Flow Force Acting on Two-Way-Cartridge Valve

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This paper deals with the free jet issuing from the metering orifice of a seat type two-way-cartridge valve. In order to estimate the flow force acting on the valve exactly, it is important to know the jet angle and the jet width from the metering orifice in the valve. The flow pattern of the free jet was analyzed using conformal mapping, where the flow was assumed nonviscous. From this analysis the relations among the jet angle, the jet width, the lift of the valve piston and the angle between the valve piston and the valve seat were obtained. Experiments were conducted to visualize the flow inside the two-way-cartridge valve. Then, to make the effect of viscosity clear, the flow pattern was numerically calculated using a streamline coordinate system, where the flow was assumed viscous. The theoretical results agreed very well with the experimental ones.

Key Words: Flow Force, Two-Way-Cartridge valve, Free Jet, Jet Angle, Contraction Coefficient

1. Introduction

Two-way-cartridge valves are often used for high speed and accurate control of a high power hydraulic drive system. Since the beginning of the 1970's two-way-cartridge valve technology has been used in West Europe and from the Institute for Hydraulics and Pneumatics in T.N.Ashen many research results were reported[3,4]. In these reports it is indicated that the movement of the two-way-cartridge valve depends on the pressures acting on the valve piston and consequently the performance of the two-way-cartridge valve system is strongly influenced by the flow force acting on the valve.

Two-way-cartridge valves are classified into two types, namely seat type valves and spool type valves. But from the viewpoint of leakage the seat type two-way-cartridge valve is widely used. A cross sectional view of the seat type two-way-cartridge valve is shown in Fig. 6. This valve resembles the conventional poppet valve. Many papers concerning the flow in the conventional poppet valve and the flow force acting on the valve, have been published[5-7]. Since the seat type two-way-cartridge valve is used for the range of higher nominal flow rates, the lift of the valve piston is made larger than in the conventional poppet valve in order to reduce energy loss at full opening. Therefore the flow pattern of the jet from the metering orifice in two-way-cartridge valve differs from the flow pattern in the conventional poppet valve. Consequently, when estimating the flow force acting on the two-way-cartridge valve, the fact should be taken into consideration that the angle of the jet from the metering orifice depends on the valve lift.

In this report, by applying the potential flow theory to the liquid jet in a two dimensional model of the seat type two-way-cartridge valve shown in Fig.6, the flow pattern, i.e. the jet angle and the jet width, and the pressure distribution on the valve piston are analyzed. Then the analytical results are verified by the experimental results. Also the effects of viscosity on the jet are examined by numerical results using a streamline coordinate system.

2. Theoretical Analysis by Potential Flow Theory

Fig.1 shows a two-dimensional model of the seat type two-way-cartridge valve. In Fig.1, \( r, s \) and \( \phi \) denote respectively the jet width, the jet angle and the velocity at infinite distance downstream, \( \psi \) the lift of the valve piston, \( \beta \) the angle between the wall \( CD \) and vertical, \( X_c \) and \( Y_c \), \( x \)-directional and \( y \)-directional distances between points A and C respectively. It is assumed that points B, E locate at infinite distance upstream, the point B at infinite distance downstream, the pressure on the free surfaces \( AB \) and \( CB \) is constant, the pressure at infinite distance upstream \( DE \) is also constant and the velocity at \( DE \) is zero. In this analysis the effects of gravity and compressibility are neglected.

Non-dimensional lift of the valve piston is defined as

\[
\eta = \frac{Y_c}{X_c} = \frac{Y_c}{X_c} \tan \beta = \cdots \cdot \cdot \cdot \{1\}
\]
By the method of conformal mapping shown in Fig. 2, the relation between the physical plane z and the t-plane is obtained as

\[ z(t) = \frac{1}{\pi t^\nu \sin \left( \frac{\pi - 2\theta}{2\pi} \cos^{-1} \frac{t}{\xi} \right)} \]

where \( Q_0 = \rho \cdot \mu \cdot \ln \) Fig. 2 denotes the flow rate per unit width perpendicular to the paper surface, \( \eta \) and \( \xi \) auxiliary complex variables which correspond to \( z \) and \( \zeta \) respectively and \( \xi \) is defined as

\[ \xi = \cos \left( \frac{\pi x}{2\pi} \right) \]

From eq. (2) the lift of valve piston \( \eta \) is expressed as a function of \( \eta \) and \( \beta \).

\[ \eta = \frac{1}{\pi t^\nu \sin \left( \frac{\pi - 2\theta}{2\pi} \cos^{-1} \frac{t}{\xi} \right)} \]

where

\[ S(\beta, \eta) = \cos \left( \frac{\pi - 2\theta}{2\pi} \cos^{-1} \frac{t}{\xi} \right) \]

Also from eq. (2) the jet width at infinite distance downstream is obtained as follows.

\[ \frac{r}{X_{CA}} = \pi \left[ \frac{1}{\pi t^\nu} \int_{t_{CA}}^{t} S(\beta, \eta, \xi) \, d\xi \right] \]

where \( \epsilon \) is introduced in order to avoid the divergence of integration at \( \xi \). Because the functions \( S \) and \( X \) are analytic functions, it is possible to change the path of integration as shown in Fig. 3. The free surface \( CD(\xi < \xi) \) is expressed as

\[ \frac{r(\xi)}{X_{CA}} = \frac{1}{\pi X_{CA}} \int_{t_{CA}}^{t} S(\beta, \eta, \xi) \, d\xi \]

The free surface \( AB(\xi > 1) \) is

\[ \frac{r(\xi)}{X_{CA}} = \frac{1}{\pi X_{CA}} \int_{t_{CA}}^{t} S(\beta, \eta, \xi) \, d\xi \]

In the case when the lift of the valve piston \( \eta \) is very small \( (\eta \ll X_{CA}) \), it is possible to consider that the wall \( CD \) is so long that the jet angle \( \alpha \) coincides with the angle of the wall \( CD \), \( \pi / 2 - \beta \) (see Fig. 4). Consequently the point B coincides with the point C in the t-plane, that is, \( \xi = 1 \). Then the normalized jet width \( r/Y \) is given as

\[ \Delta = \frac{dZ}{d\eta} \]

\[ \Omega = \ln \left( \frac{1}{\psi \cdot P} \right) \]

![Fig. 1 Model of two-way-cartridge valve (physical plane)](image)

![Fig. 2 Conformal mapping](image)
\[ \frac{x}{Y} = \frac{1}{2} \int_{-1}^{1} S_{i}(\beta, -1, \xi) d\xi + \sin \beta \left( \cos \beta - \frac{1}{2} \int_{-1}^{1} S_{i}(\beta, -1, \xi) d\xi \right) \frac{1}{\tan \beta} \] .......................... (8)

and the free surface \( AB (1 < \xi < 1) \) is expressed as
\[ \frac{y(\xi)}{Y} = -\frac{x}{Y} \int_{-1}^{1} S_{i}(\beta, -1, \xi) d\xi \] .......................... (9)

Applying Bernoulli's theorem among the infinitely upstream point, points on the wall and the infinitely downstream point, the pressure distributions on the walls \( AB \) and \( CD \) are calculated. The pressure distribution on \( AB \) \((1 < \xi < 1)\) is obtained as
\[ \frac{p}{p_{i}} = 1 - (\xi + \sqrt{\xi^2 - 1} y)^{2p_{s, s}} \sin \theta \int_{-1}^{1} S_{i}(\beta, -1, \xi) d\xi \] .......................... (10)

where \( p_{i} \) denotes the pressure at infinite distance upstream, \( X_{i} \) the distance along the wall \( AB \) measured from the point \( A \) and
\[ S_{i}(\beta, \xi, \xi) = (\xi + \sqrt{\xi^2 - 1} y)^{2p_{s, s}} / (\xi - \xi) \]

When the lift is small \((1 < X_{i} < 1)\), the pressure distribution on \( AB \) is expressed as
\[ \frac{p}{p_{i}} = 1 - (\xi + \sqrt{\xi^2 - 1} y)^{2p_{s, s}} \sin \theta \int_{-1}^{1} S_{i}(\beta, -1, \xi) d\xi \] .......................... (11)

Similarly, the pressure distribution on the wall \( CD \) \((1 < \xi < 1)\) is given as
\[ \frac{p}{p_{i}} = 1 - (\xi - \sqrt{\xi^2 - 1} y)^{2p_{s, s}} \sin \theta \int_{-1}^{1} S_{i}(\beta, -1, \xi) d\xi \] .......................... (12)

where \( z_{c} \) denotes the distance along the wall \( CD \) measured from the point \( C \) and
\[ S_{i}(\beta, \xi, \xi) = (\xi - \sqrt{\xi^2 - 1} y)^{2p_{s, s}} / (\xi - \xi) \]

When the lift is small \((1 < X_{i} < 1)\), the pressure distribution on \( CD \) is expressed by the first equation of eqs.(12), selecting the point \( C \) in Fig.4 as origin and calculating the coordinate \( z = \xi + \xi_{c} \) of the point on the wall \( CD \) by the following equations.
\[ \frac{X_{c}}{Y} = -\frac{x}{Y} \int_{-1}^{1} S_{i}(\beta, -1, \xi) d\xi + \frac{x}{Y} \cos \beta \left( \int_{-1}^{1} S_{i}(\beta, -1, \xi) d\xi \right) \] .......................... (13)

\[ \frac{Y_{c}}{Y} = 1 - \frac{x}{Y} \sin \beta \left( \int_{-1}^{1} S_{i}(\beta, -1, \xi) d\xi \right) \]

From eqs.(3)-(13), jet angle \( \theta \), jet width \( \tau \), the form of free surfaces \( AB \) and \( CD \) and the pressure distributions on the walls \( AB \) and \( CD \) can be calculated for given lift \( \mu \) and jet angle \( \beta \).

In the numerical calculation a small enough value of \( \varepsilon \) and Simpson's formula are used for integration.

3 Experimental Apparatus and Method

Fig.5 shows a schematic representation of the experimental apparatus used. The flow rate was measured by weighing method and the lift of the valve piston was measured by a dial gauge. To observe the pressure distribution, the differences between the pressures at points on the valve piston and the pressure in the down-stream overflow pipe 10 Electric motor tank 11 Platform scale
netroun overflow tank were measured by U-tube manometers.

Fig. 6 shows a cross sectional view of the two-way-cartridge valve and a schematic representation of a two-dimensional model used. As typical seat type two-way-cartridge valves, two configurations, type A valve and type B valve, were used. For the sake of flow visualization, side plates of the two-dimensional model were made of transparent acrylic. Photographs of jet from the metering orifice into water and the jet into air were taken. Then the jet angle was measured on the photograph. The jet from the metering orifice into water was visualized by the hydrogen bubble method. The width of the flow passage perpendicular to the paper surface was 50mm.

4 Results

Reynolds number \( R_e \) is defined as

\[
R_e = \frac{Q\mu}{\nu}
\]

(14)

where \( Q \) is the flow rate per unit width perpendicular to the paper surface, \( \nu \) is kinematic viscosity.

4.1 Type A valve

In the type A valve, as seen from Fig. 7, the jet from the metering orifice attaches to the valve piston and then the jet angle coincides with the taper angle of the valve piston even if the lift of the valve piston is not small. Consequently the equations for the case \( Y < X_c \) derived in Sec. 2 can be used for the type A valve.

But the equations in Sec. 2 were derived for the type B valve shown in Fig. 1. Then in order to apply the equations to the type A valve, it should be noticed that the valve piston of the type B valve corresponds to the valve seat of the type A valve and the valve seat of the type B valve to the valve piston of the type A valve (see Fig. 8). In Fig. 1 for the type B valve the valve piston moves in \( z \)-direction. But in Fig. 8 for the type A valve the valve piston moves in \( r \)-direction. The nondimensional lift of the valve piston defined in Sec. 2 is expressed for the type A valve as

![Cross sectional view of two-way-cartridge valve](image1)

![Two-dimensional model](image2)

![Flow pattern type A valve](image3)

![Typical plane for type A valve](image4)
\[ n = \frac{A_\Omega}{Xc} = \frac{1}{[(1/\beta) \tan^2 \beta + \tan \beta]} \] 

where \( y \) denotes the valve lift of the type A valve and \( \beta \) half of the difference between the outer diameter of the valve piston and the inner diameter of the valve seat.

Fig. 9 shows the control volume for this type of valve. Applying the linear momentum theory to this control volume, the flow force is given by

\[ f_n = F_n / (p_1 l) = 2 Y C_s C_e \sin \beta \cos \beta l \cdots (15) \]

where \( F_n \) denotes the flow force, \( C_s \) the contraction coefficient and \( C_e \) the velocity coefficient. Fig. 10 shows the relations between flow force and the lift of the valve piston. Experimental results are calculated from the measured pressure distributions on the valve pistons. Solid line is calculated from eqs. (5) and (15) under the assumption that \( C_e = 1 \). Broken line is calculated from eq. (15) under the assumption \( C_s < C_e \). The assumption that \( C_e = 1 \) is not adequate, that is, control volume must be taken as shown in Fig. 7.

Fig. 11 shows the pressure distribution on the wall \( CD \) calculated from eq. (11) for the case \( Y < X_c \). At the point of the minimum flow area the gradient of pressure decrease is steepest, but after the point of minimum flow area the pressure on \( CD \) continues to decrease. Therefore cut-off of the control volume at the position of the minimum flow area is inadequate to estimate the flow force exactly.

4.2 Type B valve

Fig. 12 shows a photograph of flow pattern visualized by the hydrogen bubble method. Fig. 13 shows a flow pattern calculated from eq. (6) and (7). At the point

Fig. 9 Control volume for type A valve

<table>
<thead>
<tr>
<th>Y</th>
<th>A</th>
<th>C</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 mm</td>
<td>Re = 1.6 x 10^6</td>
<td>Re = 2.4 x 10^9</td>
<td>Re = 3.06 x 10^9</td>
</tr>
<tr>
<td>5</td>
<td>2.90</td>
<td>4.84</td>
<td>5.22</td>
</tr>
<tr>
<td>7</td>
<td>4.03</td>
<td>5.68</td>
<td>7.19</td>
</tr>
</tbody>
</table>

Fig. 10 Flow force on type A valve

(\( \beta = 70^\circ \))

Fig. 11 Pressure distribution on wall CD (\( Y < X_c \))

Fig. 12 Flow pattern (type B valve, jet from metering orifice into water, \( n = 1.38, p = 294 \text{Pa} \))

Fig. 13 Shape of free surface (\( n = 1.18, \beta = 40^\circ \))
downstream of the point \( z = z_0 \times 10^{-3} \), the curvature of the free surface is zero. Fig. 13 agrees qualitatively with Fig. 12. This agreement indicates the validity of the theoretical analysis shown in Sec. 2.

Fig. 14 shows comparison of the measured jet angles with the calculated one. In the figure, two types of the measured jet angles are indicated. One is the jet into the water (symbol \( \circ \)), the other is the jet into the air (symbol \( \triangle \)). Both experimental results agree very well with the theoretical curves. It is evident from Fig. 14 that the smaller the angle \( \beta \) is, the wider becomes the region of \( \alpha \), where the jet angle \( \alpha \) is constant.

The measured and the calculated pressure distributions on the wall \( \overline{AE} \) for a small lift of the valve piston \( x \) (or \( \frac{x}{x_0} \)) and for a finite lift of the valve piston are shown in Fig. 15 and Fig. 16, respectively. \( x_0 \) in Fig. 15 denotes the distance along the wall \( \overline{AE} \) measured from point \( A \). Experimental results agree very well with theoretical curves. It is seen from Fig. 15 that as the angle \( \beta \) increases, the pressure drop due to the flow through the valve increases.

Fig. 17 shows the relation between the lift of the valve piston and the flow force, which is calculated by integration of the pressure distribution. Each curve consists of two straight parts and a transition curved part. As the lift \( n \) increases from zero, at first the flow force \( F_n \) increases linearly. After that the gradient of the curve increases gradually. As the lift increases further, the flow force increases linearly with the lift again and the gradient of the latter straight line is larger than that of the former. It is also seen from Fig. 17 that for a smaller angle \( \beta \) the flow force is smaller.

### 4.3 Effect of viscosity

In the case where the valve lift and flow rate are small, Reynolds number is finite and small. Then it seems that the effect of viscosity on the jet cannot be neglected in such case. In this section, the flow in the two-way-cartridge valve is investigated by numerical method including the viscous term in the equation of motion [14]. The following are assumed:

(a) Flow is two-dimensional, incompressible and viscous.
(b) Effects of external forces such as gravity force are neglected.
(c) Normal stress on free surface is zero.
(d) Flow at infinite distance upstream is 

Hanel's flow [10]

Under these assumptions the liquid jet from a spool valve with a right angle edge
(\beta = 0) into air has already been investigated by Takai and Takashashi using a streamline coordinate system. By the same procedure the jet from the two-way-cartridge is numerically calculated.

Fig. 18 shows the flow pattern for the case where the form of the valve piston and valve seat is symmetrical. Symmetrical configuration means that defining the point F as the intersection of the extension lines of \( \overline{EA} \) and \( \overline{FC} \), the lines \( \overline{EA} \) and \( \overline{FC} \) are symmetrical about the bisector of the angle \( \angle AFC \). Streamlines are almost linear except the region near the metering orifice. Broken lines indicate the free surface calculated by the potential theory explained in Sec. 2. Comparison of the two calculated results indicates that for the symmetrical case the effect of viscosity is to make the jet width increase.

Fig. 19 shows the relation between the angle of the valve seat and the contraction coefficient \( C_c \) for the symmetrical case. Within the range of \( 0^\circ \leq \beta \leq 60^\circ \), the contraction coefficients \( C_c \) for \( Re = 50 \) and 100 are larger than that for \( Re = \infty \).

Fig. 20 shows a flow pattern calculated under consideration of viscosity for a skew symmetrical case. When the lift of the valve piston is smaller than the lift for which the valve piston and the valve seat form a symmetrical configuration, the effect of viscosity is to reduce the jet angle.

Fig. 21 shows the pressure distribution on the wall \( \overline{AE} \) for the symmetrical case. In Fig. 21 the pressure distribution in Hamel's flow and that in a potential flow are also shown. It is seen from Fig. 21 that for the symmetrical case the effect of viscosity is to increase the pressure drop which is the cause of flow force. Then it should be noted that in the flow condition, where the effect of viscosity is dominant, the flow force acting on the valve piston is larger than the flow force calculated by the potential theory. It is also seen from Fig. 21 that the pressure...
sure drop calculated by the method of the streamline coordinate is larger than that in Hensel's flow in the region near the metering orifice. This fact shows that the effect of the downstream flow condition on the upstream condition is taken into account by the method using the streamline coordinate.

5. Conclusions

The jet from a typical seat type two-way-cartridge valve was analyzed by the potential flow theory. Flow visualization and pressure measurements showed that the theoretical results agreed well with the experimental results. By numerical calculations using a streamline coordinate system, the effects of viscosity on the jet were investigated. From this study the following can be concluded:

(a) For the type A valve, assuming that the jet angle coincides with the angle of wall and taking into account the contraction of the jet, the linear momentum theory can be applied to estimate the flow force. For this valve the flow force increases with the lift of the valve piston linearly.

(b) For type B valve, as the lift of the valve piston increases from zero, at first the flow force increases linearly. After that the gradient of the curve increases gradually. As the lift increases further, the flow force increases linearly with the lift again. For this valve, the smaller the angle is, the smaller becomes the flow force.

(c) For the type H valve, the effect of viscosity is to make the jet width larger. When the valve lift is smaller than the lift where the valve piston and the valve seat form a symmetrical configuration, the viscosity decreases the jet angle.

(d) The calculated and the measured jet angles for the liquid jet issuing into air coincide with the measured ones for liquid jet issuing into water. Therefore, the theory on liquid jet into air is found valid.

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Reference

(1) Wobben, D., Dissertation TH Aachen (1978)
(5) Tanaka, K., Report of the Aeronautical Institute of Tokyo Imperial Univ., No.51 (1929-11), 301.