Numerical Simulation of the Effects of Coriolis Force
on the Structure of Turbulence*
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Fully-developed turbulent flow in a rotating channel is simulated numerically by means of Large-eddy Simulation(LES). It is intended to supplement the difficulty of experiments of this type of flow with numerical analysis. LES is successfully applied to reasonably reproduce notable features of the effects of Coriolis force on the turbulent shear flow and its structure. Namely, the stabilization of the flow on the suction surface and the intensification of turbulence on the pressure surface by Coriolis force are confirmed. Also, modification of the time averaged flow due to the above effect is demonstrated in detail. It is also suggested that the subgrid-scale model of a very low intensity turbulence must be improved.

Key words : Fluid Mechanics, Turbulence, Numerical Simulation, Large-eddy Simulation, Rotating Channel, Coriolis Force

1. Introduction

Coriolis force has influence on the structure of a turbulent shear flow in a rotating flow field. This type of flow has attracted attention because it is a typical example of a turbulent flow affected by the body force⁽¹⁾. Also, from an industrial point of view, it is important as a flow appearing in the blade passage of a centrifugal impeller.

Therefore some results of experiments have been reported in spite of difficulty in measurement. For example, Johnston et al.⁽²⁾ treated a fully developed turbulent flow in a rotating channel and Koyama et al.⁽³⁾⁽⁴⁾ measured a turbulent boundary layer developing along the rotating plate. They say that Coriolis force increases frictional stress and turbulent energy on the pressure side while it stabilizes the flow near the suction surface. And it is also demonstrated that a secondary flow like a Taylor-Görtler type vortex is observed on the pressure side.

For numerical prediction, turbulence modelings based on Reynolds equations are widely applied, namely, mixing length model(Johnston et al.⁽²⁾), one equation model(Koyama et al.⁽⁵⁾), $k-\epsilon$ model(Wilcox et al.⁽⁶⁾), Howard et al.⁽⁷⁾), and stress equation model(Masuda et al.⁽⁸⁾).

Separately from the techniques cited above, large-eddy simulation(LES) is attracting as a numerical method with the least empirical assumption. LES is a method that separates a turbulent flow into two parts, i.e. large-scale field and small-scale fluctuation from it. Then,

three-dimensional time-dependent Navier-Stokes equations for large-scale motion are directly integrated. On the other hand, a dissipative effect of unresolvable eddies is modelled and included into the former equations. Because of advantages of universality of LES model and plentiful information obtained from the calculation, LES promises to be a useful tool for the understanding of the turbulence structure. Although LES requires large capacity of memory and computational time, it is becoming a practical method with the development of recent supercomputers. LES is successfully being applied to the turbulent flow through a nonrotating plane channel.^{(9)~(11)}

The authors have already applied LES to the turbulent channel flow and proposed a model on the mechanism of the production of wall turbulence⁽¹¹⁾. The aims of the present study are to predict the entire flow field in the rotational channel and to consider the effects of Coriolis force on the structure of turbulence. In the experiment on a rotating system, it is difficult to realize an ideal flow field and it is also hard to make an accurate measurement. As numerical simulation is a sort of experiment in which it is easy to set up a lot of conditions simultaneously, it is expected to relieve the difficulty of experiment.

In this paper, global effects of Coriolis force on a mean flow and statistical properties of turbulence are demonstrated in detail and applicability of LES to the flow affected by the body force is discussed.

2. Governing Equations

A fully developed turbulent flow of an incompressible fluid between parallel flat plates with a steady rotation rate, as shown in Figure 1, is calculated. The coordinate system rotates with plates, and x_1 is taken in the normal direction to the

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Figure 1. Rotating channel and coordinate system

plates; x_2 , in the downstream direction; and x_3 , in the spanwise direction which is the axis of rotation. Relative velocity is represented by u_i , distance from the axis of rotation by r , and angular velocity by ω .

2.1 Governing equations for the large-scale flow field In LES, along with the filtering procedure⁽¹¹⁾, each variable(u) in the flow field is decomposed as follows:

$$u = \bar{u} + u' \quad (1)$$

where $\bar{u}$ is the large-scale field and u' is the deviation from u . Furthermore, an average in the x_2 - x_3 plane parallel to the wall is denoted by $\langle \cdot \rangle$, and the fluctuation from it by f'' , i.e.,

$$u = \langle u \rangle + u'' \quad (2)$$

Substituting $u_i = \bar{u}_i + u'_i$ and $p = \bar{p} + p'$ into the governing equation on the reference frame and then applying the filtering operation, we obtain equations of motion and continuity for the large-scale flow field:

$$\begin{aligned} \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_i \bar{u}_j + \lambda_{ij}) \\ = 2\omega (\delta_{i1} \bar{u}_2 - \delta_{i2} \bar{u}_1) - \frac{\partial}{\partial x_i} (\bar{p} - \frac{r^2 \omega^2}{2}) \\ + \frac{1}{Re^*} \frac{\partial^2 \bar{u}_i}{\partial x_k \partial x_k}, \end{aligned} \quad (3)$$

$$\frac{\partial \bar{u}_k}{\partial x_k} = 0. \quad (4)$$

All variables are nondimensionalized by means of the channel width H and the effective wall frictional velocity u_e^* based on the mean friction stress on both sides defined as

$$u_e^* = \sqrt{(|\tau_0| + |\tau_1|)/2\rho}, \quad (5)$$

where ρ is the density which is assumed constant. And the time t is nondimensionalized by H/u_e^* , angular velocity ω by u_e^*/H , pressure p by ρu_e^{*2} , respectively. Re^* is the Reynolds number defined by

$$Re^* = \frac{H u_e^*}{\nu}. \quad (6)$$

The tensor λ_{ij} in eq.(3)

$$\lambda_{ij} = \overline{u'_i u'_j} + \overline{u'_i u'_j} + \overline{u'_i u'_j} \quad (7)$$

means an influence of unresolvable eddies which must be modelled to complete the equations for the large-scale motion (eq.(3),(4)).

Table 1. Computational grids

	1 (normal)	2 (stream)	3 (span)
N_i	32	32	32
H_i	1.0	4.8	1.2
h_i	0.0024~0.049	0.150	0.038

2.2 Modeling of small scale turbulence

In the present calculation, λ_{ij} is related to the velocity deformation tensor of the large-scale field

$$\bar{D}_{ij} = \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \quad (8)$$

introducing the concept of eddy viscosity as follows:

$$\lambda_{ij} = \frac{1}{3} \delta_{ij} \lambda_{kk} = -K_H \bar{D}_{ij} - K_I \langle \bar{D}_{ij} \rangle \quad (9)$$

which is the same as that used in the calculation of the stationary channel flows⁽¹¹⁾. The first term of the right hand side of the above equation is a homogeneous part(H) and the second term is an anisotropic part(I). Eddy viscosity coefficients of small-scale eddies on the resolvable motion, K_H and K_I , are given by

$$\begin{aligned} K_H &= (C_H d)^2 \sqrt{\bar{D}_{ij} \bar{D}_{ij} / 2}, \\ K_I &= (C_I d)^2 \sqrt{\langle \bar{D}_{ij} \rangle \langle \bar{D}_{ij} \rangle / 2}, \end{aligned} \quad (10)$$

where we use the value of $C_H=0.07$ and $C_I=0.14$ for the dimensionless constants. d is the representative length of the filter defined by $d = (d_1 d_2 d_3)^{1/3}$. In the region very close to the wall, because the turbulence attenuates while an effect of viscosity is predominant, d is damped following the empirical functions for each of K_H and K_I :

$$d_H = 1 - \exp\left(-\frac{x_w^+}{25}\right), \quad d_I = 1 - \exp\left\{-\left(\frac{x_w^+}{25}\right)^2\right\}, \quad (11)$$

where x_w^+ is a distance to the nearest wall measured by wall unit (ν/u_e^* or ν/u_1^*).

The model described above reflects the dissipative character of the small-scale turbulence. As the small-scale eddies have a universal feature, we guess that the influence of system rotation on an unresolvable motion is much smaller than the local rotation rate of the large-eddies and we make no modification to the model for the first approximation.

3. Numerical Method

As the computational scheme is the same as previously presented⁽¹¹⁾, we will give only an outline here.

As an actual turbulent flow field between the unlimited plates can be regarded as a field in which a similar structure appears repeatedly in the streamwise and spanwise directions, we assume a rectangular computational box and distribute grid points within it. Solving the simultaneous equation composed of equations of motion(3) and continuity(4), the turbulent eddies are simulated.

Computational domain H_i , the number of grids N_i , and grid size h_i for each direction are shown in Table 1. In the directions x_2, x_3 parallel to the walls, grids are distributed uniformly and the filter width is twice as large as the grid size. In the direction normal to the walls, grid points are non-uniformly located to make the resolution fine near the wall.

Although the number of the computational grids is limited because of the size of the memory of the available computer, the resolution ($h_i^* = h_i u^* / \nu$) is required to be enough to simulate the turbulent structure in the wall region. Using the grid system shown in Table 1, Reynolds number $Re^* = 500$, which makes the grid resolution in each direction $h_{1min}^* = 1.20$, $h_2^* = 75.0$, and $h_3^* = 18.8$ respectively for the nonrotating case, seems to be the upper limit.

On the wall, no-slip velocity boundary condition ($u_i = 0$) is applied. In x_2, x_3 directions, periodic condition is assumed.

In the time advancement, the leap-frog method is applied for the nonlinear term and Coriolis force term. For the viscous and small scale terms, the values at the preceding time-step are used. Pressure and equation of continuity are treated implicitly. Then, Fourier transforms of velocity and pressure are solved simultaneously.

An instantaneous distribution of the velocities calculated in the stationary channel⁽¹¹⁾ is used as the initial flow field and given at the beginning of the simulation. The governing equations are integrated forward in time with a slight rotation rate. It takes 1000 steps till a fully developed flow is confirmed to have been obtained. Another 2000 steps are taken to get the time average which is represented by $\langle u \rangle$. Furthermore, the calculation is continued for a larger rotation rate.

The computational time was 2.2 sec per time-step with the NEC Highspeed-Fortran-Processor (HFP) at the Computation Center of Osaka University.

4. Global Effects of Coriolis Force

The tendency in the change of entire flow field affected by Coriolis force can

(a) $\langle u_2 \rangle$ v.s. x_1

be estimated roughly by considering the production terms in the equations of the resolvable Reynolds stress tensor $\langle \bar{u}_i \bar{u}_j \rangle$. The production of shear stress $\langle u_1 u_2 \rangle$ is represented by

$$\langle P_1 \rangle = -\langle \bar{u}_1^2 \rangle \frac{\partial \langle \bar{u}_2 \rangle}{\partial x_1} + 2\omega \langle \bar{u}^* z^2 - \bar{u}_1^2 \rangle. \quad (12)$$

And those of the vertical components in three directions $\langle \bar{u}_k \bar{u}_k \rangle$ (no summation) are written as

$$\langle P_1 \rangle = 0 + 4\omega \langle \bar{u}_1 \bar{u}^* z^2 \rangle, \quad (13a)$$

$$\langle P_2 \rangle = -2\langle \bar{u}_1 \bar{u}^* z^2 \rangle \frac{\partial \langle \bar{u}_2 \rangle}{\partial x_1} - 4\omega \langle \bar{u}_1 \bar{u}^* z^2 \rangle, \quad (13b)$$

$$\langle P_3 \rangle = 0 + 0 \quad (13c)$$

respectively. So the production of the turbulent energy of the resolvable scale

$$\langle \bar{q}^2 \rangle = \frac{1}{2} \langle \bar{u}_1^2 + \bar{u}^* z^2 + \bar{u}_3^2 \rangle \quad (14)$$

becomes

$$\langle P_q \rangle = -\langle \bar{u}_1 \bar{u}^* z^2 \rangle \frac{\partial \langle \bar{u}_2 \rangle}{\partial x_1} + 0. \quad (15)$$

The second terms of the right hand side of eqs. (12), (13), (15) represent the effect of rotation. $2\omega \langle \bar{u}^* z^2 - \bar{u}_1^2 \rangle$ in eq. (12) is an additional shear stress corresponding to an anisotropy of turbulent intensity. Although production of turbulence energy is not explicitly dependent on ω , the distribution of the turbulent energy is also affected by Coriolis force implicitly through the change of turbulent shear stress. Moreover, energy exchange between the two components is caused by the rotation as shown in the second terms of eq. (13a), (13b). Of course, energy redistribution among three components caused mainly by the velocity-pressure correlation term is also remarkable.

In this chapter, keeping the relations described above in mind, we will present the results about the global flow field.

4.1 Mean velocity and turbulent intensity

4.1.1 Mean velocity distribution

Figure 2(a) shows the mean streamwise velocity profile $\langle u_2 \rangle$ obtained for some rotation rates. When the rotation is applied, the two walls $x_1 = 0$ and $x_1 = 1$ become

(b) $\langle u_2 \rangle$ v.s. x_1^*

Figure 2. Mean velocity profiles

suction and pressure surface respectively. While the velocity gradient is mild in the suction side, that on the pressure surface is steep in the region very close to the wall, reflecting the change of the shear stress described later. Although Reynolds number R_c^* is fixed to the value of $R_c^*=500$ in all the cases, Reynolds number based on the mean velocity across the walls

$$R_{cm} = \frac{HU_{2m}}{\nu} \quad (16)$$

changes since the stream flux decreases in the central area $0.3 < x_1 < 0.8$. For example, $R_{cm}=9900$ when $\omega=0$, and $R_{cm}=9100$ when $\omega=3$.

Velocity profile in the semi-log plane is shown in Fig.2(b). When rotation rate ω increases, the thickness of the viscous layer decreases and the logarithmic region becomes clear on the pressure side. On the other wall, the logarithmic region comes to disappear as the rotation increases.

4.1.2 Reynolds stresses Figure 3 shows the profiles of the resolvable Reynolds shear stress $\langle u_1 u_2^* \rangle$. As total shear stress should be rectilinear across the width to balance the mean pressure gradient, the lack of the shear stress near the wall results from the contribution by the turbulent shear stress of small-scale motion λ_{12} and the viscous shear stress. When the rotation is applied, the profile of $\langle u_1 u_2^* \rangle$ shifts towards the positive direction due to the second term of eq.(12) and the symmetry is broken. Consequently the wall shear stress decreases on the suction surface and increases on the pressure side.

The turbulent intensity in each direction is shown in Figure 4. In the suction side, since both of the Reynolds stress and the gradient of the mean velocity decrease in eq.(15), so does the turbulence production and hence the flow is stabilized. Contrarily, the turbulence is intensified remarkably in the pressure side. With respect to the streamwise velocity, the turbulent intensity increases in the region very close to the pressure surface, but away from the wall the component of this direction decreases even in the unstable side because Coriolis force acts to transfer energy from the streamwise to the vertical component as shown in the second term in eqs.(13a),(13b).

When the rotation rate increases, as

Figure 3. Resolvable turbulent shear stress $\langle u_1 u_2^* \rangle$

the vertical component of energy comes to exceed the streamwise one, the shift of the shear stress profile is saturated.

The results of the present calculation concerning the effects of Coriolis force on the mean velocity and the turbulence statistics are consistent with the estimation from eqs.(12),(13),(15) and also with the knowledge from the experimental evidence⁽²⁾ so far as the turbulence is maintained in both sides.

4.1.3 Frictional velocity on the wall

Figure 5 shows the ratio of the wall frictional velocity of the rotating channel to that of the nonrotating one. Here, to compare with the experimental data by Johnston et al.⁽²⁾, the horizontal axis is scaled by the non-dimensional rotation rate $R_0 (= \omega H / U_{2m})$. Dashed curve in Fig.5 is the estimation based on the experiment by Johnston et al. when Reynolds number is

(a) Intensity of velocity normal to the wall $\bar{u}_1$

(b) Intensity of streamwise velocity $\bar{u}_2$

(c) Intensity of spanwise velocity $\bar{u}_3$

Figure 4. Root mean square values of resolvable turbulent velocity

$R_{\omega} \approx 10000$. Although experimental data are scattered widely around the dashed line, agreement between the experimental data and the present result is roughly good up to $R_0=1$. For a large rotation rate, though a considerable fall of the wall friction caused by the relaminarization of the flow near the suction surface is observed in the experiment, there is not such a change of the situation in the flow field calculated by LES. The reason can be attributed to the fact that the model of unresolvable motion in LES is constructed on the assumption that the flow is completely turbulent. The dissipative contribution modeled by eq.(10) requires that the unresolvable-scale turbulence continue to draw energy from the large-scale eddies through the cascade process so long as the gradient of the mean velocity exists even if the flow is laminar, therefore the main flow can not be relaminarized. We guess that some modification is necessary to make it possible to realize relaminarization phenomena by LES.

Another outstanding feature of our result is that the effect of Coriolis force appears exaggeratedly when the rotation is not so fast. The reason may be that the strong anisotropy, which is mainly due to the low Reynolds number effect, in the turbulent energy in the entire flow field causes a overshift of shear stress profile through the aforementioned process.

4.2 Statistical properties of the scale of turbulence

4.2.1 Mixing length Figure 6 shows the ratio of the mixing length in rotational case to that in nonrotational case, i.e., l/l_0 . Richardson number proposed by Bradshaw⁽¹⁾

$$R_i = \frac{2\omega}{\partial \langle u_2 \rangle / \partial x_1} \left(1 + \frac{2\omega}{\partial \langle u_2 \rangle / \partial x_1} \right) \quad (17)$$

is adopted to scale the horizontal axis. The modification model⁽¹⁾ of the mixing length by the rotation using Richardson number

$$\frac{l}{l_0} = \frac{1}{1 + \beta R_i} \quad (18a)$$

Figure 5. Relation between the rotation rate and the frictional velocity on the wall

or

$$\frac{l}{l_0} = 1 - \beta R_i \quad (18b)$$

is also given in Fig.6. Johnston et al. claim that this modification is efficient so far as the flow in the wall region is completely turbulent and $|R_i| < 0.25$, and that Monin-Oboukhov constant β takes the value 6 ± 2 . The curve giving the relation between l/l_0 and R_i obtained in the present calculation shows steeper inclination near the wall than that of eq.(18), which corresponds to the fact described in Fig.5. Although it is impossible to discuss the suitability of eq.(18), we guess that it is necessary to modify the equation in the low Reynolds number flow.

4.2.2 Autocorrelation coefficients In this section, the effect of the rotation on the scale and the structure of turbulence is examined by means of the autocorrelation curve of the fluctuating velocity in streamwise and spanwise directions. Figure 7(a) shows the autocorrelation coefficients of u'_2 for the distance y in the streamwise direction,

$$R_{22}(y) = \frac{\langle \bar{u}'_2(x_1, x_2, x_3) \bar{u}'_2(x_1, x_2 + y, x_3) \rangle}{\langle \bar{u}'_2^2 \rangle} \quad (19)$$

which are taken at the location 0.0215 away from both walls. Figure 7(b) shows the autocorrelation of u'_2 for the distance z in the spanwise direction

$$R_{22}(z) = \frac{\langle \bar{u}'_2(x_1, x_2, x_3) \bar{u}'_2(x_1, x_2, x_3 + z) \rangle}{\langle \bar{u}'_2^2 \rangle} \quad (20)$$

at the same vertical distance. On the pressure surface the correlation curve descends steeply and the scale becomes smaller when the rotation is applied. So, the flow field comes to assume a character of high Reynolds number turbulence, but the structure of turbulent eddies in the wall

Figure 6. Ratio of the mixing length

Figure 7. Autocorrelation function of $\bar{u}_2'$ at the location 0.0215 away from the wall

region does not change substantially. On the suction surface, the descent of the curve becomes mild. A disappearance of the streak structure shown in Fig.7(b) is thought to be due to a marked reduction of the occurrence of a burst event. Therefore the flow field is recognized as laminar even though the complete relaminarization does not occur.

5. Conclusions

In this study, a fully developed turbulent flow in a rotating channel has been simulated numerically by LES and the global effects of the imposed rotation are investigated. Conclusions are summarized below:

- (1) Notable features of the effects of Coriolis force were successfully reproduced. Namely, the stabilization of the flow and reduction of frictional stress on the suction side and the intensification of turbulence and increase of friction on the pressure surface were confirmed.
- (2) Though the relaminarization on the suction side observed in the experiment when the rotation rate is high is not reproduced by LES, the large-scale motion reasonably reflects the characteristics of the effect of rotation, namely the reduction of the burst event in the wall.
- (3) The model of Smagorinsky type used in

LES for a very low intensity turbulence should be improved to realize the inverse transition to laminar flow exactly.

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