The Vibration of Rotating Cylindrical Shells*

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The free vibration of rotating cylindrical shells is examined in reference to that of a thin rotating ring. The results of the experiment and the theoretical analysis for four kinds of boundary conditions (i.e., (A) both ends clamped; (B) simply supported without axial constraint; (C) simply supported with axial constraint; and (D) clamped at one end and free at the other end) are presented. It is found that the dependence of the frequencies upon the rotating speeds is insignificantly affected by the boundary conditions, and thus can be represented by the simple relation for a thin rotating ring, provided that the frequencies and rotating speeds are normalized by the natural frequencies of a non-rotating cylindrical shell.

Key words: Vibration of Continuous System, Rotating Cylindrical Shell, Initial Tension, Modified Galerkin's Method, Frequency Analysis.

1. Introduction

Thin circular cylindrical shells are used in many engineering applications, such as high-speed centrifugal separators or gas turbines for high-power air-craft engine. The characteristics of the flexural vibration under high-rotating speed is a complicated problem. In previous papers, the authors derived Flugge type basic equations, reasonably considering the effects of the initial tensions due to the steady rotation, and examined the vibration characteristics theoretically, analyzing the frequencies under various boundary conditions.(1)-(3) There, we noted that the instability never happens in the case of the circumferential wave number being more than two and moreover the curves of the frequencies vs. the rotational speed of a cylindrical shell for each boundary condition are approximately represented by the simple relation for a thin rotating ring.

As for the experimental research on rotating cylindrical shells, there exist some papers which treated thin rotating rings as one-dimensional model for cylindrical shells.(4)(5) It is considered that to confirm the validity of those theoretical analyses, the experiment has to be performed, in high rotational ranges reaching the natural frequency of a non-rotating cylindrical shell. However, when both ends of a cylindrical shell are

supported, the natural frequencies are very high and so it is difficult to perform the experiment. Therefore, it seems that using a thin rotating ring, which is assumed to be an infinite length model, experiments have been carried out.

The purpose of the present paper is to perform an experiment with high rotational speed to examine the characteristics of the vibration of a rotating cylindrical shell, especially reference to that of a thin rotating ring. In previous papers, we assumed that the initial tensions were constant in the axial direction,(1)-(3) but in this paper we assume the initial tension which vary in the axial direction.

2. Experiment

2.1 Experimental apparatus

The experimental apparatus is composed of a rotor driving system, a magnetic excitation device. Figure 1 shows the experimental apparatus and the measuring system. In order to lower the natural frequencies in the non-rotating state, the circular cylindrical shell was clamped at one end and a circular disk was fitted near the free end. The frequencies of the cylindrical shell were measured by means of strain gauges through a slip ring.

2.2 Method of experiment

The rotating angular velocity Ω , the exciting radian frequency ω and the natural radian frequency p were measured, to seek the resonant point by exciting the rotating cylindrical shell with the electric magnet. For the same rotational speed and natural mode, two resonant states occurred corresponding to either of forward and backward waves. The

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relation is given as follows, (4)

$$\omega = p \pm n\Omega \quad (1)$$

where n is the circumferential wave number and the signs $+$ and $-$ correspond to forward and backward wave, respectively.

2.5 Experimental results

Table 1 shows the experimental result of the natural frequency $p/2\pi$ under the rotating speed $\Omega/2\pi$ for the circumferential wave number $n=2, 3$ and 4. We may give some remarks on the experimental results. It is reasonable that the boundary condition in the experiment may be identified approximately clamped at one end and free at the other end, but it is difficult to attain strictly a clamped condition. So, the rotating angular velocity Ω and natural angular frequencies p are normalized by the natural angular frequency p_n of a non-rotating cylindrical shell and are

Fig.1 Setup of experimental apparatus and measuring system

substituted into the following theoretical formulas for a thin rotating ring, (4)

$$\frac{p}{p_n} = \frac{2n}{n^2+1} \left(\frac{\Omega}{p_n} \right) \pm \sqrt{1 + \frac{n^2(n^2-1)^2}{(n^2+1)^2} \left(\frac{\Omega}{p_n} \right)^2} \quad (2)$$

Figure 2 (a)-(c) shows the results for the circumferential wave number $n=2, 3$ and 4. The symbol $\circ$ represents the experimental results and the solid line corresponds to Eq. (2). It is found that we can predict the relation between the rotational speeds and the natural frequencies of a rotating cylindrical shell from Eq. (2) if we know the natural frequencies of a non-rotating cylindrical shell.

3. Theoretical Analysis

It is very difficult to carry out the experiments of a circular cylindrical shell under various boundary conditions. In this chapter, the theoretical analysis will be worked out to characterize the vibration of rotating cylindrical shells in reference to Eq. (2) for a thin rotating ring. A modified Galerkin's method (6) is employed in order to consider the distribution of the initial tensions in the axial direction of a circular cylindrical shell which previous analyses neglected.

The coordinate system is shown in Figure 3 as in previous papers. The circular cylindrical shell which has mean radius a , thickness h and length l is assumed to rotate steadily at the angular

n	Natural frequency $p/2\pi$ (Hz)					
	2		3		4	
Rotating speed $\Omega/2\pi$ (Hz)	Backward	Forward	Backward	Forward	Backward	Forward
0	383	383	334	334	544	544
30	404	360	360	324	567	541
50	425	344	385	323	586	544
75	454	331	429	332	640	575
100	481	321	471	350	702	605
120	506	315	518	369	713	626
150	544	306	583	400	829	680
175	583	-	642	428	900	-
200	618	-	704	459	955	-
225	648	-	768	-	1095	-

(Material: steel, outer diameter 74.3mm, thickness 0.2mm, length 243mm)

Table 1 Natural frequencies measured in the experiment

Fig.2 Frequencies of forward and backward waves

Fig. 3 Coordinate system

velocity Ω . The steady rotation generates displacements u^* and w^* and stress resultants N_x^* and N_ϕ^* . We superimpose on u^* and w^* infinitesimal displacements $\bar{u}$, $\bar{v}$ and $\bar{w}$ due to vibration. (1)

3.1 Boundary conditions

We will treat four kinds of boundary conditions: (A) both ends clamped; (B) simply supported without axial constraint; (C) simply supported with axial constraint; and (D) clamped at one end and free at the other end. Starting from Hamilton's principle, in order to include effect of steady rotation the boundary conditions are derived as follows.

At $\xi = \pm l/2a \equiv \pm \alpha$:

$$\left. \begin{aligned} \text{(A)} \quad & u^* = w_{,\xi}^* = 0, w^* = x\Omega^2 \\ \text{(B)} \quad & N_x^* = M_x^* = 0, w^* = x\Omega^2 \\ \text{(C)} \quad & u^* = M_x^* = 0, w^* = x\Omega^2 \\ \text{(D)} \quad & \text{at } \xi = 0: \\ & u^* = w_{,\xi}^* = 0, w^* = x\Omega^2 \\ & \text{at } \xi = 2l/2a = 2\alpha: \\ & N_x^* = M_x^* = M_{x,\xi}^* = 0 \end{aligned} \right\} \dots\dots\dots (3)$$

where M_x^* are moment resultants. Here we assume that both ends of a circular cylindrical shell are fitted with circular plates of constant thickness which yields $w^* = x\Omega^2$, where $x = (1-\nu)\rho a^3/(4E)$, E and ν are Young's modulus and Poisson's ratio respectively, $\xi = x/a$ and comma (,) implies partial differentiation.

3.2 The initial tensions

The equilibrium equations in the steady rotating state are given as follows. (1)

$$\left. \begin{aligned} N_{x,\xi}^* &= 0 \\ N_\phi^* + \frac{1}{a} M_{x,\xi\xi}^* &= \rho h a \Omega^2 (a + w^*) \end{aligned} \right\} \dots\dots\dots (4)$$

where ρ is the density of the shell. Introducing displacements u^* and w^* into Eq. (4), we have solutions as follows.

$$\begin{aligned} u^* &= A_1 + A_2 \xi + A_3 \sinh \lambda_1 \xi + A_4 \cosh \lambda_1 \xi \\ &\quad + A_5 \sinh \lambda_2 \xi + A_6 \cosh \lambda_2 \xi \\ w^* &= \frac{1}{1+k-\Gamma} (\Gamma a - \nu A_2) \\ &\quad + \frac{\lambda_1}{k\lambda_1^2 - \nu} (A_3 \cosh \lambda_1 \xi + A_4 \sinh \lambda_1 \xi) \\ &\quad + \frac{\lambda_2}{k\lambda_2^2 - \nu} (A_5 \cosh \lambda_2 \xi + A_6 \sinh \lambda_2 \xi) \end{aligned} \dots\dots\dots (5)$$

where

$$\begin{aligned} \left(\frac{\lambda_1^2}{\lambda_2^2} \right) &= -\frac{\nu}{1-k} + \sqrt{\left(\frac{\nu}{1-k} \right)^2 - \frac{1+k-\Gamma-\nu^2}{k(1-k)}}, \\ k &= (0.5h/a)^2/3 \\ \Gamma &= (1-\nu^2)\rho a^2 \Omega^2/E \end{aligned}$$

Here, $A_i (i=1, \dots, 6)$ are arbitrary constants. Dimensionless initial tensions $n_x^* = N_x^*/P$ and $n_\phi^* = N_\phi^*/P$ can be expressed as follows:

$$\left. \begin{aligned} n_x^* &= \frac{1}{a} \left[A_2 + \frac{\nu(\Gamma a - \nu A_2)}{1+k-\Gamma} \right] \\ n_\phi^* &= \frac{1}{a} \left[\nu A_2 + (1+k) \frac{\Gamma a + \nu A_2}{1+k-\Gamma} + f(\xi) \right] \end{aligned} \right\} \dots\dots\dots (6)$$

where

$$\begin{aligned} f(\xi) &= \lambda_1 \left(\nu + \frac{1+k}{k\lambda_1^2 - \nu} \right) (A_3 \cosh \lambda_1 \xi \\ &\quad + A_4 \sinh \lambda_1 \xi) + \lambda_2 \left(\nu + \frac{1+k}{k\lambda_2^2 - \nu} \right) (A_5 \cosh \lambda_2 \xi \\ &\quad + A_6 \sinh \lambda_2 \xi) \dots\dots\dots (7) \\ P &= Eh/(1-\nu^2) \end{aligned}$$

Since arbitrary constants for each boundary condition can be determined easily, we will omit their definite expressions.

3.3 Frequency equation

The Flugge type basic equations which include the effect of the initial tensions due to the steady rotation are given as follows: (1)

$$\begin{aligned} E_1 &\equiv (1+n_x^*) \bar{u}_{,\xi\xi} + \left\{ \frac{1-\nu}{2} (1+k) + n_\phi^* \right\} \bar{u}_{,\phi\phi} \\ &\quad - \gamma \bar{u}_{,\xi\xi} + \frac{1+\nu}{2} \bar{v}_{,\phi\phi} + \nu \bar{w}_{,\xi\xi} - k \bar{w}_{,\xi\xi\xi} \\ &\quad + \frac{1-\nu}{2} k \bar{w}_{,\phi\phi\phi} = 0 \\ E_2 &\equiv \frac{1+\nu}{2} \bar{u}_{,\xi\xi} + (1+n_\phi^*) \bar{v}_{,\phi\phi} + (\gamma \Omega^2 - n_\phi^*) \bar{v} \\ &\quad + \left\{ \frac{1-\nu}{2} (1+3k) + n_x^* \right\} \bar{v}_{,\xi\xi} - \gamma \bar{v}_{,\xi\xi} \\ &\quad + (1+2n_\phi^*) \bar{w}_{,\phi\phi} - \frac{3-\nu}{2} k \bar{w}_{,\xi\xi\phi} - 2\gamma \Omega \bar{w}_{,\xi} = 0 \\ E_3 &\equiv -\nu \bar{u}_{,\xi\xi} + k \bar{u}_{,\xi\xi\xi} - \frac{k}{2} (1-\nu) \bar{u}_{,\phi\phi\phi} \\ &\quad + \frac{k}{2} (3-\nu) \bar{v}_{,\xi\xi\phi} - (1+2n_\phi^*) \bar{v}_{,\phi} + 2\gamma \Omega \bar{v}_{,\xi} \\ &\quad - k \bar{w}_{,\xi\xi\xi} - 2k \bar{w}_{,\xi\xi\phi} - (2k - n_\phi^*) \bar{w}_{,\phi\phi} \\ &\quad - k \bar{w}_{,\phi\phi\phi} - (1+k+n_\phi^* - \gamma \Omega^2) \bar{w} \\ &\quad + n_x^* \bar{w}_{,\xi\xi} - \gamma \bar{w}_{,\xi\xi} = 0 \end{aligned} \dots\dots\dots (8)$$

where $\gamma = \rho h a^2/P$. The boundary conditions based on Hamilton's principle are given as follows. At the end of the circular cylindrical shell,

$$\begin{aligned} \left[\left(\bar{N}_x + \frac{1}{a} N_x^* \bar{u}_{,\xi} \right) \delta \bar{u} \right]_e &\equiv \frac{P}{a} [B_1 \delta \bar{u}]_e = 0 \\ \left[\left(\bar{N}_{x\phi} - \frac{1}{a} \bar{M}_{x\phi} + \frac{1}{a} N_x^* \bar{v}_{,\xi} \right) \delta \bar{v} \right]_e &\equiv \frac{P}{a} [B_2 \delta \bar{v}]_e = 0 \\ \left[\left(-\frac{1}{a} \bar{M}_{x,\xi} - \frac{1}{a} (\bar{M}_{x\phi} + \bar{M}_{\phi x})_{,\phi} + \frac{1}{a} N_x^* \bar{w}_{,\xi} \right) \delta \bar{w} \right]_e \\ &\equiv \frac{P}{a} [B_3 \delta \bar{w}]_e = 0 \\ \left[\frac{1}{a} \bar{M}_{x\phi} \delta \bar{w}_{,\xi} \right]_e &\equiv \frac{P}{a} [B_4 \delta \bar{w}]_e = 0 \end{aligned} \dots\dots\dots (9)$$

Since Eq. (8) are differential equations with variable coefficients, they cannot be solved analytically. And we cannot find such trial functions that satisfy boundary conditions (D) considered in §5.1. So, a modified Galerkin's method is utilized to obtain the approximate solutions. If we choose the trial functions which satisfy the geometrical boundary conditions at least, Galerkin's equations are as follows:

$$\begin{aligned} \int_{\xi} \int_{\varphi} E_1 \delta \tilde{u} d\xi d\varphi dt - \int_{\xi} \int_{\varphi} [B_1 \delta \tilde{u}]_e d\varphi dt &= 0 \\ \int_{\xi} \int_{\varphi} E_2 \delta \tilde{v} d\xi d\varphi dt - \int_{\xi} \int_{\varphi} [B_2 \delta \tilde{v}]_e d\varphi dt &= 0 \\ \int_{\xi} \int_{\varphi} E_3 \delta \tilde{w} d\xi d\varphi dt - \int_{\xi} \int_{\varphi} [B_3 \delta \tilde{w}]_e d\varphi dt \\ - \int_{\xi} \int_{\varphi} [B_4 \delta \tilde{w}_s]_e d\varphi dt &= 0 \end{aligned} \quad \dots\dots\dots (10)$$

So, we approximate $\tilde{u}$, $\tilde{v}$ and $\tilde{w}$ with M terms as follows:

$$\left. \begin{aligned} \tilde{u} &\approx \tilde{u}_M = \cos(n\varphi + \rho t) \sum_{m=1}^M a_m U_m(\xi) \\ \tilde{v} &\approx \tilde{v}_M = \sin(n\varphi + \rho t) \sum_{m=1}^M b_m V_m(\xi) \\ \tilde{w} &\approx \tilde{w}_M = \cos(n\varphi + \rho t) \sum_{m=1}^M c_m W_m(\xi) \end{aligned} \right\} \quad \dots\dots\dots (11)$$

where a_m , b_m and c_m are arbitrary constants and U_m , V_m and W_m are the functions of the coordinate $\xi (=x/a)$ which satisfy the given geometrical boundary conditions. The variational functions are chosen as follows:

$$\left. \begin{aligned} \delta \tilde{u}_s &= \cos(n\varphi + \rho t) \delta a_s U_s(\xi) \\ \delta \tilde{v}_s &= \sin(n\varphi + \rho t) \delta b_s V_s(\xi) \\ \delta \tilde{w}_s &= \cos(n\varphi + \rho t) \delta c_s W_s(\xi) \end{aligned} \right\} \quad \dots\dots\dots (12)$$

As δa_s , δb_s and δc_s are arbitrary, simultaneous linear equations with 3M unknowns a_m , b_m and c_m are obtained as follows:

$$\sum_{m=1}^M [\Phi_{sm}^{(i)} a_m + \Psi_{sm}^{(i)} b_m + \Theta_{sm}^{(i)} c_m] = 0 \quad \dots\dots\dots (13)$$

$$(i = 1, 2, 3; s = 1, 2, \dots, M)$$

where

$$\begin{aligned} \Phi_{sm}^{(1)} &= \int_{\xi} \left[(1+n_x^*) U_{m,ee} + \left(\gamma \rho^2 - n^2 \left\{ \frac{1-\nu}{2} (1+k) + n_x^* \right\} \right) U_m \right] U_s d\xi - [(1+n_x^*) U_{m,e} U_s]_e \\ \Psi_{sm}^{(1)} &= \int_{\xi} \frac{1+\nu}{2} n V_{m,e} U_s d\xi - [\nu n V_m U_s]_e \\ \Theta_{sm}^{(1)} &= \int_{\xi} \left[-k W_{m,ee} + \left\{ \nu - \frac{k}{2} (1-\nu) n^2 \right\} W_{m,e} \right] U_s d\xi - [(\nu W_m - k W_{m,ee}) U_s]_e \\ \Phi_{sm}^{(2)} &= - \int_{\xi} \frac{1+\nu}{2} n U_{m,e} V_s d\xi + \left[\frac{1-\nu}{2} n U_m V_s \right]_e \\ \Psi_{sm}^{(2)} &= \int_{\xi} \left[\{ \gamma (\rho^2 + \Omega^2) - n^2 - (1+n^2) n_x^* \} V_m + \left\{ \frac{1-\nu}{2} (1+3k) + n_x^* \right\} V_{m,ee} \right] V_s d\xi \\ &\quad - \left[\left\{ \frac{1-\nu}{2} (1+3k) + n_x^* \right\} V_{m,e} V_s \right]_e \\ \Theta_{sm}^{(2)} &= \int_{\xi} \left[(2\gamma \rho \Omega - n - 2n n_x^*) W_m + \frac{k}{2} (3-\nu) n W_{m,ee} \right] V_s d\xi - \left[\frac{3}{2} k n (1-\nu) W_{m,e} V_s \right]_e \\ \Phi_{sm}^{(3)} &= \int_{\xi} \left[k U_{m,ee} - \left\{ \nu - \frac{k}{2} (1-\nu) n^2 \right\} U_{m,e} \right] W_s d\xi - \left[\left(k U_{m,ee} + \frac{k}{2} (1-\nu) n^2 U_m \right) W_s \right]_e + [k U_{m,e} W_{s,e}]_e \\ \Psi_{sm}^{(3)} &= \int_{\xi} \left[\frac{k}{2} (3-\nu) n V_{m,e} + (2\gamma \rho \Omega - n - 2n n_x^*) V_m \right] W_s d\xi - \left[\frac{k}{2} (3-\nu) n V_{m,e} W_s \right]_e + [k \nu n V_m W_{s,e}]_e \\ \Theta_{sm}^{(3)} &= \int_{\xi} \left[-k W_{m,ee} + \{ 2k n^2 + n_x^* \} W_{m,e} + \{ \gamma (\rho^2 + \Omega^2) - k(n^2 - 1)^2 - 1 - (n^2 + 1) n_x^* \} W_m \right] W_s d\xi \\ &\quad - [(-k W_{m,ee} + \{ n_x^* + k n^2 (2-\nu) \}) W_{m,e} W_s]_e - [k (W_{m,ee} - \nu n^2 W_m) W_{s,e}]_e \end{aligned} \quad \dots\dots\dots (14)$$

We use the trial function for each boundary condition as follows:

$$\begin{aligned} \text{(A) both ends clamped} \\ \text{SM: } U_j &= \sin \frac{j}{a} \pi \xi & \text{AS: } U_j &= \cos \frac{j-1/2}{a} \pi \xi \\ V_j &= \cos \frac{j-1}{a} \pi \xi + \cos \frac{j}{a} \pi \xi & V_j &= \sin \frac{j-1/2}{a} \pi \xi + \sin \frac{j+1/2}{a} \pi \xi \\ W_j &= \cos \frac{j-1}{a} \pi \xi + \cos \frac{j}{a} \pi \xi & W_j &= \sin \frac{j-1/2}{a} \pi \xi + \sin \frac{j+1/2}{a} \pi \xi, \\ \text{(B) simply supported without axial constraint} \\ \text{SM: } U_j &= \sin \frac{j-1/2}{a} \pi \xi & \text{AS: } U_j &= \cos \frac{j}{a} \pi \xi \\ V_j &= \cos \frac{j-1/2}{a} \pi \xi & V_j &= \sin \frac{j}{a} \pi \xi \\ W_j &= \cos \frac{j-1/2}{a} \pi \xi & W_j &= \sin \frac{j}{a} \pi \xi, \\ \text{(C) simply supported with axial constraint} \\ \text{SM: } U_j &= \sin \frac{j}{a} \pi \xi & \text{AS: } U_j &= \cos \frac{j-1/2}{a} \pi \xi \\ V_j &= \cos \frac{j-1/2}{a} \pi \xi & V_j &= \sin \frac{j}{a} \pi \xi \\ W_j &= \cos \frac{j-1/2}{a} \pi \xi & W_j &= \sin \frac{j}{a} \pi \xi, \\ \text{(D) clamped at one end and free at the other end} \\ U_j &= C_{1j} (-\sin \tau_j \xi - \sinh \tau_j \xi) + C_{2j} (\cos \tau_j \xi - \cosh \tau_j \xi) \\ V_j &= C_{1j} (\cos \tau_j \xi - \cosh \tau_j \xi) + C_{2j} (\sinh \tau_j \xi - \sin \tau_j \xi) \\ W_j &= C_{1j} (\cos \tau_j \xi - \cosh \tau_j \xi) + C_{2j} (\sinh \tau_j \xi - \sin \tau_j \xi) \end{aligned}$$

$$\begin{aligned}
C_{1j} &= (\cos 2\tau_j \alpha + \cosh 2\tau_j \alpha) / C_j, & C_{2j} &= (\sin 2\tau_j \alpha - \sinh 2\tau_j \alpha) / C_j \\
C_j &= (\cos 2\tau_j \alpha + \cosh 2\tau_j \alpha)(\cos 2\tau_j \alpha - \cosh 2\tau_j \alpha) + (\sin 2\tau_j \alpha - \sinh 2\tau_j \alpha)(\sin 2\tau_j \alpha - \sinh 2\tau_j \alpha) \\
&+ \cos 2\tau_j \alpha \cosh 2\tau_j \alpha = 0
\end{aligned}
\tag{15}$$

where we separate the symmetric modes SM form and the anti-symmetric modes AS with respect to the mid-span of a circular cylindrical shell owing to the fact that the boundary conditions (A)-(C) are identical at both ends. And the trial functions for the boundary condition (D) are taken from normal modes of a cantilevered beam. Substituting Eqs.(6) and (15) into Eq.(14) and integrating yields homogeneous equations with respect to a_m , b_m and c_m . If the determinant of the coefficient matrix is set equal to zero, the following frequency equation is obtained:

$$D_i \equiv [\phi_{im}^{(1)}, \psi_{im}^{(1)}, \theta_{im}^{(1)}] = 0 \tag{16}$$

The value of ρ which satisfies Eq.(16) is the natural radian frequency. In the actual numerical computation, the dimensionless form of radian frequency ρ and rotating angular velocity Ω are used in the same way as in previous papers and they are transformed in terms of γ as follows:

$$\bar{\rho} = \sqrt{\gamma} \rho, \quad \bar{\Omega} = \sqrt{\gamma} \Omega \tag{17}$$

Eq.(16) is solved by the determinant search method and the natural frequencies may be obtained.

3.4 Numerical results

Figure 4 (a)-(c) show the numerical results for the case where $\alpha=5.0$, $k=h/2a=0.005$ and $n=2, 3$ and 4 , and $\nu=0.3$. To identify boundary conditions (A)-(D), the symbols $\circ$ for (A):, $\triangle$ for (B):, $\square$ for (C): and $\bullet$ for (D): are used and the solid line is used to indicate Eq.(2). As might be found in previous papers, we assume that Eq.(2) holds good and natural frequencies are hardly influenced by boundary conditions. Here, it is certain that the natural frequencies of a rotating cylindrical shell can be predicted precisely if we know the natural frequencies of the non-rotating cylindrical shell.

4. Conclusions

The vibration of rotating cylindrical shells was experimentally and theoretically examined in reference to that of a thin rotating ring. The conclusions are summarized as follows:

- (1) From both the experimental results and theoretical analysis for four kinds of boundary conditions (A) both ends clamped; (B) simply supported without axial constraint; (C) simply supported with axial constraint; and (D) clamped at one end and free at the other end, it is found that the dependence of the frequencies upon the rotating speed is approximately represented by a simple relation for a thin rotating ring, provided that the frequencies and rotating speeds are normalized by the natural frequencies of a non-rotating cylindrical shell.
- (2) From the results in (1), we can say that the frequencies of a rotating cylindrical shell for any boundary condition can be predicted precisely if we know natural frequencies of the non-rotating cylindrical shell.

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Fig.4 Frequencies of forward and backward waves