Study on Mechanical Interface between Head and Media in Flexible Disk Drive*
(2nd Report, Dynamic Response Characteristics Due to a Concentrated Force)

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In order to get a better understanding of head to flexible disk interface problems, Green's function for a spinning flexible disk deflection is derived by extension of the analytical method proposed for a stationary flexible disk in the previous paper. As a result, it is found that a flexible disk has many resonance speeds where each circumferential mode component is excited by a stationary force. The external stiffness has an effect to increase these resonance speeds, whereas the external damping makes the disk deflection asymmetric with respect to the applied force. Restraining effect of a liner used in an available floppy disk cartridge on the disk deflection is clarified by the numerical analysis of a disk model supported partially by a set of external springs. Theoretical results are roughly verified by experiments.

Key words: Business Equipment, Dynamic Response, Flexible Disk, Resonance Speed, Green's Function

1. Introduction

Floppy disk storage is playing an important role in data processing systems, especially in small and personal computer systems, owing to its merits of low cost, high capacity, and interchangeability. Advanced technology for higher density recording has been continuously utilized to provide a larger capacity disk without increase of disk size or a smaller size disk and drive without decrease of storage capacity. A new product for higher density recording has been realized by optimizing physical and dimensional parameters of head and medium for better contact condition as well as by employing higher density recording medium and head.

There have been many papers studying the mechanical interface between head and flexible disk [1]-[8]. Some of them concern air bearing characteristics [2], [5]-[8], and others deal with the fundamental dynamic behavior of a rotating floppy disk produced by a concentrated force or mass spring system [3], [4]. However, the dynamic behavior of a commercially available flexible disk in contact recording is much more complex than revealed in the above mentioned papers. More detailed analytical and experimental studies should be done, if we desire to obtain some useful data to design a higher density contact recording floppy disk drive.

In order to establish an analytical method which can estimate quantitatively the contact force and contact area of head, the authors determined the contact force and disk deflection by solving the nonlinear contact problem by means of Green's function of non-rotating disk deflection which is derived by the finite element method in the radial direction and Fourier series expansion in the circumferential direction[9]. The analytical results are found in good agreement with the experimental ones, if the amount of the disk deflection is less than that of the disk thickness.

In this paper, Green's function for a spinning flexible disk deflection is first derived by applying the previous paper's method. The spinning disk deflections produced by a single stationary concentrated force is expressed as a sum of the responses of each circumferential mode which has individually a resonance speed. The effects of the external damping and external stiffness on the disk deflection are analyzed in detail. The restraining effect of a liner used in an available floppy disk cartridge on the disk deflection is also numerically analyzed using Green's function. Theoretical results are roughly verified by experiments.

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2. Nomenclature

- $a$: inner radius of disk, mm
- $b_r$: penetrating depth of point contact head from clamping height, mm
- $c$: external radius of disk, mm
- $d$: external damping, $\text{Ns/m/\text{mm}^3}$
- $D = \frac{B^{1/2}}{12(1-v^2)}$, N/mm
- $E$: Young's modulus, Pa
- $F_r$: concentrated force, N
- $G$: disk deflection due to a concentrated unit force, mm/N
- $k$: stiffness density of external spring, N/mm
- $K_o$: stiffness density of partial external spring, mm
- $S_o$: subdivided disk area, mm
- $\omega$: disk deflection, mm
- $\rho$: density of disk, kg/mm
- $\sigma_r$, $\sigma_\theta$: radial and circumferential stresses in disk, Pa
- $\nu$: Poisson's ratio
- $\omega$: rotating speed of disk, rpm

3. Analysis

3.1 Green's function of disk in transverse deflection

Figure 1 shows the geometry of a spinning flexible disk which is clamped at $r=a$, free at $r=b$, and spins with angular velocity. The unit concentrated force is applied at $r=\xi$ and $\theta=\phi$. When including the external stiffness $K$ and the external damping $c$ by air or base plate, the differential equation for disk deflection is written in the form,

$$D \frac{\partial^4 G}{\partial r^4} - H \frac{\partial G}{\partial r} = \frac{1}{r} \left[ \delta(r-\xi) \delta(\theta) - \delta(r-\xi) \delta(\phi) \right]$$

where $\delta(\theta)$ and $\delta(r-\xi)$ are Dirac delta functions. $\sigma_r$ and $\sigma_\theta$ are the radial and circumferential stresses in the disk caused by centrifugal force. They are given by

$$\sigma_r = \frac{c_r^2 + c_r^2}{c_r^2 + c_r^2}$$

$$\sigma_\theta = c_r^2 + c_r^2$$

$$c_r = \frac{1+\nu}{2\rho G} \left[ (\nu-1) \mu^2 - (3+\nu) \mu \right]$$

$$c_\theta = \frac{1-\nu}{8\rho G} \left[ 3(\nu-1) \mu^2 - (3+\nu) \mu \right]$$

Substituting Eq. (3) into Eq. (1), we obtain a set of ordinary differential equations for the deflection functions $G$ and $K_0$ of the forms,

$$D \frac{\partial^4 G}{\partial r^4} + K_o \frac{\partial G}{\partial r} + F_r \frac{\partial G}{\partial r} + S \frac{\partial G}{\partial r}$$

where,

$$Q = \frac{2D\nu}{r^2}$$

$$R_c = - \left( \frac{D + 2D_1}{r^2} + H \right)$$

$$S = \frac{D}{r^2} - \frac{H}{r} H$$

$$T_r = \frac{D}{r^2} - \frac{H}{r} H$$

$G$ and $K_0$ represent sine and cosine components of the amplitude for $l$-th circumferential mode, respectively. The FEM based on Galerkin's method is applied to Eqs. (4) and (5) in the same manner as described in the previous paper to obtain the numerical solutions of $G$ and $K_0$.

3.2 Analysis of disk deflection partially supported by external springs

In section 3.1, Green's function is derived under the assumption that the external stiffness is homogeneous over the disk. However, the flexible disk in practical use is supported partially by a flexible liner for the purpose of removing contamination from the disk surface and restraining disk vibration. In order to clarify the liner's effect on disk vibration, we next analyze the spinning disk deflection supported partially by liner, by assuming the liner to be a set of discrete springs.

![Fig.1 Analytical model of flexible disk](image)

![Fig.2 Analytical model of disk supported partially by a set of external springs](image)
Figure 2 shows an analytical model, where the deflection of the disk and the force of the liner are assumed to be expressed approximately by the discrete state variables at mesh points. If \( w_0 \) is defined as a deflection of the liner and disk from the equilibrium position without head, the restoring force of the liner \( f_0 \) at the mesh point \((i,j)\) is given by

\[
f_0 = -K_{ij}w_0 \tag{7}
\]

where \( K_{ij} \) is the effective stiffness density of the liner and it can take different values at each mesh point. If the contact force of the head is simplified as a concentrated force \( F_0 \) applied at the mesh point \((m_l)\), the disk deflection at point \((k, l)\) is written as

\[
w_{kl} = \sum G_{kl}w_0 \tag{8}
\]

where \( G_{kl} \) is a discrete Green's function, i.e., an influence coefficient from the point \((i,j)\) to the point \((k,l)\). Substitution of Eq.(7) into Eq.(8) yields,

\[
w_{kl} = G_{kl}F_0 \tag{9}
\]

We can get the disk deflection \( w_{kl} \) for a given head load \( F_0 \) by solving Eq.(9). If we want to calculate the head load \( F_0 \) for a given head penetration \( B_0 \), we have only to interchange these two terms after putting \( B_0 \) into \( w_{kl} \).

4. Theoretical Result

4.1 General features of transverse deflection of the spinning disk

Let us first examine a general view of disk deflection and the effect of inertia force and external damping on disk deflection. Figure 3 shows a transverse deflection of an 8-inch flexible disk produced by a concentrated force when the inertia or external damping is excluded or included. In each case, the disk speed is 500 rpm and the concentrated force of 1 mN is applied at \( \xi = 80 \text{ mm} \) and \( \theta = 90^\circ \). In the numerical calculation, the number of subdivisions of the disk is 15 in the radial direction and 50 terms are taken into account in Fourier's series expression. It is shown from Fig.3 that the inertia force makes the circumferential deflection distribution a rippled form and that the external damping makes it asymmetric with respect to loading position and decreases the deflection particularly in the upper region of the concentrated force. Inertia term is always included in the calculated results described in the following.

4.2 The effect of rotating speed and disk size

Figure 4 shows the amplitude \( \sqrt{G_1 + R^2} \) of \( i \)-th order circumferential mode at \( r = \xi \) versus rotating speed when \( \omega = 0 \). The amplitude is equal to \( |G_1| \), because all coefficients \( K_0 \) for Fourier sine series are zero when \( \omega = 0 \). The dotted lines in the figure indicate the region where \( G_1 \) is negative. Resonances of from 2nd to 7th mode appear successively. The phase of the modal amplitude changes from 0\(^\circ\) to 180\(^\circ\) above the resonance. However, resonance speeds in zero and 1st mode are not known.

Figure 5 shows the circumferential deflection distributions of an 8-inch flexible disk at the radial positions where and , for various rotating speeds. It is seen from this figure that the phase of each mode reverses at the resonance speed as mentioned above.

Figure 6 (a) and (b) show the

(a) Exclude inertia and external damping.

(b) Include inertia and exclude damping.

(c) Exclude inertia and inclucde damping.

(d) Include inertia and damping.

\[
\begin{align*}
\omega &= 25 \text{ mm}, b = 100 \text{ mm}, H = 0.078 \text{ mm}, \omega_0 = 500 \text{ rpm} \\
K &= 4.9 \times 10^4 \text{ N/m mm}^2, \mu = 1.3 \times 10^4 \text{ kg/m}^2, \xi &= 10 \text{ mm} \\
e &= 1 \times 10^6 \text{ N sec/m}^2, k &= 100 \text{ N/mm}, \xi = 80 \text{ mm}
\end{align*}
\]

Fig.3 Effect of inertia and external damping.
increases with an increase in the external stiffness. The increasing rate increases with a decrease in the mode number. Figure 9 shows a representative circumferential deflection distribution at 360 rpm for various values of the external stiffness. The 5th, 2nd, 3rd, and 4th modes of the circumferential deflection are predominant in Figs.9 (b), (c), (e), and (f), respectively. Referring to Fig.8, it will be understood that the resonance speed of the corresponding mode coincides with the rotating speed of 360 rpm when the external stiffness takes the value described in Figs.9. As the external stiffness increases, the phase of the resonant mode changes from 180° to 0° at the resonance, because resonance of the related mode changes from supercritical condition to subcritical condition. In cases of 3.5- and 5.25-inch disks, the resonance naturally does not appear at the speed of 360 rpm even if k is varied.

Fig.9 Circumferential deflection distributions for various values of external stiffness.

4.4 Effect of external damping

Figures 10 (a) and (b) show the i-th mode amplitude $\sqrt{G_i + K_i}$ of the 8-inch disk at $r=\xi$ when $c=1.0 \times 10^4$ and $1.10^4 N/s/mm^2$, respectively. According to Fig.10 (a), the amplitude of the disk at the resonance speed becomes finite by the effect of the external damping. If the external damping increases further by a factor of ten as shown in Fig.10 (b), the resonance phenomena can not be observed and mode deflection of order zero becomes dominant. Figure 11 shows the representative circumferential deflection distributions at 360 rpm for various values of the external damping. It is seen that the external damping makes the circumferential disk deflection asymmetric with respect to the loading position because of the term $K_i$. It is further seen from Fig.11 (c) that a large damping decreases remarkably the amplitude of each mode except mode zero. The same feature as
amplitudes of individual circumferential modes versus rotating speed for 5.25- and 3.5-inch flexible disks, respectively. Figure 7 shows the circumferential deflection distribution at the rotating speeds of 300 and 500 rpm for both disks. The lowest resonance speed of the 5.25-inch disk occurs at 587 rpm in the second mode, whereas that of the 3.5-inch disk appears at 1466 rpm in the third mode. Therefore, a large deflection peculiar to resonance is not observed except at around 500 rpm for the 5.25-inch disk. It is noted that the disk rotation has little effect on the deflection distributions due to a stationary force in case of the 3.5-inch flexible disk, when the rotating speed is less than 500 rpm.

4.3 Effect of external stiffness

Figure 8 shows the effect of the external spring stiffness on the resonance speeds of each mode for the 8-inch disk when $\omega = 360$ rpm. The resonance speed

![Graph showing the effect of external stiffness on resonance speeds](image)

Fig. 4 Amplitudes of individual circumferential modes versus rotating speed for 8-inch disk.

![Graph showing the circumferential deflection distribution of 8-inch disk at various rotating speeds](image)

Fig. 5 Circumferential deflection distribution of 8-inch disk at various rotating speeds.

![Graph showing amplitudes of individual modes for 5.25 and 3.5-inch disks](image)

Fig. 6 Amplitudes of individual circumferential modes for 5.25 and 3.5-inch disks.

![Graph showing circumferential deflection distributions of 5.25 and 3.5-inch disks](image)

Fig. 7 Circumferential deflection distributions of 5.25 and 3.5-inch disks.
Fig.11 Circumferential deflection distributions for various values of external damping.

![Graphs showing deflection distributions](image)

Fig.12 Circumferential deflection distribution when disk is supported partially by external springs.

- Described above can be seen in the case of 3.5- and 5.25-inch disks.

4.5 Effect of partial elastic support

In order to examine the effect of elastic support by liner on disk deflection, the disk area is divided into ten parts in the radial direction and 36 parts in the circumferential direction. Figure 12 (a) shows the case where the external stiffness is zero at every mesh point, i.e., $K_\mu=0$ and $k=0$. The available 3.5-inch flexible disk is put in a hard plastic case and its both sides are supported by liner sheet except the region of head access window. Then, the calculated results of the second model including elasticity of liner are shown in Fig.13 (b), where $K_\mu=0$ at $j=1,2$ and 36 and $K_\mu=1 \times 10^7 \text{N/mm}^2$ at $j=3$ to 35. Due to constraining force of external stiffness, disk deflection is seen to be slightly decreased. The commercially available 3.5-inch disk has an elastic plate spring at the upper stream region with respect to the head, whose restoring force is applied to the liner at about right angles in front of the head. Then, the calculated deflection of the third model is shown in Fig.12 (c), where the additional external stiffness $K_\mu$ of $2 \times 10^7 \text{N/mm}^2$ is taken into account in the region where $j$ is 3 to 9 and $j=27$ to 29 in Fig.2. With an increase of the local additional external stiffness at about $-90^\circ$, it is seen that the negative deflection in the circumferential regions around $-90^\circ$ is remarkably reduced.

![Graphs showing comparison between experimental and theoretical results](image)
5. Experimental Study

The disk driving mechanism described in the previous paper[9] is used in this study. Figure 13 (a) shows comparison between the calculated and measured circumferential deflection distributions of the 8-inch disk at the typical radial positions where r=7 and 98 mm. In experiment, the spherical head with 20 mm radius of curvature protrudes into the spinning disk by 100 μm at the radial position where φ=8.7. Disk rotates at 1000 rpm. The clamping height at the disk inner radius is 50 μm above the base plate. The spinning disk is supported by air film over the base plate produced by disk rotation. Since the identification of the external stiffness and damping is not easy, two typical calculated results are shown in Fig.13 (a). It is seen that the calculated results agree qualitatively with the experimental ones but not quantitatively. It is considered that the qualitative disagreement results from the heterogeneous characteristics of the external elastic and damping coefficients over the disk surface, because the thickness of air film is not uniform in the radial direction. Further the base plate has a 10 mm wide head access window in the radial direction.

Figure 13 (b) shows the comparison between the experimental and calculated results of the 3.5-inch flexible disk. Since the gravitational deflection of the 3.5-inch disk is small, the clamping height from the base plate is set large enough to eliminate the effect of air film developed between the base plate and the rotating disk. Experimental penetrating depth of the spherical head is about 50 μm. It is seen from this figure that the calculated results are in fairly good agreement with the experimental ones because the external stiffness and damping effects are small and uniform.

6. Conclusions

Analytic-numerical method which combines the finite element method and Fourier series expansion is applied to the analysis for fundamental deflection characteristics of a spinning flexible disk. As a result, the circumferential deflection distribution due to a stationary concentrated force is found to have the following characteristics:

(1) Each circumferential mode component more than the first order has individually a resonance speed. The phase of each mode changes from zero to 180° above the individual critical speed.

(2) The external stiffness has an effect to increase the resonance speed in each mode. The increasing rate of the critical speed increases with a decrease in the mode number.

(3) The external damping makes the circumferential deflection asymmetric with respect to the loading point and reduces the amplitude of every mode except mode zero. Then, the disk deflection due to a concentrated force tends to be uniform in the circumferential direction with an increase in the external damping.

By using Green's function derived above, the restraining effect of a liner on disk deflection was analyzed by modeling the liner as a set of linear springs supporting the disk at discretized mesh points. It is found from experimental study that the theory can fairly estimate the steady deflection distribution of a spinning disk if the external stiffness and damping can be accurately identified.

References