Pulsating Flow in Curved Pipes*

(3rd Report, Axial Velocity Profile)

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The axial velocity of a laminar, fully-developed pulsating flow in curved circular pipes has been measured by means of a laser doppler velocimeter in a wide range of various parameters, i.e. radius ratio of pipes R/a , mean Dean number $\bar{D}$, Womersley number α and flow rate ratio η . The experimental results agree well with the solutions of the numerical analysis. The instantaneous velocity measured is classified into a time-mean component and varying ones and the effects of parameters on them are discussed. Explanations are given for the complicated behavior of the curved pipe pulsating flows.

Key Words: Fluid Mechanics, Curved Pipe, Pulsating Flow, Velocity Profile, Dean Number, Womersley Number, Flow Rate Ratio, Laser Doppler Velocimeter

1. Introduction

In recent years, oscillating and pulsating flows in curved pipes are attracting particular attention in connection with the flow in the circulatory system, the heat exchangers and the piping system in chemical reactors. For example, Smith⁽¹⁾ analyzed theoretically a pulsating flow in a curved pipe with arbitrary cross-section. Lin et al.⁽²⁾ studied numerically a pulsating flow in a curved circular pipe. Chandran et al.⁽³⁾ reported on the velocity profiles obtained experimentally in the pulsating flow through a curved circular pipe like the blood flow. However, these works were carried out under some limited conditions, and so they are not satisfactory from a practical point of view.

For the above reason, the authors⁽⁴⁾ analyzed numerically the pulsating laminar flow in curved pipes with circular cross-section and found that the velocity profiles and the distribution of wall shearing stresses were complicated. In this study, the axial velocities of a laminar, fully-developed pulsating flow in the curved circular pipes are measured by means of a laser doppler velocimeter in a wide range of various parameters dominating the flow. The experimental results are compared with the numerical ones to demonstrate their validity. The instantaneous velocity measured is classified into a time-mean component and varying ones, and the effects of the parameters on them are made clear.

2. Nomenclature (see Fig.1)

a : radius of a pipe
 D : Dean number = $Re \sqrt{a/R}$
 (r, θ) : coordinates
 R : curvature radius of a pipe
 Re : Reynolds number = $2aU/\nu$
 t : time
 U : instantaneous cross-sectional average velocity [Eq.(1)]
 w : instantaneous axial velocity [Eq.(2)]
 α : nondimensional frequency (Womersley number) = $a\sqrt{\omega/\nu}$
 η : flow rate ratio = $\bar{U}/\bar{U}$
 $\Theta = \omega t$
 ν : kinematic viscosity
 ϕ : phase difference [Eq.(2)]
 ω : angular frequency
 Superscripts
 $-$: time-mean value
 $\sim$: amplitude value

3. Experimental Apparatus and Procedure

The experimental apparatus is shown schematically in Fig. 2. The system is same as one employed for the measurement⁽⁵⁾ of the pressure losses except a test section and measuring devices of velocity, so we describe only the test section and the measuring devices.

Fig.1 Coordinate system

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The curved pipes are made of glasses and vinyl tubes as shown in Table 1. The vinyl tubes, whose elastic modulus and ratio of thickness to inside diameter are above 24MPa and 0.3 respectively, are shaped in a half circle by fitting them into a groove cut on acrylate plates. The tube No.1 is turned in two folds (pitch angle of about 4°) and ones No.2-4 are bent into a U-shape. A straight pipe, whose length is more than 160a, is connected with the entrance of the curved pipe so that a fully-developed, straight-pipe flow comes into it. A sufficiently large effluent tank is attached to the exit of the pipe to remove the influence of water level variation.

A laser doppler velocimeter (LDV) is used to measure the axial velocity on the plane including the pipe axis and the center of curvature ($\theta = \pm 90^\circ$). The optical system of the LDV (NIHON KAGAKU KOGYO: 27Ser.), the laser source of which is 5mW He-Ne gas laser, is a fringe mode and is mounted on a bench which is movable in the direction of the optical axis (0.015 mm/step). The doppler signal received through a frequency tracker (KANOMAX, 8015) is recorded in a recorder together with a time-marker signal which indicates the position of the piston. The tracker, however, can not follow the doppler signals being generated in an adverse flow, and so the velocity at that time is measured by reading directly the frequency of the doppler signal which is recorded in a digital memory scope. The cross angle of the laser beam on air is 30.0° and the measuring volume in the testing pipe is about 0.001 mm^3 . The working fluid is a city water and a small amount of aluminum powder is added to it as a seeding material.

The experimental conditions are shown in Fig. 3: nondimensional frequency $\alpha = 2 \sim 28$ (frequency 0.05~2Hz), mean Dean number $\bar{D} = 30 \sim 300$ (mean Reynolds number $Re = 80 \sim 1000$, mean velocity $\bar{U} = 0.4 \sim 10 \text{ cm/s}$) and flow rate ratio $\eta = 0.5$ and 1. The numerical values given in Fig. 3 denote test run number.

1:Variable speed motor, 2:Oscillatory flow generator, 3:Head tank, 4:Effluent tank, 5:Flowmeter, 6:Time marker device, 7:Laser doppler velocimeter, 8:Frequency tracker, 9:Recorder, 10:Band pass filter, 11:Digital memory scope, 12:Universal counter

Fig.2 Schematic diagram of experimental apparatus

The reports ⁽⁶⁾⁻⁽⁸⁾ on the entrance flow in a curved pipe suggest that the length of entrance region for an unsteady curved pipe flow is less than that in a steady one. So, using an empirical expressions ⁽⁹⁾ for the entry length in the steady flow, we first calculate the mean velocity at which the fully developed flow starts at the section of 90° from the inlet of the curved pipe. Then the measurements for the Nos.2-4 curved pipes (U-shape) are carried out under the condition that the maximum value of the cross-sectional mean velocity does not exceed the calculated one. As seen from Fig. 4, the velocity distributions which were obtained in the way described above agree approximately with the results measured at the section of 360° from the inlet of the No.1 pipe. From these results, we can consider that the measurements in the Nos.2-4 pipes were performed in a fully-developed flow. More-

Tab.1 Dimensions of test pipe

No.	Material	2a (mm)	R (mm)	R/a	Thick (mm)	Refractive Index
1	Glass	10.2	57	11.2	1.5	1.46
2	Glass	17.8	80	9.0	1.6	1.46
3	Vinyl	6.9	105	30.4	1.5	3.2
4	Vinyl	15.4	255	33.1	2.3	4.4

Fig.3 Scope of experimental condition

$\theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ$	R/a	α	$\bar{D}$	η	Test channel
$\circ \triangle \square \diamond$	11.2	55	29	0.49	No.1
$\bullet \blacktriangle \blacksquare \blacklozenge$	9.0	56	31	0.49	No.2
$\bullet \blacktriangle \blacksquare \blacklozenge$	33.1	55	27	0.47	No.4

Fig.4 Influence of radius ratio R/a on flow ($\alpha = 5.5$, $\bar{D} = 30$, $\eta = 0.5$)

over, we can recognize that the radius ratio has little effect on the flow within the range of the present experiment ($R/a = 9 \sim 33$).

4. Results and Discussion

In this experiment, a sinusoidally pulsating flow-rate is produced in the pipe by driving the piston periodically. So the instantaneous velocity averaged over the cross section can be represented as

$$U = \bar{U} + \bar{U} \sin \theta = \bar{U}(1 + \eta \sin \theta) \cdots \cdots (1)$$

Figure 5 shows some examples of the variation of axial velocity. The curves showing the wave-form deviate temporarily from a sine curve, since the frequency tracker can not follow the velocity change at $U \approx 0$. However, it can be assumed that the velocity changes sinusoidally on the whole. So, the local instantaneous velocity is expressed by the Fourier series with respect to time as

$$w(r, \theta, t) = \bar{w}(r, \theta) + \sum_{n=1}^{\infty} \bar{w}_n(r, \theta) \cdot \sin \{n\theta + \phi_n(r, \theta)\} \cdots \cdots (2)$$

where ϕ_n is the phase difference of the n -th component of velocity variation from that of flow rate given by Eq.(1). Here the first and the second components of the Fourier series are obtained by using more

than twelve experimental results which are measured at 30° intervals or more limited ones in a cycle.

The characteristics of the pulsating flow in the curved pipes depend on four nondimensional parameters of radius ratio R/a , mean Dean number $\bar{D}$ (or mean Reynolds number $\bar{Re}$), Womersley number α and flow rate ratio η (or oscillating Dean number $\bar{D}$, or oscillating Reynolds number $\bar{Re}$)⁽⁵⁾. But we don't refer to the effects of the radius ratio R/a on the flow hereafter since very little is known about its

Fig.5 Wave form of axial velocity
(Run 19; $\alpha=5.6$, $\bar{D}=297$, $\eta=1.07$, $R/a=11.2$)

Fig.6 Profiles of velocity and phase difference
(Flow at low Womersley number ($\alpha=5.5$, $\eta=1$))

effects in this experiment as described in the chapter 3.

4.1 Instantaneous velocity profile

Some of the velocity profiles are shown in Figs. 6-8(a)-(c) for $\alpha \approx 5.5, 10, 18, \bar{D} \approx 30, 90, 300$, and $\eta \approx 1.0$. The ordinate of the figures denotes $w/(\bar{U}+\bar{U})$, where $(\bar{U}+\bar{U})$ is the maximum value of the cross-sectional mean velocity. The abscissa shows a nondimensional distance r/a on the x axis. So the left and right sides of the figures correspond to the inside and outside walls respectively. The velocity profiles are illustrated for four phases, i.e. the maximum accelerative and decelerative phases ($\theta=0^\circ, 180^\circ$), and the phases of the largest and smallest flow rates ($\theta=90^\circ, 270^\circ$). The experimental results, except for a part of the results for the high mean Dean numbers, are in good agreement with the numerical solutions indicated by the solid lines in Figs. 6-8 (a)-(c).

At the end of the decelerative phase ($\theta=270^\circ$), the fluid flows in the adverse direction near the inside wall or near the inside and outside walls, and the profile of the velocity exhibit a striking unsymmetry with respect to the axis of $\theta=0$. The reversed flow, however, disappears at the latter half of the accelerative phase ($\theta=0^\circ$) and the velocity profiles for the low mean Dean number ($\bar{D} \approx 30$) become rela-

tively symmetrical and are convex in the central region of the pipe. But the velocity profiles become similar to the Dean-type one with an increase in the mean Dean number, in which the location of the maximum velocity is shifted to the outside wall, due to the remarkable effects of the curvature of the pipe on the flow. When the Womersley number is high, the velocity drops near the central region of the pipe at the latter half of the accelerative phase or between that stage and the first half of the decelerative phase ($\theta=0^\circ \sim 180^\circ$) [see Figs. 7(b), (c) and 8(c)].

As mentioned above, the profile of the axial velocity is different from that observed in the steady flow and varies complicatedly in phase during a cycle. So, in order to clarify the reason why the velocity profile curve shows the peculiar behavior and the effects of the Womersley and mean Dean numbers on the velocity profile, the instantaneous velocity is classified into a time-mean component and varying ones ($\bar{w}$ and $\bar{w}_n$ respectively) on basis of Eq.(2). Then we discuss how $\bar{w}$, $\bar{w}_n$ and the phase difference ϕ_n influence the velocity profile.

4.2 Time-mean component, amplitude and phase difference

The results of the time-mean component $\bar{w}/\bar{U}$, the amplitude $\bar{w}_1/\bar{U}$ and the phase difference ϕ_1 of the first component are

Fig.7 Profiles of velocity and phase difference
[Flow at moderate Womersley number ($\alpha=10, \eta=1$)]

shown in Figs. 6~8(d)~(f). Though the second component $\tilde{w}_2$ generally increases with the mean Dean number $\bar{D}$, it is small compared with the first component, being under about 7% of the first one. Therefore, we describe only the fundamental wave component ($n=1$) and the suffix 1 is omitted for simplicity. The solid lines in Figs. 6~8(d)~(f) denote the results of the numerical analysis described in the preceding report⁽⁴⁾. In the figures (d)~(f), the numerical results of both $\tilde{w}$ and ϕ on the y axis are indicated by the broken lines to compare with those on the x axis.

$\bar{w}/\bar{U}$ shows the Dean-type profile, in which the maximum velocity occurs near the outside wall independently of the Womersley number α . The location of the maximum velocity is shifted toward the outside wall with an increase in the mean Dean number $\bar{D}$. From the results, it is seen that the Womersley number α has little significance for $\bar{w}$.

The first component $\tilde{w}/\bar{U}$ has the maximum value near the inside wall when the Womersley and mean Dean numbers are low [see Fig. 6(d)]. As the mean Dean number $\bar{D}$ increases, the location of maximum $\tilde{w}/\bar{U}$ for low Womersley number, however, is shifted towards the outside wall and its profile becomes similar to that of $\bar{w}/\bar{U}$. On the other hand, with an increase in the mean Dean number $\bar{D}$ the profile of $\tilde{w}/\bar{U}$ on the y axis changes from the convex one [Fig. 6(d)] to the annular one, in which

the maxima is located near the wall as seen in Fig. 6(f). Conversely, in the case of high Womersley numbers, the inertial effect is very significant for the flow variation and makes the profile of $\tilde{w}/\bar{U}$ a plug flow type in which the velocity has almost the same value except in the vicinity of the wall [see Fig. 8(d) and (e)]. As the mean Dean number increases, the plug-flow profile for high α is distorted and the velocity on the x axis has the maximum value near the inside wall as shown in Fig. 8(f). For the moderate Womersley numbers α , the profile of $\tilde{w}/\bar{U}$ is approximately uniform in the central region of the pipe though it swells a little near the inside wall [see Fig. 7(d) and (e)]. With an increase in the mean Dean number $\bar{D}$, $\tilde{w}/\bar{U}$ profile shows a swelling at the central region of the pipe and becomes nearly symmetric with the pipe axis [see Fig. 7(f)]. From the observation of the profile $\tilde{w}$, it is found that the reversing-flow phenomenon near the inside wall or near the inside and outside walls depends mainly on $\tilde{w}$.

The phase difference ϕ for the low Womersley number ($\alpha \approx 5.5$) has a positive value at the inner region and a negative one at the outer region. In other words, the phase of $\tilde{w}$ leads at the inner region and delays at the outer one. On the other hand, the phase of $\tilde{w}$ on the y axis leads near the wall. For the low mean Dean and Womersley numbers, the phase delays at the

Fig.8 Profiles of velocity and phase difference
(Flow at high Womersley number ($\alpha=18, \eta=1$))

Fig.9 Profiles of velocity and phase difference
(Flow at low flow rate ratio ($\eta=0.5$, $\bar{D}=300$))

central region of the pipe but it comes to lead with an increase in $\bar{D}$. Contrarily, for the high Womersley number ($\alpha \approx 18$), the phase difference is approximately a negative constant value in the cross section irrespective of the mean Dean number $\bar{D}$, and is a positive value in the extreme vicinity of the wall. In case of the moderate Womersley number ($\alpha \approx 10$), ϕ shows a profile approximately similar to that for the high Womersley number, but the phase lag in the central part of the pipe is remarkable. From the above discussion on ϕ profile, the occurrence of a depression in the curves of w profile [see Figs. 7(b), (c) and 8(c)] can be explained by considering the phase lag in the central region of the pipe and the profile of $\tilde{w}$.

Some examples of the results for $\eta \neq 0.5$ are shown in Fig. 9. When the flow rate ratio decreases, the mean flow affects greatly the phenomena and the velocity profile becomes similar to that in the steady flow throughout a cycle, though the reversed flow appears near the inside wall at the end of the decelerative phase for the moderate and high Womersley numbers ($\alpha \approx 10, 18$). The profiles of $\bar{w}$, $\tilde{w}$, and ϕ , with the exception of the high Womersley and mean Dean numbers as in Fig. 9(f), are close to the profiles for the case of $\eta=1$.

5. Conclusions

The axial velocity profile of a

fully-developed pulsating flow in curved pipes was measured under the conditions of radius ratio $R/a=9\sim33$, nondimensional frequency $\alpha=2\sim28$, mean Dean number $\bar{D}=30\sim300$ and flow rate ratio $\eta=0.5$ and 1. As typical examples of the results, the results for $\alpha \approx 5.5, 10, 18, \bar{D} \approx 30, 90, 300$, and $\eta \approx 1$, and a part of the results for $\eta \neq 0.5$ are illustrated. The relations between the instantaneous velocity and the various parameters are made clear by means of the experimental results and numerical ones in our preceding report⁽⁴⁾. Here the instantaneous velocity w is classified into a time-mean component $\bar{w}$ and varying ones $\tilde{w}_n$, and the effects of the various parameters on $\bar{w}$, $\tilde{w}_n$ and the phase difference ϕ_n are discussed. The explanations are given for the complicated behaviors of the pulsating flows in curved pipes, for example, the phenomena of a reversing flow, a depression in the velocity profile at the central region of the pipe and so on.

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