THREE DIMENSIONAL CRACK GROWTH ANALYSIS BY THE T.N.P. METHOD?

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The T.N.P. method (Translation of Nodal Points method) is applied to the three dimensional crack growth analysis. The method is compared with the conventional one for two dimensional problems and good agreement is recognized. CT specimens with and without side groove are analyzed in elastic and elastic-plastic states. It is shown that the crack grows and equalizes the J integral distribution along the crack front. The resultant crack front configurations agree with those obtained in many experiments.

Key Words: Fracture, Crack Growth Analysis, Finite Element Method, Three Dimensional J Integral, CT Specimen

1. Introduction

In these several years, the stable crack growth problem, so-called J controlled growth, has been studied by many researchers both numerically and experimentally. New fracture criterion, T-modulus by Paris(1), CTOA by Shih(2) are proposed in these studies, and three dimensional crack growth condition is also studied by Kanninen(3,4) for cracked pipe. In these studies, the finite element method is used widely in elastic-plastic analyses and J crack growth analyses. In general, crack growth is well simulated by releasing the nodal reaction force at the crack tip and creating new crack surfaces in the two dimensional fields(5). Round robin analyses are also carried out by JSME(6).

However, in the three dimensional space, the same method is not available to simulate the crack growth as shown in Fig.1. In Fig.1(a), a straight crack front exists parallel to the z axis. The crack growth begins when some part of the crack front satisfies the fracture criterion. By releasing the nodal reaction force at the crack tip, element which includes the same part of the crack front, new crack front configuration is obtained as shown in Fig.1(b), which has a discontinuity in the crack front line. It is obvious that this result is due to the numerical method and is not a realistic one.

In this paper, the T.N.P. method, proposed by Nakagaki et al.(7), is used to simulate the three dimensional crack growth. In this method, the nodal points at the crack front are moved forward in the crack growth direction, and then a smooth crack front configuration is obtained as shown in Fig.1(c). In the following, the T.N.P. method is introduced briefly and applied to the two dimensional crack growth problem. The results are compared with those by conventional method. Then, CT specimens with and without side groove are analyzed and a three dimensional J integral is obtained and discussed.

Fig.1 Growth of three-dimensional crack front.

2. T.N.P. Method

Fig.2 shows a structure which includes a crack and is in equilibrium state. At the crack tip, small length ∆a is closed by the surface forces, (C1a)n and (C2a)n, acting on both sides of ∆a. The virtual work principle of this system is expressed as follows:

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\[ \int_{\partial d} \sigma_{ij} \varepsilon_{ij} \, d\sigma - \int_{\partial d} \beta_{ij} \, d\sigma - \int_{V} \nabla \cdot \mathbf{F} \, dV = 0 \]  \hspace{1cm} (1)

where \( \sigma_{ij} \) is stress tensor, \( \varepsilon_{ij} \) is strain tensor, \( \beta_{ij} \) means displacement, \( \mathbf{F} \) is body force, \( \partial d \) denotes surface force acting on \( S_0 \) and \( Su \) is a surface on which the displacement is determined, \( V \) is the volume of the structure, \( n \) is the unit normal vector, \( u_i \) is virtual displacement increment which is zero on \( Su \), and \( \delta e_{ij} \) is virtual strain due to \( \delta u_i \).

In the following, the state before crack growth is denoted by \( p \), and \( p+1 \) means the state after crack growth. Shape function \( N_{ij} \), and \( B_{ij} \), which relates strain with displacement, are expressed as follows:

\[ \epsilon_{ij} = N_{ij} u_i \]  \hspace{1cm} (2)
\[ \delta e_{ij} = B_{ij} \delta u_i \]  \hspace{1cm} (3)

where \( u_i \) means the nodal displacement. Using eqs. (2) and (3), and deleting \( \delta u_i \), eq. (1) is rewritten as follows for state \( p \):

\[ \int_{\partial d} \sigma_{ij} B_{ij} \delta u_i \, d\sigma - \int_{V} N_{ij} \delta e_{ij} \, dV = 0 \]  \hspace{1cm} (4)

First, the conventional method which releases the nodal reaction force at the crack tip is discussed. With the crack growth, the element shape does not change, then the virtual work principle in \( p+1 \) is expressed by the next equation.

\[ \int_{\partial d} \sigma_{ij} B_{ij} \delta u_i \, d\sigma - \int_{V} N_{ij} \delta e_{ij} \, dV = 0 \]  \hspace{1cm} (5)

Deleting eq. (4) from eq. (5), the following equation is obtained.

\[ \int_{\partial d} \sigma_{ij} B_{ij} \delta u_i \, d\sigma = \int_{V} N_{ij} \delta e_{ij} \, dV \]  \hspace{1cm} (6)

where \( \delta \sigma_{ij} \), \( \delta \beta_{ij} \), and \( \delta \mathbf{F} \) mean the changes of stress, body force, and crack force from \( p \) to \( p+1 \), respectively.

\[ \int_{\partial d} \sigma_{ij} B_{ij} \delta u_i \, d\sigma - \int_{V} N_{ij} \delta e_{ij} \, dV = 0 \]  \hspace{1cm} (7)

After crack growth, the virtual work principle is written as:

\[ \int_{\partial d} \sigma_{ij} B_{ij} \delta u_i \, d\sigma - \int_{V} N_{ij} \delta e_{ij} \, dV = 0 \]  \hspace{1cm} (8)

By deleting eq. (7) from eq. (8), we have

\[ \int_{\partial d} \sigma_{ij} B_{ij} \delta u_i \, d\sigma = \int_{V} N_{ij} \delta e_{ij} \, dV \]  \hspace{1cm} (9)

Eq. (9) is the same as eq. (6), but in this case, the third term on the right side of eq. (9) is not obtained because \( \Delta \alpha \) can be determined arbitrarily. Then eq. (9) is rewritten using eq. (7):

\[ \int_{\partial d} \sigma_{ij} B_{ij} \delta u_i \, d\sigma = \int_{V} N_{ij} \delta e_{ij} \, dV \]  \hspace{1cm} (10)

The right side of eq. (10) is obtained numerically and crack growth analysis is possible by releasing these forces gradually by considering the unloading effect. In this method, stresses and strains in eq. (7) are calculated by interpolation and numerical errors may occur. These errors are cancelled by the iteration during the release of the reaction forces and CPU time is the same as that of the conventional method.

3. Two dimensional crack growth analysis.

Crack growth analyses are carried out for a CCT specimen as shown in Fig. 3. Due to the symmetry, a quarter part of the specimen is analyzed. Fig. 4(a) is a mesh pattern for the conventional method, and (b) shows one for T.N.P. method. \( \Delta \alpha \) is lam in each crack growth step, and two steps are analyzed. Material constants
for the analysis are: Young's modulus $E=206$ GPa, yield strength $\sigma_Y=490$ MPa, Poisson's ratio $\nu=0.3$, and hardening ratio $H=581$ MPa.

Fig. 5 shows the change of the J integral value with the crack growth, and Fig. 6 shows the load-displacement curves obtained by the two methods. Both results agree very well in these two figures. Fig. 7 shows the crack opening profile during crack growth. For $\Delta a=1$ mm, results of both methods agree well, and for $\Delta a=2$ mm, a little differences are recognized. These are due to the differences of mesh patterns in this step.

4. Three dimensional crack growth analysis of CT specimen.

In the following, two kinds of standard ICT specimens, one without side groove and another with side groove, are considered. The material of specimens is assumed to be A533B steel. The material constants of A533B steel and the results of elastic-plastic analysis before crack growth are published by authors in the previous paper (9). In this chapter, the results of crack growth analyses are shown. Three dimensional J integral is evaluated along the crack front, and it is assumed that the crack growth occurs at first where the J value is the maximum.

4.1 ICT specimen without side groove.

(i) Elastic analysis. Fig. 8 shows a quarter part of ICT specimen. In the thickness direction, the mesh is divided into three layers. Crack growth analysis is carried out keeping a constant displacement where an external load is subjected (which is denoted by the arrow). Fig. 9 shows the results of elastic analysis. At first, a straight crack front is assumed as shown in Fig. 9(a)-1. In this figure, dotted lines show mesh pattern along the thickness direction. Fig. 9(a)-2 shows the J integral distribution of (a)-1. In this case, J value is the maximum at the center of the specimen, and it is assumed that the crack growth occurs at the center of the specimen. The resultant crack front configuration is shown in Fig. 9(b)-1, and the corresponding J integral distribution is shown in (b)-2. The J value is the largest where the curved crack front intersects with the straight one. Then the crack growth occurs here, and the same procedures are repeated until the J integral values become nearly equal along the crack front, as shown in Fig. 9(d)-2. In this step, the crack length at the center of the specimen is 0.9 mm larger than that near the free surface. As these analyses are carried out in elastic states, these processes are considered to

Fig. 3 CCT specimen.

(a) For the conventional method.

Fig. 4 Mesh breakdown.

Fig. 5 J- a curves. Fig. 6 Load-displacement curves.

Fig. 7 Crack opening profiles.

Fig. 8 Size and mesh pattern of CT specimen.
simulate the fatigue precracking. In ICT specimen, it is well known that the fatigue precrack grows faster in the center of the specimen than in other regions. Fig.10(a) shows the fracture surface obtained experimentally, Fig.10(b) shows the shape of the fatigue precrack of Fig.10(a), and Fig.10(c) shows the corresponding J integral distributions of Fig.10(b). By comparing Fig.10(c) with Fig.9(d)-2, it is noticed that both distributions are similar to each other except nearby the free surface. These results show that the fatigue precrack grows and equalizes the J integral values along the crack front, which coincides with the results of Bakker(10).

(i) Elastic-plastic analysis. Using the shape of the fatigue precrack, shown in Fig.10(b), ICT specimen is analyzed elastic-plastically until the maximum value of the J integral becomes nearly equal to 100kJ/m. Fig.11(a) shows the final stage of the elastic-plastic analysis. J value is the maximum in the center part of the specimen, and crack growth is assumed to occur smoothly along the crack front. The maximum Δa is 0.25mm at the center of the specimen. Some procedures are repeated until Δa=1.65mm, where the J integral distribution becomes nearly uniform along the crack front. In the final step, the difference between the maximum and the minimum J values is 10.3%, which is 22.8% before crack growth analysis. The same results are usually obtained experimentally, which is called tunneling effect.

Now the crack growth process in the ICT specimen is summarized as follows. At first, a saw cut notch is introduced as a straight crack as shown in Fig.9(a)-1. By the fatigue precracking, the crack front configuration changes and equalizes the J distribution along the specimen thickness, as shown in Fig.9(d)-2. Due to the
elastic-plastic deformation, the J distribution changes again and reaches the maximum at the center of the specimen, as shown in Fig.11(a)-1. After the fracture criterion is satisfied, a stable crack growth occurs at the center of the specimen mainly, and finally the J distribution becomes again uniform along the crack front.

Fig.12 shows the differences in J values of each integration contour during the crack growth analysis. Five paths are used, and the maximum and the minimum values are shown in this figure. The differences in the J values between different paths are under 4% before crack growth, and they become a little larger(6%) during the elastic-plastic analysis. Shih(3) showed that the path independency of the J integral holds when $\Delta a$ is small in case of two dimensional analysis. In this three dimensional study of 4.2 ICT specimen with 25% side groove.

Fig.13(a) shows a quarter part of ICT specimen with side groove, and Fig.13(b) shows the shape of it. At first, it is assumed that the fatigue precrack is made after introducing the side groove. When the crack front is straight, the J value is much larger at the free surface, as shown in Fig.14(a). Then the crack growth is assumed to occur at the free surface. After several steps, Fig.14(b) is obtained. The shape of the crack front is qualitatively similar to the results of experiments, shown in Fig.15. The crack front configuration is measured from Fig.15, and the result is used for the elastic analysis. The corresponding J value distribution is shown in Fig.16. Fig.14(b) and Fig.16 coincide with each other well except near the free surface.

In the experiment, side groove is introduced after the fatigue precracking, and it can be assumed that the crack front is straight along the specimen thickness. Fig.17(a) shows the result of the elastic-

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Fig.12 J values of each integration path.

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Fig.13 ICT specimen with side groove.

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Fig.14 Crack growth of side grooved CT specimen in elastic state.

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Fig.15 Fracture surface of side grooved CT specimen.

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Fig.16 J integral distribution of Fig.15.

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Fig.17 Crack growth of side grooved CT specimen in elastic-plastic state.
plastic analysis using the straight crack front. Except nearby the free surface, J values are equal along the specimen thickness. Crack growth analysis is carried out assuming that the crack grows at first where the J value is the maximum. The final stage of the analysis is shown in Fig.17(b), where the crack front is not smooth and the J distribution is not uniform either along the crack front. This is not considered appropriate. Then the crack is assumed to grow uniformly along the specimen thickness. The results are shown in Fig.18, where $\Delta a=0.5$mm. The J distribution does not change largely and is similar to that of the initial state. This is more natural than that in Fig.17(b). It is now concluded that with 25% side grooved CT specimen, it is natural for the crack to grow uniformly along the crack front as many experiments([11,12]) indicate.

![Graph](image)

**Fig.18** J integral distribution when the crack front grows uniformly along the specimen thickness.

5. Summary

T.N.P. method is used for the three dimensional crack growth analysis. This method is compared with the conventional one, and is applied to the crack growth problem of CT specimen. Following results are obtained:

1. The results of T.N.P. method agree with those of the conventional method for two dimensional crack growth problem.
2. The fatigue precrack of ICT specimen grows and equalizes the J integral distribution along the specimen thickness.
3. By the tunneling effect, the J integral distribution becomes nearly uniform along the crack front due to the stable crack growth in elastic-plastic state.
4. Crack growth occurs uniformly along the specimen thickness in 25% side grooved CT specimen.

References

(3) Shih, C.F. et al., ASTM STP 668, (1979), p.65
(7) Nakagaki, M. et al., ASTM STP 668 (1979), p.185
(12) Hirano, et al., Ibid., No.820-2, (1982), p.4