point of the vane and the i-coordinate giving the leading edge position (corresponding to the inlet setting radius of the cascade on the Z̄ plane) are fixed beforehand, to get one solution. As the leading edge position is movable freely in the \( \theta \)-direction (corresponding to the angular direction), however, the necessary deflection angle and the desirable vane profile may be derived. Besides, the roundness of the leading and trailing edges is also given beforehand.

Now, it is assumed that the velocity distribution is given as a function of non-dimensional length \( l/w \) on the suction side and \( l/w \) on the pressure side, where, \( l \) and \( w \) are the local and total length measured from the rear stagnation point on the vane contour (subscripts s and p denote the values for suction side and pressure side). At first, assuming an arbitrary vane contour (Contour O) on the Z̄ plane, the strength of the vortex distributed on its contour is derived by substituting the given velocity into Eqs. (4) and (7). The vertical velocity component \( \omega \) is not equal to zero, as an arbitrary vane contour is assumed, and its component can be determined by substituting the derived vortices into Eq.(5). In this procedure, the contour assumed on the Z̄ plane must be transformed into the Z̄ plane, to get the correspondence of the contour length on the Z̄ plane to \( l/w \) and \( l/w \) on the Z̄ plane.

As the vane contour must be one of the stream lines (\( \omega = 0 \)), Contour O is modified by turn in the \( \theta \)-direction from the rear stagnation point so as to make the tangent of the new vane contour coincide with the direction of the resultant velocity \( \omega \) (Fig.3). That is, the j-th calculation point on Contour O is moved by \( \Delta \theta \) obtained from Eq.(3) vertically against its contour. Here, the positive value means to move in the outward direction of the contour.

\[ \Delta \theta = \frac{1}{2} \frac{\partial^2 u}{\partial i^2} \]  

where, \( \Delta \theta \) is the segment length between k-th and k-th calculation points on Contour O. Then, new coordinates of the modified point are written as follows.

\[ x^j = x^i + \Delta \theta \sin \theta \]  

\[ y^j = y^i + \Delta \theta \cos \theta \]

The modified n-th calculation point \( S(x, y) \), however, does not coincide with the fixed coordinates \( S(x, y) \), because the numerical error accumulates in proportion to increase of \( j \). Thus, the correction of its error is carried out in proportion to the length measured from the rear stagnation point on Contour O. For the \( x \)-coordinate, the j-th coordinate \( x^j \) is remodeled by the relation of

\[ x^j = x^j + \Delta x_0 \frac{\partial u}{\partial \theta} \]

where, \( \Delta x_0 \) denotes the length measured from the n-th to 1st calculation point indicating the rear stagnation point. The \( \theta \)-coordinate is also recomputed by the same procedure (\( \partial y/\partial \theta \)).

The coordinate \( x^j \) is moved further to the coordinate given by the relation of

\[ x^j = x^j + \Delta x_0 \frac{\partial u}{\partial \theta} \]

when the \( x^j \)-coordinate giving the leading edge position does not coincide with the fixed \( x \)-value. The first approximate vane contour is formed by joining smoothly the coordinates \( S(x^j, y^j) \).

The vane profile having the given velocity distribution is designed successfully, when all the calculation points do not move even though the above procedures are repeated.

2.3 Numerical example

Validity of the developed inverse method is checked in this paragraph. The full-line and the dash-line in Fig.4 show the pressure coefficient \( C_p \) distributions on the suction and pressure surfaces of Aerofoil B shown in Fig.6. These values were calculated directly by the singularity method presented in 2.1. The vane profile was derived from the velocity distributions given from these \( C_p \) values. The deriving process of the vane contour on the Z̄ plane is shown in Fig.3. The final contour (mark ●) coincides exactly with Aerofoil B (full-line). Accordingly, the
$C_{o}$-distributions on the final contour, calculated directly by the singularity method (marks o and • in Fig.4), coincide naturally with the given values.

3. Determination of the Rear Stagnation Point

It is important for the cascade design to predict well the downstream flow angle. When its angle is to be predicted by the above singularity method, the rear stagnation point must be located on the reasonable position. A method to find out this point, however, has not been established. Then, the position is sought by trial and error in this chapter.

3.1 Preliminary experiments

Figure 6 shows the return vanes used in the preliminary experiments for a return channel. The cascade is composed of 20 vanes. The maximum thickness of each vane is 21 mm (at 44% of the chord length for Aerofoil A, 42% for Aerofoil B). The vane is formed by circular arc surface and its height varies straight in the radial direction. The inlet setting angles of Aerofoils A and B are 62° and 51° measured from the radial direction, which are decided due to the outlet flow angle of the conventional vaneless-diffuser. The outlet angles of Aerofoils A and B are 8° and 6°, which are decided roughly by data on the straight cascade. When the effect of the vane thickness is neglected and the passage between neighboring canter lines is a two-dimensional diffuse, the divergent angles of Aerofoils A and B are 12.5° and 6.9°. The equivalent diffusion ratios $D_{a} = \frac{v_{a}}{v_{r} - v_{m}}$, $v_{m}$: maximum velocity on the suction surface, $v_{r}$ mean velocity at the cascade outlet section) for Aerofoils A and B are 1.77 and 1.48.

Those cascades were installed in the return channel shown in Fig.1 of the 2nd Report, and the flow patterns at Section IV in the channel and the pressures on the middle-span vane surface were measured. Figure 7 shows the velocity and flow angle distributions at Section IV. Where, $\theta = 0°$ and 18° give the sections corresponding to the trailing edges. The wake flow due to the flow separation on the vane surface is observed and its velocity defect is remarkable near the middle-width section (z/h = 0.5). Moreover, the flow angle varies markedly on account of the secondary flow and wake flow. The pressure distributions on the vane surface at the middle-span section, however, coincide well with those calculated by the above mentioned singularity method, except the region of separation (Fig.8). Where, $\varepsilon$ gives the position of the rear stagnation point (see Fig.9) and the separation point is one obtained visually by the oil-film method.

1.2 Rear stagnation point

The reasonable position of the rear stagnation point is discussed by using the mean flow angle $\beta$ at Section IV for $\theta = 0°$, as the outlet flow condition is very complicated. The full-lines in Fig.9 give the relations between the position of the rear stagnation point $\varepsilon$ and $z$ calculated by the singularity method. Moreover,

![Fig.4 Pressure distributions on the surface of Aerofoil B](image)

![Fig.5 Modifying process of vane contour on the mapping plane (Z'-plane)](image)

![Fig.6 Aerofoils used for the preliminary experiments](image)

the marks o and • indicate the experimental mean flow angles which are plotted on each full-line. As recognized in this figure, to make the calculated flow angle coincide with the experimental one, it is necessary to choose the angle $\varepsilon$ putting $\theta = 0°$ for $\varepsilon = 0°$, as the outlet flow condition is very complicated. The full-lines in Fig.9 give the relations between the position of the rear stagnation point $\varepsilon$ and $z$ calculated by the singularity method. Moreover, $\varepsilon$ and $z$ are determined based on some data for small incidence angle.

$\theta = 0°$ : The mean flow angle at Section V is always the same as that at Section IV, as shown in the 2nd Report.
the rear stagnation point nearly equal to the flow angle $\beta$. Such relation is shown by the dash-line in this figure. This method seems to be applicable to a circular cascade having a large deflection angle, such as one accompanied with an intense secondary flow and a little flow separation.

When the cascade for the return channel is designed by the above presented inverse method, the velocity distribution ought to be given such that the calculated outlet flow angle $\beta$ becomes zero (radially flow out) when $\varepsilon = 0^\circ$.

4. Cascade Design

4.1 Design conditions

The same design conditions as the preliminary experiments were applied, to facilitate comparison with the performances for Aerofolds A and B. That is, the inlet of the cascade is at $r_1 = 234.72\text{mm}$ and the outlet is at $r_2 = 202.5\text{mm}$. The vane height is varied straight from $h = 20.44\text{mm}$ at $r_1$ to $h = 28.18\text{mm}$ at $r_2$. When the width variation is approximately given by Eq.(1) in the inverse method, we have $\beta = \alpha$, at $r_1 = r_2$, and $h = h_2$ at $r_1 = r_2$. Its error from the actual vane height is smaller than 3.7%. For the flow angle estimation at $r = 160\text{mm}$, we have $\beta = 29.4\text{mm}$ ($r = 160\text{mm}$) and its error is smaller than 6.5%. The maximum thickness of the vane is about 20mm, and the radius giving roundness to the leading and trailing edges are 4mm and 2mm to get good performance over a wide operating range.

The cascade is also composed of 20 vanes. The flow angle at the datum section is $\beta = 50^\circ$ (nearly mean value for Aerofolds A and B) and the flow angle downstream of the cascade is $\beta = 0^\circ$ (radial direction).

Fig.8 Pressure distributions on the vane surface

Fig.9 Relation between the rear stagnation point of the vane and the downstream flow angle

Though the desirable vane has to suppress the flow separation well and to give the radial flow out condition, it is difficult to obtain the velocity distribution satisfying both demands at the same time. Accordingly, the following procedure is taken in the present paper.

4.2 Design of the cascade

The relation between the velocity distribution and the vane profile was examined using the inverse method (the position of the rear stagnation point is at $\varepsilon = 0^\circ$). Where, the velocity on the suction side is expressed as a negative value, from the analytical standpoint. The velocity distribution I showing a straight deceleration along the vane surface was given as the first step (full-line in Fig.10(a)), referred to those of Aerofolds A and B. The obtained vane is thick as shown in Fig.10(b). The abrupt variations are caused to the velocity distribution I near the leading and trailing edges (velocity distribution II, dot and dash-lines in Fig.10(a)), the obtained vane becomes thin thickness (considerably close to the trailing edge) as shown in Fig.10(b). Then, the velocity
distribution III with an abrupt variation near the leading edge and small roundness on the suction side was given [dashed line in Fig.10(a)]. Its obtained vane possesses roughly the desired thickness.

Furthermore, the distribution III was modified, to get the desirable velocity distribution satisfying the above demands for the cascade design. That is, the velocity distribution on the suction side was modified such that the gradient and region of the pressure rise (velocity deceleration) were held as low as possible and the equivalent diffusion ratio became smaller too. In this case, the distribution on the pressure side was also modified slightly, to get the desired maximum thickness of the vane. Such modifications were repeated till the flow angle $\phi$ downstream of the cascade composed of the obtained vanes, estimated directly by the singularity method, becomes zero (radial direction).

Two of some velocity distributions determined by the above mentioned modification are illustrated in Fig.11(a). The obtained vane profiles and the outlet flow angles are shown in Figs.11(b) and (c), respectively. Though the profiles differ from each other, both mean outlet flow angles are almost zero (radial direction). The equivalent diffusion ratio of the velocity distribution V is a little lower than that of IV, as the former maximum velocity on the suction side is smaller than the latter one. To evaluate the performance over a wider operating range, the pressure distributions were calculated di-
rectly by the singularity method for three kinds of the incidence angles (Fig.12). Where, marks o and A indicate the pressure coefficients estimated from the given velocities shown in Fig.11(a), and their values coincide exactly with those calculated directly for $\alpha=0^\circ$. The pressure gradient along the suction surface for the given velocity distribution $V$ is smaller than that for IV when $\alpha=6^\circ$, and its region is also narrower when $\alpha=0^\circ$.

Judging from these evaluations, the vane profile obtained from the velocity distribution $V$ was adopted for verifying experimentally to obtain the expected performances. The specifications and profile are shown in Fig.13. The maximum camber is 20% of the chord length and the maximum thickness is 20mm (at 55% chord length). The outlet excess angle is $17^\circ$ and its value is greater than 9.6° estimated from data on the straight cascade (9). The equivalent diffusion ratio is 1.65, the decelerating ratio is 0.69 and the corresponding two-dimensional divergent angle (9) is 9.2°.

5. Conclusions

An inverse method based on the singularity method, considering the vane height variation, was developed. And the cascade for the return channel of a high-pressure large-sized centrifugal turbomachine was designed using this method. The optimum position of the rear stagnation point on the round trailing edge was also derived. The main conclusions are as follows.

(1) By using the developed inverse method, the vanes of the circular cascade, which have the given velocity distribution on the contour, can be designed successfully.
(2) For designing the return vanes of the centrifugal turbomachine, the velocity distribution on the vane contour should be given such that the outlet flow runs in the radial direction where the rear stagnation point is located at $x=0^\circ$ (Fig.9).
(3) A desirable thick-vane for the return channel, which can suppress well the flow separation on the surface and discharge the flow into the radial direction was designed.

The flow patterns and performances of the cascade composed of a desirable vane designed in this report will be examined in the 2nd Report.

Acknowledgement

The authors are much indebted to Mr. K.Kurai and Mr. T.Kida for help in this paper. The authors also would like to thank Mr. T.Kohi and Mr. K.Kura of Ishikawajima Harima Heavy Industry Co. Ltd., who suggested the present study and arranged the test rig. A part of this study has been carried out with Grant-in-Aid for Co-operative Research appropriated from the Ministry of Education of Japan.

References


(7) Shigesi,H., Japan National Aerospace Laboratory, TR-57 (1979), 1.