Theoretical Analysis of the Finish Rolled Gear Tooth Profiles*

(Third Report, Rolled Tooth Profiles and Modified Tooth
Profiles of the Die of Helical Gears)

By Tatsuo OZAKI**, Kazuyuki FUNATSU*** and Ikuro FUCHI****

A deformation model of finish gear rolling of helical gears is proposed considering deflections of the teeth, local elasto-plastic indentations of the tooth surfaces at the contact points and edge effects near the side edges and the tip of the teeth.

The rolled tooth profiles of works and the modified tooth profiles of dies which produce the finish rolled gears with a true involute tooth profile are analyzed theoretically by this model.

This model is confirmed to be valid by the experiments using dies with a standard tooth profile and ones with a modified tooth profile determined theoretically.

Key Words : Finish Gear Rolling, Rolled Tooth Profile, Modified Tooth Profile, Helical Gear, Deformation Model

1. Introduction

The rolled tooth profiles and the modified tooth profiles of dies in finish gear rolling of spur gears were analyzed theoretically assuming a simple deformation model in the former reports⁽¹⁾⁽²⁾.

In this report, the tooth profiles of finish rolled helical gears and the modified tooth profiles of the die are analyzed theoretically assuming a similar deformation model. The die is assumed to be elastic and the work to be elastic, perfectly-plastic. Deflections of the teeth, local elasto-plastic indentations of the tooth surfaces at the contact points and edge effects near both the side edges and the tips of the teeth are considered.

The experiment is done to confirm the validity of the theoretical analysis using the finish gear rolling apparatus of our own making⁽³⁾.

2. Theoretical Analysis of the Finish Rolled Tooth Profiles of Helical Gears

2.1 Deformation model During rolling, normal contact loads

are distributed along the lines of contact of the die and the work as shown in Fig.1. Total elasto-plastic displacement ΔS at an arbitrary point of contact $Q(a,b)$ is the sum of depth of plastic indentation of the work ΔS_p and elastic displacements of the work and the die ΔS_e .

$$\Delta S = \Delta S_p + \Delta S_e \quad \dots\dots\dots (1)$$

The relations between depth of plastic indentation and contact load are obtained by the slip line field assuming that a cylindrical, rigid, perfectly-plastic work is indented by a cylindrical, rigid die with a parallel axis in the first report⁽¹⁾, and the relations are assumed to hold at all contact points. ΔS_p near the side edges of the tooth, however, is actually deeper than one near the middle of the tooth. Figure 2 shows some results of the indentation experiment of cylindrical works 15, 20 mm in radius and 15, 20, 25 mm long by a cylindrical die 35.93 mm in radius and 40 mm long. The relation between depth of indentation and contact load obtained experimentally near the middle of the work agrees well with

Fig. 1

* Received 2nd October, 1984.

** Professor,
Faculty of Engineering, Kyusyu
University, Hakozaki 6-10-1
Higashiku, Fukuoka.

*** Kyushu Matsushita Electric Co.,
Ltd., Minoshima 4-1-62
Hakataku, Fukuoka.

**** Canon Inc.,
Shimomaru 3-30-2 Otaku, Tokyo.

Fig. 2 Increase in depth of plastic indentation near the end surface of a cylindrical work (k: yield shear stress, ρ : relative radius of curvature)

Fig. 3

that obtained theoretically by the slip line field. ΔS_p is deeper than one near the middle to the extent of about 4 mm apart from the end surface of the work within the range of contact loads commonly imposed in finish gear rolling.

In calculation of the rolled tooth profiles, a tooth of the work is divided into ten equal widths as shown in Fig. 3. Contact load at the middle of a divided tooth is assumed to act uniformly over the width of the divided tooth. The face width of the work is 20 mm here, so the edge effect on ΔS_p is considered at the first and the second divided tooth from both side edges of the tooth ($M=1, 2$ and $9, 10$). The ratios α_1, α_2 of ΔS_p at the points 1, 3 mm apart from the end surface of the work to one at the middle point of the work are obtained from the experimental results of indentation for each magnitude of contact loads. Assuming that a change of friction at the contact point does not cause the edge effect changes, ΔS_p at $M=1, 10$ and $2, 9$ are determined here to be the products of α_1, α_2 and ΔS_p theoretically obtained for each value of the coefficient of friction μ in the second report⁽²⁾. The dimensionless depth of plastic indentation is plotted on a parabolic curve in terms of the dimensionless contact load for each value of μ as shown in Fig. 4.

Fig. 4 Relations between depth of plastic indentation and contact load for each divided tooth

Table 1 Coefficients in Eq. 2

M	μ	a_i	b_i	c_i
3-8	0.1	18.38	-0.113	0.006
	0.0	19.27	-0.122	0.007
	-0.1	20.28	-0.139	0.008
2,9	0.1	17.15	0.500	0.003
	0.0	17.98	0.519	0.003
	-0.1	18.92	0.540	0.003
1,10	0.1	18.22	1.095	0.018
	0.0	19.01	1.141	0.019
	-0.1	20.00	1.193	0.020

$$\left(\frac{\Delta S_p}{\rho}\right) \times 10^3 = a_i \left(\frac{F}{k\rho B}\right)^2 + b_i \left(\frac{F}{k\rho B}\right) + c_i \quad \dots\dots\dots (2)$$

Values of a_i, b_i , and c_i are tabulated in Table 1. Considering the sliding direction and the moving direction of the contact point on the tooth surface of the work during rolling, the equation for $\mu=-0.1$ is used on the tooth face, that for $\mu=0.1$ on the tooth flank and that for $\mu=0.0$ at the pitch point for both surfaces of the tooth.

Elastic displacement is obtained by integrations of the products of the distributed contact load and the effect functions of deflection and elastic indentation at $Q(a, b)$ along the line of contact. The approximated equation proposed by Umezawa⁽⁴⁾ is used for the effect function of deflection, and that proposed by Umezawa and Ishikawa⁽⁵⁾ for the effect function of elastic indentation. When K_b^W and K_b^D are the effect functions of deflection of the work and the die, U_z^W and U_z^D are those of elastic indentation of the work and the die and the distributed contact loads act as shown in Fig. 1, Equation (1) becomes

$$\begin{aligned} 10^{-3} \rho \left[a_i \left\{ \frac{p(a, b)}{k\rho(a, b)} \right\}^2 + b_i \left\{ \frac{p(a, b)}{k\rho(a, b)} \right\} + c_i \right] &+ \iint_{x,y} \{ K_b^W(a, b, x, y) + U_z^W(a, b, x, y) \} p(x, y) dx dy \\ &+ \iint_{x,y} \{ K_b^D(a, b, x, y) + U_z^D(a, b, x, y) \} p(x, y) dx dy - \iint_{x,y} K_b^W(a, b, x, y) q(x, y) dx dy \\ &- \iint_{x,y} K_b^D(a, b, x, y) r(x, y) dx dy = \Delta S \quad \dots\dots\dots (3) \end{aligned}$$

In calculation, a tooth is divided into N equal widths (Fig.3.). Contact load at the middle (x_{mi}, y_{mi}) of a divided tooth is assumed to act uniformly over the width λ ($=B/N$) of the divided tooth. Let (x_{mi}, y_{mi}) be the representative contact point of the divided tooth, and Equation (3) is transformed into Eq. (4).

$$\begin{aligned} 10^{-3} \rho \left[a_i \left\{ \frac{p(a, b)}{k\rho(a, b)} \right\}^2 + b_i \left\{ \frac{p(a, b)}{k\rho(a, b)} \right\} + c_i \right] + \sum_{i=1}^n \{ K_b^w(a, b, x, y) + U_z^w(a, b, x, y) \} p(x_{mi}, y_{mi}) \frac{\lambda}{\cos \beta_g} \\ + \sum_{i=1}^n \{ K_b^p(a, b, x, y) + U_z^p(a, b, x, y) \} p(x_{mi}, y_{mi}) \frac{\lambda}{\cos \beta_g} - \sum_{i=1}^n K_b^w(a, b, x, y) q(x_{mi}, y_{mi}) \frac{\lambda}{\cos \beta_g} \\ - \sum_{i=1}^n K_b^p(a, b, x, y) r(x_{mi}, y_{mi}) \frac{\lambda}{\cos \beta_g} = \Delta S \end{aligned} \quad (4)$$

where β_g : base helix angle.

Equation (4) at all contact points yields simultaneous equations in terms of unknown contact loads.

Rolling load is obtained by equating the radial components of forces at each contact point as follows

$$P = \sum_{i=1}^n \{ \sin \alpha_b + (-1)^l \mu_i \cos \alpha_b \} F_i \quad (5)$$

where l : $l=1$ when i th contact point is on the driven side of the work and $l=2$ when it is on the follower side.

n : number of the contact points of all divided teeth.

F_i : $p(x_{mi}, y_{mi}) \lambda / \cos \beta_g$.

In the case of free work rotating, rolling torque is zero, we get

$$\sum_{i=1}^n \sum_{j=1}^m [(-1)^l (-1)^j \{ r_g - (-1)^j \mu_i \rho_i' \} \cos \alpha_b \cdot \cos \beta / \cos \alpha_s] F_i = 0 \quad (6)$$

where α_b : normal operating pressure angle.

α_s : transverse operating pressure angle.

β : helix angle.

r_g : transverse base radius of the work.

ρ_i' : transverse radius of curvature of the work at the contact point.

m : number of contact points of each divided tooth.

j : $j=1$ when the first contact point as viewed from the direction of rotation of the work is on the driven side of the work and $j=2$ when it is on the follower side.

2.2 Method of analysis

ΔS given at all contact points on both surfaces of the work teeth are equal at each moment when the die and the work rotate at a constant velocity ratio in the case of external work driving. Since ΔS is determined by the amount of radial feed of the die into the work in the rolling for a constant center distance, contact load F_i can be obtained from Eq.(4). ΔS is unknown in the rolling for a constant rolling load, and Equation (5) is added.

In the case of free work rotating, ΔS at each contact point is equal only on the same side of the work teeth at each moment. Since the sum of ΔS given on both sides of the work teeth at each moment is known in the rolling for a constant center distance, F_i can be obtained from Eqs.(4) and (6). ΔS given on both sides of the work teeth are unknown in the rolling for a constant rolling load, and F_i can be obtained from Eqs.(4), (5) and (6).

Depth of plastic indentation i.e. amount of stock rolled ΔS_p at each contact point is obtained by substituting the value obtained above for F_i into Eq.(2). Calculation is repeated rotating the work stepwise for an equally divided small

angle from beginning to end of contact for one surface of a certain tooth of the work. The rolled tooth profile is determined by plotting ΔS_p at each contact point along the tooth profile of each divided tooth.

When the work has a tooth profile error $(e_w)_i$ before rolling, ΔS is considered to vary by $(e_w)_i$ at each contact point in calculation.

2.3 Results of theoretical analysis, experiment and discussion

Dimensions of the works and the dies used in calculation and experiment are tabulated in Table 2. The teeth of the dies used in the experiment are ground to the standard tooth profile with JIS 0 grade accuracy by Maag gear grinder after hobbed and hardened. Works (S45C) are pre-hobbed.

The finish gear rolling apparatus is a two opposed dies-plunge feed type of our own making⁽³⁾. A die with a fixed axis is driven. The die is radially fed and works rotate freely. The rolling experiments for constant center distances and for constant rolling loads can be done by this apparatus. Rolling loads and center distances between the die and the work are measured and recorded during rolling.

The rolling experiment is done for constant rolling loads in the case of free work rotating. The die is driven by hand. The rolled tooth profiles are inspected by Hamai gear testing machine and measurements over pins are done to check the changes in the tooth thickness after every time of rolling. The work is rolled four times for a same rolling load. Measurements of center distance between the die and the work during rolling experiment show that they are kept nearly constant at each time of rolling, and that they shorten stepwise after every time of rolling.

Table 2 Dimensions of dies and works

	Die	Work	Die	Work
Module	3		3	
Pressure angle	20°		20°	
Helix angle	20°		30°	
Number of teeth	66	34	66	32
Addendum modification coefficient	0.0	0.0	0.0	0.0
Diameter of tip circle	216.196	114.006	234.201	116.071
Diameter of root circle	204.169	100.797	222.137	102.985
Face width	30	20	30	20
Contact ratio	2.19		2.34	

Therefore, the tooth profiles actually rolled after each time of rolling are compared with those calculated for such a value of constant center distance that ΔS_p calculated at the pitch point of the middle of the face width of the work is equal to one measured. For the first time of rolling, the tooth profile error of the pre-hobbed work is taken into calculation, and from the second time of rolling, the calculated tooth profile error of each divided tooth for the preceding time of rolling.

Figures 5 and 6 show the tooth profiles calculated and actually rolled for the rolling loads 13000N and 9000N.

ΔS_p decreases after every time of rolling due to the work hardening of the work. For the rolling load 13000N, the features of the pre-hobbed tooth profiles still remain leaving small irregularities on the rolled tooth profiles after the first time of rolling, though big projections on them are depressed. The rolled tooth profiles are smoothed gradually after each time of rolling, and the rolled tooth profiles of the follower side become concave. The rolling load 9000N seems to be so low that the tooth profile errors of the pre-hobbed works are not eliminated leaving small irregularities on the rolled tooth surfaces even after several times of rolling.

(a) Rolled tooth profiles at the middle of face width after each time of rolling

(b) Rolled tooth profile of each divided tooth after 4th time of rolling

Fig. 5 Rolled tooth profiles calculated and actually rolled ($\beta=20^\circ$, $P=13000$ N)

Fig. 6 Rolled tooth profiles at the middle of face width after each time of rolling ($\beta=20^\circ$, $P=9000$ N)

The features of the tooth profiles calculated agree well with those actually rolled.

3. The Modified Tooth Profiles of the Die

3.1 Method of analysis

In order to produce a true involute profile on the rolled tooth surfaces in rolling the work with a true involute profile, ΔS_p must be equal at all contact points over the tooth surfaces of the work. For this purpose, the tooth profiles of the die are usually modified.

The modified tooth profiles of the die can be determined theoretically using the deformation model mentioned in 2.1 for a constant center distance in the case of external work driving. The procedure is as follows.

(1) The amount of stock rolled ΔS_p at the pitch point of the middle divided tooth of the work is obtained for the given rolling stock ΔS when the work with a true involute profile is rolled by the die with a true involute profile.

(2) The required contact load F_i at each contact point is obtained by substituting the values obtained in (1) for ΔS_p into Eq. (2).

(3) Total elasto-plastic displacement at each contact point is obtained by substituting the values obtained in (2) for F_i into Eq. (4). The necessary amount of modification at each contact point is a difference between ΔS and the value obtained above. When the work has a tooth profile error $(e_w)_i$ before rolling, ΔS is considered to vary by $(e_w)_i$ at each contact point.

Since the edge effect in the direction of the tooth profile near the tip of the tooth is not considered so far in this deformation model, the amounts of stock rolled ΔS_p calculated near the tip are relatively small, while ΔS_p actually rolled near the tip are rather large and the tip is rounded off. This results in excess amounts of modification calculated near the root of the die tooth, and a preliminary experiment with this die shows that the tip is excessively rounded off after rolling.

The indentation experiment of rectangular works 45 mm long, 25 mm thick and

Fig. 7 Increase in depths of elastic and plastic indentations near the end surface of a rectangular work

20, 25, 30 mm wide by cylindrical tools 10, 15 mm in radius is done near the end surface in the longitudinal direction of the work under various loads. Figure 7 shows an example of the experimental results. The solid curves in Fig. 7 are the curves of hexagonal equations approximated by the least squares method. Depths of elastic and plastic indentations are greater near the end surface. In this chapter, this end effect in the direction of the tooth profile is taken into consideration for the deformation model. Ratios of depths of indentation near the end surface to ones at the middle of the work are expressed as functions of the distance from the end surface.

Components of displacement calculated using this new deformation model are shown in Fig. 8 as an example. (ΔS_p : depth of plastic indentation of the work, ΔS_{kbw} : deflection of the work, ΔS_{uzw} : depth of elastic indentation of the work, ΔS_{kbd} : deflection of the die, ΔS_{uzd} : depth of elastic indentation of the die), where the die and the work are assumed to have no tooth profile error. Amounts of deflection are small except near the tip, because deflections due to contact loads of both surfaces compensate each other. Depths of elastic and plastic indentations are large near the tip.

3.2 Results and discussion

Dimensions of the die and the work used in calculation and experiment are tabulated in Table 3.

An example of the modified tooth profile of the die calculated is shown in

Fig. 8 An example of calculation of components of displacement (external work driving, constant center distance $\Delta S = 40 \mu\text{m}$)

Fig. 9 Modified tooth profile of a die calculated for $\Delta S_p = 1 \mu\text{m}$

Table 3 Dimensions of dies and works

	Die with standard profile	Die with modified profile	Work
Module	3.0	3.0	3.0
Addendum modification coefficient	0.0	0.0	0.0
Pressure angle	20°	20°	20°
Number of teeth	66	66	31
Helix angle	20°L.H.	20°L.H.	20°R.H.
Diameter of reference circle	210.707	210.359	98.969
Diameter of tip circle	216.196	216.727	104.969
Diameter of root circle	204.169	201.350	91.469
Diameter of base circle	196.483	196.483	92.288
Face width	30	30	20
Material	SKD11	SKD11	S45C
Hardness	HRC60	HRC60	HV215

Fig. 10 Tooth profiles and tooth traces of dies

Fig. 11 Rolled tooth profiles (constant rolling load)
(A: before rolling, B: actually rolled, C: calculated)

Fig.9. The modified tooth profile of the die calculated for small amount of stock rolled is used in this experiment, referring to the preliminary experimental results. The tooth profiles and tooth traces of the dies used in the experiment are shown in Fig.10. A die without modification of the tooth profile (the standard tooth profile) is also used for comparison. There are little differences between the modified tooth profiles calculated on both the driven side and the follower side, so an identically modified tooth profile is produced on both sides of the die tooth used in the experiment to facilitate the die making.

The experiments are done for constant rolling loads and for constant center distances in the case of free work rotating. The works are rotated five times during rolling.

There are little differences between the experimental results for constant rolling loads and constant center distances. The results for $2\Delta S = 30, 45, 60 \mu m$ correspond to ones for $P = 6000, 9000, \text{ and } 12000N$ respectively. The rolled tooth profiles calculated and actually rolled are shown in Fig.11 (A:tooth profile before rolling, B:tooth profile actually rolled, C:rolled tooth profile calculated). In rolling for a low rolling load, 7000N, surface roughness is smoothed almost perfectly. The features before rolling are left on the surface of the follower side and the tooth profile of the driven side becomes slightly concave by the die with standard profile, while fairly true involute profiles are produced on the surfaces of both sides by the die with a modified profile. Increasing the rolling load causes the tooth profile accuracy to worsen after rolling by the die with standard profile; tip is rounded off and profile near the middle becomes concave. In rolling by the die with a modified profile under a high rolling load, 12000N, fairly true involute profiles are produced on the surfaces of the driven side, and slightly concave profiles on the surfaces on the follower side due to the small amount of modification of the tooth profile of the die, but the degree of concavity is lower than by the die with a standard profile.

In any case, the rolled tooth profile accuracy is found better by the die with a modified profile than by the die with a standard profile. The proposed method to determine theoretically the modified tooth profiles of the die is proved valid. The features of the tooth profiles calculated agree well with those actually rolled. Some differences, however, still remain between the profiles calculated near the tip and the root and those actually rolled, and this deformation model does not explain adequately the differences between the features of the profiles of the driven side and those of the follower side. These are left for study in the future.

Table 4 Amounts of stock rolled
(S:by the die with standard profile
M:by the die with modified profile)

Rolling condition		Experiment		Calculation
Constant rolling load	P=6000N	S	6	6
		M	4	5
	P=9000N	S	11	11
		M	7	8
	P=12000N	S	14	16
		M	11	13
Constant center distance	$2\Delta S=30\mu m$	S	9	7
		M	5	6
	$2\Delta S=45\mu m$	S	13	11
		M	8	12
	$2\Delta S=60\mu m$	S	17	15
		M	14	16

The sum of the amounts of stock rolled on both surfaces (the amount of decrease in tooth thickness) calculated agrees well with one actually rolled (Table 4). The method to choose the rolling conditions in calculation of the modified tooth profiles of the die and the method to determine the optimum conditions in actual rolling are also the future problems to be studied.

4. Conclusions

(1) A deformation model is proposed to analyze theoretically the rolled tooth profiles under various rolling conditions considering plastic indentation of the work, deflection and elastic indentations of the die and the work, and edge effects near the side edge and the tip of the tooth.

(2) The rolled tooth profiles calculated using this model are experimentally confirmed to agree well with those actually rolled.

(3) A method to determine the modified tooth profiles of the die based on this model is proposed, and its validity is experimentally proved.

Acknowledgement

The authors wish to thank Messrs. Y. Ooba, I. Nakagawa, and M. Miyaguchi for their contributions in part to this study and Nachi-Fujikoshi Corp. for its assistance in making the rolling dies.

References

- (1) Ozaki, T., et al., Bull. of the JSME, Vol. 20, No. 147 (1977-9), 1206.
- (2) Ozaki, T., and Imamura, A., Bull. of the JSME, Vol. 23, No. 180 (1980-6), 1003.
- (3) Ozaki, T., et al., Tech. Rep. of the Kyushu Univ. (in Japanese), Vol. 51, No. 5 (1978-10), 671.
- (4) Umezawa, K., Bull. of the JSME, Vol. 15, No. 90, (1972-12) 1632.
- (5) Umezawa, K. and Ishikawa, J., Bull. of the JSME, Vol. 16, No. 97 (1973-7), 1085.