Vibration Analysis of Disk-spindle Due to Ball Bearing Excitation*

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Large capacity magnetic disks require a head positioning accuracy in the order of a few micro-meters. This implies a technology to master the difficult problems of the disk-spindle assembly vibration. The following subjects are studied in this paper:

- (1) The gyroscopic effect on the natural frequency of the rotor-bearing system is experimentally made clear by exciting a rotating spindle.
- (2) The whirling force acting on the spindle system is dominantly caused by slight deformations on the ball bearing races.
- (3) It is shown numerically as well as experimentally, considering whirling direction, that the resonance causing a head positioning error results from a coincidence of rotor natural frequency and the ball bearing excitation frequency.

Key Words : Vibration of Rotating Body, Precision Machine, Disk Spindle, Ball Bearing Excitation and Resonance, Identification of Vibration System

1. Introduction

The trend toward large capacity and high speed processing of main frame computer system demands similar improvements of the peripheral equipment, especially the magnetic disk file called head disk assembly(HDA). The HDA enables so powerful capability for the data storage and the fast data handling that its market is expanding as main data file equipment.

The process of the development for the HDA is historically shown in Fig.1, concerning the memory capacity and some corresponding specifications. The pitch density has been made remarkably precise, increasing the storage capability. It is achieved by reducing the coating thickness for thin film of media and the flying height of the head over the disk. Furthermore, such higher pitch density requires more accurate head positioning of less than a micron meter^(1,2). The reduction of vibration of the head access mechanism⁽³⁾ and the disk spindle in the HDA, with the internal structure as shown in Fig.2, is then one of key technologies.

Fig. 1 Trend of specifications of HDA

* Received 3rd April, 1984

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The disk spindle is a two-shaft rotor, which consists of a main shaft supported by ball bearings and an outer hub having multi-level disks. The hub is tightly overhung on the main shaft at the top. The rated speed of the rotor is 60 rps. Its operational speed is not so fast.

Fig. 2 Configuration of Head Disk Assembly (HDA)

However strong gyroscopic effects appear due to the thin disk with large diameter, generating the same behaviors as in rotor dynamics of high speed rotating machines. Conventional studies deal with vibration problems⁽⁴⁾ in which the stiffness of the ball bearing supporting the spindle is estimated. They do not consider rotor dynamics characterized by the gyroscopic effect, i.e. the whirling rotor motion.

In this paper, the spindle vibration caused by ball passing excitation due to race deformation in ball bearing is made clear by considering the gyroscopic effect. The actual HDA treated in this study is one of our prototypes for the development of a new spindle with capability of 600 MBites memory.

First, to clarify vibration characteristics of the spindle itself, the harmonic external excitation test is made of the rotor while it is nonrotating or rotating. Rotational speed for the test is adjusted to the rated speed and around it. The ball passing excitation and corresponding resonance conditions of the rotor are experimentally examined.

Then, numerical simulations are executed to verify the experimental data concerning the following points:

- (1) how to make an accurate dynamic model of the spindle,
- (2) analysis of natural frequencies and eigen mode shapes,
- (3) resonance conditions induced by ball passing excitation, and
- (4) response curve of vibration amplitude versus speed.

Finally, comparison between the experimental data and the numerical results proves the effectiveness of our approach.

Many studies on the ball passing excitation due to deformation of inner race and outer race of ball bearing have been done. The exciting frequency is introduced in many handbooks⁽⁵⁾⁽⁶⁾⁽⁹⁾. However, unfortunately only the values of these frequencies are given, and the whirling direction is not shown. The novelty of this paper is to clearly indicate the values and whirling direction of the exciting frequency.

Fig. 3 Excitation test rig

2. Experimental Analysis of Disk Spindle Vibration

2.1 Excitation test rig

The disk spindle used in our experiment, shown in Fig.3, is operated at variable rotational speeds around the rated speed of 60 rps. The rotating spindle is externally excited by a moving coil type shaker (MB Ltd.), acting horizontally upon the basement of the HDA. The harmonic excitation force is adjusted in the frequency range of 100~800 Hz and magnitude of about 3.4 N.

The spindle radial vibration is detected by eddy current type displacement sensors (AEC Ltd. Type 225-05) placed at the upper and lower portions of the shaft. The hub radial vibration is picked up by capacitive gap sensors (ONO SOKKI Ltd. VT-510). In this experiment, the actual cover made of a thin steel plate is replaced by an aluminum cover and these sensors are mounted on it. To avoid runout noise in the displacement sensing, particular caps (called master ring in Fig.3) manufactured in perfect circle configuration are fixed at the both ends of a shaft, made of SUS 304 (diameter $\phi 30\text{mm}$, eccentricity less than $20\mu\text{m}$).

The basement vibration is measured by piezo acceleration meters (ENDEVCO Type 22).

The rotational speed of the driving motor is controlled by the magnitude of the input voltage. It is transmitted to the spindle rotation by a belt and pulley, in the range of the spindle rotational speeds $\Omega = 30\sim 70$ rps. This paper is not concerned with unbalance vibration, but with higher frequency vibration. Therefore, all displacement and acceleration signals, immediately upon detection, are filtered by a high pass filter (cut off freq. 100HZ).

2.2 Response curves under harmonic excitation

The harmonic response curve of the detected vibration signals is made by a tracking filter with the exciting frequency ν . Figure 4 shows the harmonic response curves obtained at the upper portion of the shaft with certain rotational speeds. Two resonance peaks appear at the exciting frequencies $\nu = 173$ and 483 Hz at no rotation $\Omega = 0$ rps. One of them, a peak at

$\nu = 173$ Hz for $\Omega = 0$, separates into two peaks as the rotational speed increases, e.g. two peaks at $\nu = 127$ and $\nu = 257$ Hz for the maximum speed $\Omega = 69$ rps. The other peak at $\nu = 483$ Hz for $\Omega = 0$ rps is divided in a similar way into two peaks at $\nu = 456$ and $\nu = 530$ Hz for the maximum speed. These separations of the natural frequencies are due to the gyroscopic effect which is proportional to the rotational speed.

Fig. 4 Response curves in harmonic excitation test (top of main shaft)

Fig. 5 Response curves in harmonic excitation test (hub and basement)

The other peaks for $\Omega = 0$ rps seen in Fig. 4, at $\nu = 212$ and $\nu = 406$ Hz, remain at fixed frequencies in spite of the increase of rotational speed. It is then interpreted that these peaks are caused by resonance of nonrotational structure.

The response curves measured by the other sensors are shown in Fig. 5. With respect to the vibration of the upper portion of the main shaft, we can see a resonance peak around $\nu = 380$ Hz at the high speed rotation, $\Omega = 69$ rps. The corresponding peak of the hub vibration appears at $\nu = 392$ Hz for no rotation $\Omega = 0$, at $\nu = 381$ Hz for the maximum rotation $\Omega = 69$ rps. This peak frequency slightly decreases in the hub vibration as the rotational speed increases. This resonance corresponds to the basement vibration at $\nu = 384$ Hz.

Therefore, it can be predicted that the mode shape corresponding to this resonance is caused mainly by bending deformations of the nonrotating structure, i.e. the basement, plus a small influence of the rotating structure, i.e. the spindle.

2.3 Head positioning accuracy

The head follows the servo tracking signal written on the disk as reference. The error between the head position and the reference is called the head positioning signal. It is used as a feedback signal to the controller of the head access mechanism. Therefore, the amplitude of the vibration of this error signal should be as small as possible, as it determines the head positioning accuracy.

The vibration of the head positioning signal is shown in Fig. 6(a) in frequency domain for the normal operation, i.e. rated rotational speed and no external excitation. The response curve of the head positioning signal under the external excitation is shown in Fig. 6(b).

According to Fig. 6(a), the head positioning error is dominantly caused by the vibration component of $\nu = 380$ Hz. Therefore, improvement of the positioning accuracy requires consideration of this resonance in designing the spindle and basement system.

According to Fig. 6(b), some remarkable peak amplitudes appear at $\nu = 517$ Hz, 461 Hz and 381 Hz. It implies that the design of the basement structure is considered, as no vibration around these frequencies is transmitted from the basement to the spindle.

2.4 Vibration characteristics of disk-spindle system

(1) All of resonance frequencies measured in the previous excitation test are plotted in Fig. 7(a), marked by $\circ$, $\bullet$ and Δ . Lines through these marks show the natural frequencies, i.e. forward and backward natural frequencies shown by solid lines with superscript f and dotted lines with superscript b, respectively. In 1st and 5th modes, the separation of the forward and backward natural frequencies can be accepted for increase of the rotational speed. Otherwise, the natural frequencies of 2nd, 3rd and 4th modes are invariable in spite of increase of the rotational speed,

Fig. 6 Response curves in harmonic excitation test (positioning signal)

as indicated by solid lines, because of coincidence between the forward and backward natural frequencies. The vibration characteristics of the disk spindle system is then made clear.

(2) The vibration response curve of the top of the main shaft is shown in Fig.7(b) for the rotational speed range $\Omega = 20 \sim 70$ rps and without external excitation. Two peaks at about 32 rps and a peak at 60 rps can be seen. At each rotational speed of these peaks, the dominant frequency ω is obtained by FFT analyzer; $\omega = 199, 146$ and 384 Hz at $\Omega = 31.3, 32.8$ and 60.5 rps, respectively. These correspondences are marked by O on Campbell's diagram as shown in Fig.7(a).

(3) Frequencies of the ball passing excitation due to slight deformations of inner and outer races of the ball bearings are now discussed. This machine employs a single row ball bearing (NSK 6207 and 6206 placed at the upper and lower portions, respectively). Referring to details shown in Chap.4, the outer race deformation of this bearing induces an excitation with the 1st frequency of $\pm 3.6\Omega$ and the inner race deformation with the one of $\pm 6.42\Omega$ and -4.4Ω . These positive and negative ratios to the rotational speed imply forward and backward excitations, respectively. The previous two peaks at $\Omega = 31.3$ and 32.8 rps

Fig. 7 Experimental results

correspond with respective intersections, on Campbell's diagram of Fig.7(a), between the 1st forward natural frequency and the excitation $+6.42\Omega$ and between the 1st backward one and the excitation -4.4Ω .

The other peak at $\Omega = 60.5$ rps corresponds with the intersection between the 3rd natural frequency and the excitation $+6.42\Omega$. All these intersections relate to the excitation frequencies defined by the inner race deformation. It can be estimated that this machine contains imperfections of the inner race.

(4) Each peak amplitude on the response curve of Fig.6 is described in Fig.7(c), as straight bar graph on the right for no excitation and on the left for excitation at the rated rotation. The peak with the frequency 380 Hz stands out regardless of the excitation. The response at the frequencies of 458 and 517 Hz, i.e. the resonance of 5th mode is very sensitive to external excitations.

The servo tracking signal is written on the lowest disk. The head position on the upper disks is defined by a cylindrical shape standing on the lowest disk. Therefore, the inclination mode shape of the hub to which the disks are attached induces the observed head positioning error.

The vibration mode shapes are shown in Fig.7(c). The mode with the frequency 170 Hz is mainly formed by deformation of the disk itself with one nodal diameter, which has some influence on the head positioning accuracy. On the other hand, the hub inclination, as shown in 3rd mode of the frequency 380 Hz, has a large influence on the head positioning error. Therefore, it can be deduced that the stiffness of the basement supporting the spindle must be increased in order to avoid the resonance of the inclination mode.

3. Dynamic Model of Disk Spindle System

The disk spindle system consists of two level shafting; the main shaft supported by bearing and the hub stacking 13 disks with equal pitch. Both are connected by the baking tie procedure. The motor driving force is transmitted to the pulley fixed at the lowest portion of the shaft by a belt. The bearing housing containing the upper and lower bearings stands on the basement of uniform material.

In this chapter, FEM technique (mainly using beam elements) is applied to the disk spindle system for the set up of an equivalent computational model for the rotor dynamics analysis. The dynamic model for the vibration simulation is determined step by step in the manner of constructing this machine, in order to agree with experimental data.

3.1 System connecting main shaft and hub

First, the dynamic model on the basis of beam theory is formulated for the system consisting of a spindle and a hub (See Fig.8). The baking tie of both shafts is modelled by a rotational spring $K_{\theta hs}$. The value of this rotational spring is determined through comparison of the com-

puted natural frequencies with experimental values obtained by the harmonic excitation test for the system under free-free boundaries. The comparison gives value $K_{\theta hs} = 5.9 \times 10^{18}$ N·mm/rad as the rotational spring.

3.2 System connecting main shaft, hub and pulley

Next, a pulley is attached to the previous system (See Fig.9) at the lowest portion. The connecting stiffness between the shaft and the pulley is also modelled by a rotational spring. Its value is estimated by $K_{ps} = 5.9 \times 10^{13}$ N·mm/rad in similar manner by comparing computational natural frequencies with experimental ones under free-free boundaries.

Fig. 8 Identification of hub supporting stiffness $K_{\theta hs}$

Fig. 9 Identification of pulley supporting stiffness K_{ps}

Fig.10 Vibration characteristics of disk spindle (free-free boundary)

3.3 Disk spindle system

Thirteen disks are then added to the previous system. In this simulation, group of three disks are attached to the nodal points Nos. 4, 7, 9 and 11. The last 13th disk is put on the nodal point No.12, with the moment of inertia I_d connected by a rotational spring $K_{\theta dh}$, in relation to $I_d = \omega_n^2 K$, where ω_n is assumed by the natural frequency of one nodal diameter, i.e. $\omega_n = 170$ Hz. The computational results of the natural frequencies obtained by these assumptions are compared with experimental values measured in the free-free excitation test in Fig.10. There is good agreement. The dynamic model is obtained from FEM beam elements as shown in the figure.

3.4 Rotational stiffness of bearing housing against basement

The bearing housing is placed on the basement in the same structure with uniform material, as shown in Fig.11. It is not easy to measure the rotational stiffness of the housing directly. Instead of the experiment, the static deformation calculation is done by FEM technique (shell elements). The deformation under unity load in horizontal direction yields an equivalent rotational stiffness $K_{\theta b} = 2.0 \times 10^9$ N.mm/rad.

Fig.11 Stiffness of bearing housing on basement

Fig.12 Vibration characteristics of assembled disk spindle ($\omega = 0$ rps)

3.5 Vibration characteristics of nonrotating disk spindle system

The disk spindle previously mentioned in paragraph 3.3 is built in the housing through bearings on the basement. The equivalent dynamic model is shown in Fig.12 with a stiffness of the bearing $K_B = 80000$ N/mm, usually assumed.

The figure shows the comparison between the numerical values and measured natural frequencies under the free-free excitation test while nonrotating. Their good agreement through 2nd to 4th natural frequencies guarantees a reasonable dynamic model for the disk spindle system.

Obviously, in Fig.12 the pulley mainly vibrates in the 1st mode. The deformation of one nodal diameter on the disk is dominant in the 2nd mode, the relative deformation of the hub to the main shaft in the 3rd mode, and the whole bending inclination of the shaft and hub system against the basement in the 4th mode.

4. Simulation of Rotor Vibration due to Ball Passing Excitation

4.1 Whirling motion of ball

The ball passing excitation is due to deformations of inner and/or outer races, which are caused by uneven race surfaces, imperfect roundness of the shaft, tieing malfunction of ball bearings with the shaft and so on. In this section, the frequency of the ball passing excitation is discussed.

The exciting frequencies are shown in many articles on the dynamics of ball bearings. However, these conventional studies do not show the whirling direction of the exciting force. Their data is therefore insufficient for rotor-ball bearing systems with strong gyroscopic effect. This circumstance is made clear by this study.

In the case of a generally used ball bearing, the inner race is rotating and the outer race is not, as shown in Fig.13.

Fig. 13 Ball bearing

The whirling speed of the balls, Ω_r is determined by the following formula for the rotational speed of the inner race, Ω .

$$\Omega_r = \frac{\Omega}{2} \left(1 - \frac{D}{d} \cos \alpha \right) \equiv \beta \Omega \quad (1)$$

where D = diameter of ball
 d = rolling diameter of ball
 α = touch angle

Generally speaking, the value of β is about 0.4. The balls are whirling in the positive direction (rotational direction) with a speed of about 40 % of the rotational speed Ω .

4.2 Exciting force due to deformation of outer race

It is simply assumed that a protruding deformation, i.e. bump exists at a portion on the outer race, as shown in Fig. 14. Everytime the balls pass the bump on the outer race, an exciting force acts upon the nonrotating outer race in a certain direction with the ball passing frequency, i.e. (the whirling speed Ω_r) * (number of balls Z). The force is then expressed by the whirling form;

$$F = 2F_0 \cos Z\Omega_r t \\ = F_0 (e^{iZ\Omega_r t} + e^{-iZ\Omega_r t}) \quad (2)$$

Fig. 14 Excitation mechanism due to deformed races

Therefore, the bump on the outer race produces two whirling exciting forces, a forward one $+Z\Omega_r$ and a backward one $-Z\Omega_r$.

In general case of m bumps, the following formulas may be presented, as the whirling exciting forces:

$$F = F_0 (e^{iZ\Omega_r t} + e^{-iZ\Omega_r t}) \quad (3)$$

For instance, the elliptical deformation of outer race is here equivalent to the number of bumps $m=2$.

4.3 Exciting force due to deformation of inner race

A bump on the inner race is assumed in same manner, as shown in Fig. 14. The balls are in rotation on the inner race with the relative speed $\Omega - \Omega_r$ in the backward direction. Everytime the balls pass the bump on the inner race, an exciting force acts upon the rotating inner race in a certain direction. The ball passing frequency is given by the frequency of (relative speed of balls $\Omega - \Omega_r$) * (number of balls Z). Therefore, the exciting force is expressed by the following formulas on the rotational coordinate axes.

$$F_r = 2F_0 \cos Z(\Omega - \Omega_r)t \\ = F_0 (e^{iZ(\Omega - \Omega_r)t} + e^{-iZ(\Omega - \Omega_r)t}) \quad (4)$$

This exciting force is transformed in a fixed coordinate system, in which the equation of motion of the rotor is expressed.

$$F = F_r e^{i\Omega t} \\ = F_0 [e^{i((Z+1)\Omega - Z\Omega_r)t} + e^{-i((Z-1)\Omega - Z\Omega_r)t}] \quad (5)$$

This formula indicates the exciting whirling force acting on the rotor with two frequencies, i.e. forward frequency $(z+1)\Omega - Z\Omega_r$ and backward one $-(z-1)\Omega - Z\Omega_r$.

In case of an elliptical deformation of the number of bumps is $m=2$. Using m bumps for general case, the exciting force is expressed in the rotating coordinate system:

$$F_r = 2F \cos mZ(\Omega - \Omega_r)t \quad (6)$$

then transformed into the fixed coordinate system:

$$F = F_0 [e^{i((mZ+1)\Omega - mZ\Omega_r)t} + e^{-i((mZ-1)\Omega - mZ\Omega_r)t}] \quad (7)$$

4.4 Resonance of rotor vibration caused by ball passing excitation

The frequencies of the ball passing excitation due to deformed races are summarized as whirling exciting forces in Tab.1.

Otherwise, the natural frequencies of the rotor ω depend upon the rotational speed Ω and the gyroscopic effects γ .

$$\frac{\omega(\Omega)}{\omega_n} = \pm \sqrt{1 + \left(\frac{\gamma}{2} \frac{\Omega}{\omega_n} \right)^2} + \frac{\gamma}{2} \frac{\Omega}{\omega_n} \quad (8)$$

The gyro factor γ is estimated by the ratio of modal gyro coefficient per modal mass. Roughly speaking, it means physically the ratio of translational moment of inertia per polar moment of inertia of the rotor.

In the range of low rotational speeds compared with natural frequencies, e.g. at the rated speed of the disk spindle, the natural frequency is approximated as follows:

$$\omega(\Omega) \approx \pm \omega_n + \frac{\gamma}{2} \Omega \dots \dots \dots (9)$$

Tab. 1 Exciting frequency with whirling direction

Deformation		Excitation Frequency	
Bearing Race	Number	Forward	Backward
INNER	1	$(Z+1-Z\beta)\Omega$	$-(Z-1-Z\beta)\Omega$
	m	$(mZ+1-mZ\beta)\Omega$	$-(mZ-1-mZ\beta)\Omega$
OUTER	1	$+Z\beta\Omega$	$-Z\beta\Omega$
	m	$-mZ\beta\Omega$	$-mZ\beta\Omega$

The behavior of the natural frequencies of the rotor depending on the rotational speed Ω is described in Fig.15, called natural frequency map. The forward natural frequency ω_f (solid lines) increases as the rotational speed increases. On the other hand, the backward natural frequency ω_b (dotted lines) decreases to zero.

Fig.15 Resonance condition caused by ball passing excitation

On these natural frequency maps of Fig.15, the ball passing exciting frequencies are given by straight lines with consideration of the whirling directions. The upper and lower figures correspond with the excitations due to deformations of outer and inner races, respectively. Solid straight lines are forward excitations, i.e. $+Z\beta\Omega$ and $+(Z+1-Z\beta)\Omega$, and dotted lines are backward excitations, i.e. $-Z\beta\Omega$ and $-(Z-1-Z\beta)\Omega$.

Response occurs when the exciting frequency meets the natural frequency of the rotor in value as well as in whirling direction.

Therefore, intersections between solid lines shown in the positive frequency region indicate rotational speeds at which forward resonances occur. Intersections between dotted lines in the negative region indicate backward resonances. Here, the forward and backward frequencies are separately described in the positive and negative regions, respectively.

4.5 Resonances due to ball passing excitation in the disk spindle system

The resonance criteria for the forward and backward conditions introduced in the previous section are applied on the disk spindle of Fig.12. The natural frequencies of the disk spindle, defined by the computational model of Fig.12 at 0 rps, are calculated depending on the rotational speed as shown in Fig.16. The lowest frequency of 70 Hz at 0 rps is here omitted, because of no experimental data. The order of natural frequencies is designated as 1st of 170Hz corresponding to the thin disk vibration to 4th.

With respect to the 1st mode (one nodal disk vibration) and 3rd mode (rocking mode of the spindle), calculated natural frequencies in Fig.16 well agree with measured ones in Fig.7.

In the former case, the natural frequency is clearly divided into forward and backward component as the rotational speed increases, due to a strong gyroscopic effects ($\gamma = 2$). At low rotational speed = 0 ~ 70 rps compared with the natural frequency $\omega_n = 170$ Hz, the approximate formula is available for the behavior of the separation. The evidence for $\gamma = 2$ is that the difference between the forward and backward natural frequencies equals to twice the rotational speed.

In the latter case, the gyroscopic effects is estimated at $\gamma = 0.18$, which is not so large because of the rocking mode of the spindle. The corresponding separation of the natural frequency is small in the calculated as well as in the measured results.

The calculated second mode indicates the vibrating hub against the spindle, with the frequency of $\omega_n = 212$ Hz and $\gamma = 1.4$. The separation is rather large in Fig.16 while the measured one is not so large in Fig.7.

The calculated fourth mode of Fig.16 corresponds to the measured 5th mode of Fig.7. The gyroscopic factor $\gamma = 0.83$ is experimentally estimated by the separation of the natural frequency, which is greater than the calculated one $\gamma = 0.4$.

That is why the separation in the calculated curves is smaller than in the measured one.

The measured 4th mode with no separation does not appear in the calculation. It means that the corresponding dynamic model for the 4th mode is not included in the calculation of this rotor dynamics. This mode is, therefore, due to vibration of local structures ignored in this disk spindle system of Fig.12.

Fig.16 Numerical results of resonance condition

The ball passing exciting frequencies are drawn on the natural frequency map of Fig.16 obtained by the calculation. Specifications of the ball bearing used in the experiment are shown in Fig.13. According to the formula in Tab.1, the exciting force due to deformed inner race has a frequency of $+6.42\Omega$ in the forward direction and -4.4Ω in the backward direction. Deformations of the outer race generate an excitation with the frequencies of $\pm 3.5\Omega$ in the forward and backward directions.

Resonance speeds caused by deformation of the inner and outer races are marked by, ○ and □, respectively. According to this calculation, the forward resonance is predicted at the normal operation of this machine because the intersection 3F marked by ○ is situated at about the rated speed 60 rpm. It is confirmed by the experimental data concerning the vibration with 380Hz on Campbell's diagram of Fig.7. On

the rotational speed $\Omega = 33.3$ rps, at which the servo tracking signal is written, the resonances of 1F and 1B are also predicted.

4.6 Simulation of ball passing vibration

The equation of motion for rotor vibrations is commonly formulated in the non-rotating coordinate system. Putting rotor displacements denoted by x and y in X and Y directions respectively, a complex displacement z is defined as:

$$z = x + iy \quad (10)$$

The equation of motion of rotor is in the following form:

$$M\ddot{z} + i\Omega C_g \dot{z} + C\dot{z} + Kz = F \quad (11)$$

where M =mass matrix, C_g =gyroscopic matrix, C =damping matrix and K =stiffness matrix.

External force F on the right hand is assumed by rotational ratio excitation whose frequency is proportional to the rotational speed Ω , as shown in Eq.(2) and (5):

$$F = F_0 e^{in\Omega t} \quad (12)$$

$$F = M\delta(n\Omega)^2 e^{in\Omega t} \quad (13)$$

The former means constant amplitude of the force. The latter is estimated in the same manner as unbalance force, where the deformation δ of the race corresponds to eccentricity of the rotor mass. In our opinion, the latter is reasonable for the presentation of the ball passing excitation. The force acts on the rotor at the location at which ball bearings are placed.

Harmonic vibration simulations under the ball passing excitation are shown in Fig.17, where $M\delta$ =unit load = 9.8×10 N.s (equivalent to $\delta = 0.7 \mu\text{m}$). The upper and lower parts of Fig.17 correspond to deformations of the inner and outer races, respectively. Because of small damping, peak amplitudes appear sharply, corresponding to the resonance points shown by the marks ○ and □. The correspondences are indicated by numbers, e.g. 1F, 2B and so on, beside peak amplitudes.

According to the calculated results of Fig.17, it is obvious that the deformation of the inner race induces greater vibration than the deformation of the outer race. In the figure, peak amplitudes at 1F, 1B, and 3F are higher than at 2F.

The disk spindle used in the experiment contains deformed inner races. The reason is that the peak amplitudes at 1F, 1B and 3F except 2B occur both in the calculated results of the lower part of Fig.17 and in the experimental data of Fig.7(b). The resonance around 3F must be particularly avoided for this kind of machine. Improvements of the basement stiffness K_{ϕ} , the hub weight, number of balls and so forth are recommended in order to avoid resonances. The simulation program developed here is effective for this purpose.

Fig.17 Numerical simulation of vibration response due to ball passing excitation

5. Conclusions

Experimental and numerical studies on rotor dynamics for the disk spindle are done with respect to the ball passing vibration caused by deformed races in ball bearings. Following conclusions are drawn:

- (1) By introducing variable rotational speed operation beside the rated one and harmonic exciting test while rotating, the impact of gyroscopic effects on natural frequencies is clearly demonstrated. In this machine, deformed inner races dominantly cause rotor vibrations and head positioning errors with a frequency of 380 Hz at the rated rotational speed, corresponding to the rocking mode of the spindle.
- (2) The disk spindle system is assembled step by step. At the same time, reasonable dynamic models of the system are completed by data obtained in the exciting test under free-free boundaries.
- (3) Simulations of the vibration induced by ball passing excitation agree well with the measurement.
- (4) Exciting frequencies due to deformations of the inner and/or outer races of ball bearings are made clear including the whirling directions. Therefore, the mechanism of the forward and backward resonances represents a major improvement on the description of such systems.

Finally, the authors would like to acknowledge the cooperation of Dr. Hayama, Dr. Kikuchi, managers of Mechanical Engineering Research Laboratory of Hitachi Ltd. and Mr. Katoh, manager of Odawara Works of Hitachi Ltd.

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