An Analysis of Water Hammering in Bubbly Flows

(2nd Report, Transient Pressure Profiles in One-component Flows)

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Water hammer phenomena caused by a sudden valve closure are investigated in one-component two-phase flows. The phenomena in one-component two-phase flows are more complicated than those in two-component two-phase flows because of the presence of mass transfer between liquid and vapor. Basic partial differential equations based on a one-dimensional homogeneous flow model are solved analytically by linearization and iterated Laplace transformation. The profile of the pressure transients at the closing valve are given. It is shown that the mass transfer between the two phases has a great influence on the profile of the pressure transients, and that the magnitude of the influence depends on the relaxation time τ_M of the interphase mass transfer.

Key Words: Multiphase Flow, Water Hammering Analysis, One-component Bubbly Flow, Laplace Transformation

1. Introduction

The water hammer phenomena caused by a rapid valve closure in gas-liquid two-phase flows must be clarified for the safety of boilers and chemical plants, and the pipeline transport of petroleum and natural gas, and especially for the safety analysis of LOCAs in reactors. The pressure characteristics in two-phase flows are greatly different owing to the higher compressibility of gas from those in a liquid single-phase flow. Furthermore, they change more largely in one-component two-phase flows than those in two-component two-phase flows since an interfacial mass transfer occurs between liquid and vapor. But, only a few investigations have been done on water hammer in one-component two-phase flows. The authors have reported on experimental and analytical results about one-component two-phase flow using F-113. This analysis is a numerical solution obtained on a computer under a given condition; therefore, it is difficult from it to grasp the physical meaning of the water hammer phenomenon.

From the above viewpoint, in the present report, an approach to a linearized analysis is presented using a one-dimensional homogeneous bubbly flow model with distributed-parameter in order to clarify the profile of transient pressures qualitatively. In the previous report (1), the results about two-component two-phase flows have been described, but in this report the results about one-component two-phase flows on the profiles of pressure transients, propagation velocity, and the effect of valve closing time will be expressed in an analytical form. It is shown that the pressure transients can be correlated with the relaxation time τ_M, which signifies the rate of the mass transfer between two phases. Comparison with our experimental results will be made in another report.

2. Nomenclature

A_0 : Coefficient (= (3Pr/3x)(W|M|^(n-1))
A_0 : Sound velocity
F : Total surface area of vapor bubbles
L : Total length of test section
x : Arbitrary distance from stop valve
M_E, M_G : Evaporation and condensation rates per unit volume and hour
n : Polytropic index
s, z : Laplace transform of time t and distance x
T : Temperature
t : Time
U : Unit step function
V : Total volume of vapor bubbles in unit volume
W : Velocity
x : Distance
q_x : Void fraction
ζ : Evaporation and condensation coefficient
ρ : Density
τ : Valve closing time
τ_M : Relaxation time of interphase mass transfer

Subscripts
g : Gas phase, l : Liquid phase
o : Steady state, TP : Two-phase

3. Fundamental Equations

A one-dimensional homogeneous bubbly-flow model is shown in Fig. 1, which consists of a rapid closure valve (1), a rigid
pipe \( \tau \) and a reservoir tank with compressible volume \( \delta \). The following assumptions are made:

1. There is no slip motion between the bubbles and the liquid.
2. The effects of the pipe wall elasticity and the liquid compressibility are negligible compared with those of gas compressibility.
3. The friction loss is negligible. (But this is taken into account in Section 4.2.3.)
4. The pressure in the bubbles is equal to that in the surrounding liquid.
5. The liquid temperature is constant owing to the high heat content of the liquid.

Under the above assumptions, equations of continuity for gas or liquid phase hold as follows:

\[
\frac{\partial}{\partial t}(\alpha W) + \frac{\partial}{\partial x}(\alpha W) = M_e - M_e \tag{1}
\]

\[
\frac{\partial}{\partial t}[(1 - \alpha)\rho] + \frac{\partial}{\partial x}[(1 - \alpha)\rho W] = M_e - M_e \tag{2}
\]

where the mass transfer rates \( M_e \) and \( M_e \) per unit volume and time can be given from the gas molecular kinetic theory by putting \( \xi \) as a condensation (or evaporation) coefficient.

\[
M_e - M_e = \frac{\xi P T_a}{\sqrt{2\pi R T_a \lambda}} \left[ 1 - \frac{P}{P_T} \right] \tag{3}
\]

Here, \( T_a \) is an interphase boundary temperature on the inside of bubble and \( T_e \) is assumed to equal \( T_a \) since the heat discharged at the boundary between gas and liquid may be transferred to the liquid side of a higher heat transfer coefficient than that of the vapor side.

The momentum equation for the two-phase mixture can be formulated as

\[
-\frac{\partial}{\partial t}(\alpha W^2) + \frac{\partial}{\partial x}(\alpha W^2) + \frac{\partial}{\partial x}[(1 - \alpha)\rho W^2] + \frac{\partial P}{\partial x} = 0 \tag{4}
\]

The vapor bubble is assumed to change polytropically.

\[
P = c_p \rho \tag{5}
\]

where \( c \) is a constant and \( n = 1 \) for an isothermal change and of \( n \) for a reversible adiabatic (isentropic) change.

4.1 Solution by linearization

The state quantities in Eqs. (1) to (5) are each considered to consist of a steady-state value (represented by subscript \( 0 \)) and an infinitesimal fluctuation (represented by \( \delta \)), like \( P_0, M_0, W_0, \alpha_0, \delta_0 \). By substituting the above equations into Eqs. (1) to (5), neglecting the infinitesimal fluctuation...

### Fig. 1 A total system of water hammer

Terms of more than the second order, using dimensionless forms such as \( \tilde{p} = P'/P_0, \tilde{w} = \delta W/\delta_0, \delta = \delta_0/\delta_0, \) and \( \delta = \delta_0/\delta_0, \) performing iterated Laplace transformation of time \( t \) and distance \( x \), the following equations can be obtained:

\[
(W_0 + s) \bar{\delta}(s, z) + (W_0 + s) \bar{\delta}(s, z) + W_0 \bar{\delta}(s, 0) - W_0 \bar{\delta}(s, 0)
\]

\[
-W_0 \bar{w}(s, 0) = -\bar{\delta}(s, z)/\bar{w} \tag{6}
\]

\[
-(1 - \alpha_0) \alpha_0 \bar{p}(s, z) + \bar{p}(s, 0) = \rho_0 \bar{p}(s, z)/\bar{w} \tag{7}
\]

\[
\rho_0 \bar{w}(s, z) = \rho_0 \bar{w}(s, z) + \rho_0 \bar{w}(s, 0)
\]

\[
\bar{w}(s, 0) = \bar{w}(s, 0) + \bar{w}(s, 0)
\]

\[
\bar{p}(s, z) = \bar{p}(s, z) + \bar{p}(s, 0)
\]

where \( \rho_0 \bar{w}(s, 0) = (1 - \alpha_0) \rho_0 \bar{w}(s, 0) + \alpha_0 \delta_0 \bar{w}(s, 0) \) is the mean density of a two-phase flow in the initial state, \( \rho_0 = \bar{\rho}_0/\bar{\rho}_0 \) is the density, and \( \bar{\rho}_0 \) is the velocity in homogeneous two-phase flows, and \( \bar{\rho}_0 = \rho_0 \bar{\rho}_0 + \rho_0 \bar{\rho}_0 \) for a spherical bubble. This signifies the relaxation time required for the mass transfer to settle into a state of thermodynamic equilibrium.

By eliminating \( \bar{\rho}_0 \) and \( \bar{w} \) from Eqs. (6) to (9), the following equations of \( \bar{p} \) and \( \bar{w} \) are given.

\[
[(a_0 - W_0^2)z^2 - W_0(s + k_0)z - s(s + k_0)] \bar{p}(s, z) = -[(a_0 - W_0^2)z - W_0(s + k_0)] \bar{p}(s, 0) + (n W_0(s + k_0)) \bar{w}(s, x) \tag{10}
\]

\[
[(a_0 - W_0^2)z^2 - W_0(s + k_0)z - s(s + k_0)] \bar{w}(s, z) = -[(a_0 - W_0^2)z - W_0(s + k_0)] \bar{w}(s, 0) + (n W_0(s + k_0)) \bar{w}(s, x) \tag{11}
\]

where \( k_0 = \frac{n \rho_0}{\bar{\rho}_0 (k_0 + k_0)} \).

By applying inverse Laplace transformation of Eqs. (10) and (11) with...
respect to \( s \), the following equations (12) and (13) can be obtained.

\[
\begin{align*}
\hat{\rho}(s, 0) &= \frac{-2nw_{w_s}}{a_0 - Kw_{w_s} + \sqrt{C_0}} \hat{\omega}(s, 0) \\
\hat{\omega}(s, 0) &= \frac{-2nw_{w_s}}{a_0 - Kw_{w_s} + \sqrt{C_0}} \\
&+ \frac{1}{2C_0} \left( 1 - Kw_{w_s} + \sqrt{C_0} \right) e^{-\nu s} \hat{\rho}(s, 0) \\
&+ \frac{1}{2C_0} \left( 1 - Kw_{w_s} + \sqrt{C_0} \right) e^{-\nu s} \hat{\omega}(s, 0)
\end{align*}
\]

where \( C_0 = a_0 \left( 2n + K_0 \right)^2 - K_0 \left( a_0^2 - W_0^2 \right) \)
and
\( a_1, b_1 = \{-2b + K_0 W_0 + \sqrt{C_0} / 2(a_0^2 - W_0^2) \}

The boundary condition is given as \( \hat{\omega}(s, 0) = 0 \) in Fig. 1. Substituting this into Eqs. (12) and (13), the pressure transient \( \hat{\rho}(s, 0) \) at the closing valve is given as

\[
\begin{align*}
\hat{\rho}(s, 0) &= \frac{-2nw_{w_s}}{a_0 - Kw_{w_s} + \sqrt{C_0}} \hat{\omega}(s, 0) \\
&+ \frac{2C_0}{\left( 1 - Kw_{w_s} + \sqrt{C_0} \right)} e^{-\nu s} \hat{\omega}(s, 0) \\
&+ \frac{1}{\left( 1 - Kw_{w_s} + \sqrt{C_0} \right)} e^{-\nu s} \hat{\rho}(s, 0)
\end{align*}
\]

That is, the transient pressure response at the downstream valve can be expressed by

\[
\begin{align*}
\frac{-2nw_{w_s}}{a_0 - Kw_{w_s} + \sqrt{C_0}} \hat{\omega}(s, 0) \\
+ \frac{2C_0}{\left( 1 - Kw_{w_s} + \sqrt{C_0} \right)} e^{-\nu s} \hat{\omega}(s, 0) \\
+ \frac{1}{\left( 1 - Kw_{w_s} + \sqrt{C_0} \right)} e^{-\nu s} \hat{\rho}(s, 0)
\end{align*}
\]

4.2 The pressure transients at downstream valve

The initial transient pressure profile, until the rarefaction wave returns to the downstream valve, is given in brackets [ ] at the right hand side in Eq. (14).

That is,

\[
\begin{align*}
\hat{\rho}(s, 0) &= \frac{-2nw_{w_s}}{a_0 - Kw_{w_s} + \sqrt{C_0}} \hat{\omega}(s, 0) \\
&+ \frac{2C_0}{\left( 1 - Kw_{w_s} + \sqrt{C_0} \right)} e^{-\nu s} \hat{\omega}(s, 0) \\
&+ \frac{1}{\left( 1 - Kw_{w_s} + \sqrt{C_0} \right)} e^{-\nu s} \hat{\rho}(s, 0)
\end{align*}
\]

\[
\begin{align*}
\hat{\omega}(s, 0) &= \frac{-2nw_{w_s}}{a_0 - Kw_{w_s} + \sqrt{C_0}} \hat{\omega}(s, 0) \\
&+ \frac{1}{2C_0} \left( 1 - Kw_{w_s} + \sqrt{C_0} \right) e^{-\nu s} \hat{\rho}(s, 0) \\
&+ \frac{1}{2C_0} \left( 1 - Kw_{w_s} + \sqrt{C_0} \right) e^{-\nu s} \hat{\omega}(s, 0)
\end{align*}
\]

The transient pressure \( \hat{\rho}(s, 0) \) is solved analytically by applying inverse Laplace transformation of Eq. (15) with respect to \( s \) for an initial condition of \( \hat{\omega}(s, 0) \).

4.2.1 The initial response for step change of valve closing

The pressure response \( \hat{\rho}(s, 0) \) for a sudden valve closure of \( \hat{\omega}(s, 0) = -1/s \) as shown by a solid line in Fig. 2 is derived from Eq. (15), assuming \( 1 - Kw_{w_s} + \sqrt{C_0} \approx 1 \).

\[
\begin{align*}
\hat{\rho}(s, 0) &= \frac{-2nw_{w_s}}{a_0 - Kw_{w_s} + \sqrt{C_0}} \hat{\omega}(s, 0) \\
&+ \frac{1}{2C_0} \left( 1 - Kw_{w_s} + \sqrt{C_0} \right) e^{-\nu s} \hat{\rho}(s, 0) \\
&+ \frac{1}{2C_0} \left( 1 - Kw_{w_s} + \sqrt{C_0} \right) e^{-\nu s} \hat{\omega}(s, 0)
\end{align*}
\]

The inverse Laplace transformation with respect to time \( t \) is given as

\[
\begin{align*}
\rho(t, 0) &= \frac{-2nw_{w_s}}{a_0 - Kw_{w_s} + \sqrt{C_0}} \omega(t, 0) \\
&+ \frac{1}{2C_0} \left( 1 - Kw_{w_s} + \sqrt{C_0} \right) e^{-\nu t} \rho(t, 0) \\
&+ \frac{1}{2C_0} \left( 1 - Kw_{w_s} + \sqrt{C_0} \right) e^{-\nu t} \omega(t, 0)
\end{align*}
\]

where \( I_0 \) is a modified Bessel function of 0th order, that is, \( I_0(t) = 1 + t^2/4 + t^4/64 + \cdots \). and

\( K_0 = (n/\sqrt{\pi})(l - \rho_{w_0})(l - \rho_{w_0}) \).

The numerical values of Eq. (17) are given in Table 1 and plotted by a solid line against a dimensionless time \( t_0 \) in Fig. 3. A broken line in Fig. 3 shows the profile of the pressure rise at \( t = \tau \) in the case of the valve closing time \( \tau \). That is, the transient pressure rises sharply due to a sudden valve closure \( (t = 0) \), attaining a maximum value \( \Delta P_{\text{max}} = a_0 \rho_{w_0} K_0 \) and decreases.

\[ \hat{\rho}(s, 0) = \frac{-2nw_{w_s}}{a_0 - Kw_{w_s} + \sqrt{C_0}} \hat{\omega}(s, 0) \]

\[ \hat{\omega}(s, 0) = \frac{-2nw_{w_s}}{a_0 - Kw_{w_s} + \sqrt{C_0}} \hat{\omega}(s, 0) \]

\[ \hat{\rho}(s, 0) = \frac{-2nw_{w_s}}{a_0 - Kw_{w_s} + \sqrt{C_0}} \hat{\omega}(s, 0) \]

\[ \hat{\omega}(s, 0) = \frac{-2nw_{w_s}}{a_0 - Kw_{w_s} + \sqrt{C_0}} \hat{\omega}(s, 0) \]

Fig. 2 Initial condition \( \hat{\omega}(t, 0) \)

<table>
<thead>
<tr>
<th>( t_0 )</th>
<th>( \frac{1}{1 - e^{-t_0}} )</th>
<th>( \frac{\hat{\rho}(s, 0)}{a_0} )</th>
<th>( \frac{\hat{\omega}(s, 0)}{a_0} )</th>
<th>( \frac{\hat{\omega}(s, 0)}{a_0} )</th>
<th>( \frac{\hat{\rho}(s, 0)}{a_0} )</th>
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<td>0.2</td>
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</tr>
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</tr>
<tr>
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</tr>
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<td>0.309</td>
<td>10.0</td>
<td>1.000</td>
<td>0.128</td>
</tr>
</tbody>
</table>
exponentially as shown by a solid line in Fig. 3. The degree of the decrease depends on \( \kappa_0 \), that is, the relaxation time of the mass transfer \( \gamma_M \). The shorter the relaxation time \( \gamma_M \) is, the more the pressure decreases. When \( \gamma_M \) is infinite, that is, there is no phase change, \( \overline{P}(t,0) \) is equal to \( \alpha_0 \overline{P}_m \overline{X}_0 / \overline{P}_0 \) and independent of time \( t \), as \( \kappa_0 = 0 \), and the pressure does not decrease with time \( t \). This agrees with the analytical result in the previous report (8).

4.2.2 The effect of the valve closing time \( \tau \)

In general, the valve involves the time lag \( \tau \) until the closing end. To make clear the effect of such valve closing time \( \tau \) on the pressure transients, the critical condition shown by a broken line in Fig. 2 is considered. Substituting \( \mathcal{W}(s,0) = (1 - e^{-\kappa_0 s}) / \tau s^2 \) into Eq. (15) and applying the inverse Laplace transformation with respect to time \( t \), the pressure transient \( \overline{P}(t,0) \) at the downstream valve (\( \kappa = 0 \)) is obtained.

\[
\begin{align*}
\overline{P}(t,0) &= \frac{1}{(\alpha_0 \overline{P}_m \overline{X}_0 / \overline{P}_0)} \left[ I_0 \left( \frac{K_d(t-t)}{2} \right) + I_1 \left( \frac{K_d t}{2} \right) \right] \\
&= \left( -t - \frac{1}{2} \right) \left[ I_0 \left( \frac{K_d(t-t)}{2} \right) + I_1 \left( \frac{K_d t}{2} \right) \right] \\
&+ \frac{1}{2} \left[ \frac{K_d(t-t)}{2} \right] U(t-t) + \frac{1}{2K_d(t-t)} U(t-t) \\
&= \frac{1}{2K_d} \left( -e^{-K_d t} - 1 + K_d(t-t) \right) U(t-t)
\end{align*}
\]

where \( I_1(t) \) is a modified Bessel function of 1st order. \( I_1(t) = t^2 + t^3/16 + t^4/384 + \ldots \).

The result is shown in Fig. 4 for the case of \( N_0/a_0 = 0 \). The solid lines represent the profiles of the transient pressure under the constant condition of \( 1/\kappa_0 = \gamma_M / a_0 = 0.025 \) s and the broken lines represent those of \( \gamma_M / a_0 = 0.2 \) s for five variables, \( \gamma = 0, 0.025, 0.05, 0.1, 0.15 \) s. The locus connecting the values of maximum pressure rise \( \overline{P}(t,0) \) at \( t = \tau \) is shown by chain dotted lines for \( 1/\kappa_0 = \gamma_M / a_0 = 0.005, 0.025 \) and 0.2 s and against \( \kappa_0 \tau / 2 \) by a broken line in Fig. 3. It is seen that the maximum pressure rise occurs at \( t = \tau \) and the pressure becomes smaller with an increasing \( \tau \) as shown by a dotted line in Fig. 4 (this corresponds to Eq. (19) or a broken line in Fig. 3). This phenomenon differs from that in two-component two-phase flows where the maximum pressure rise is constant independently of \( \tau \). After the maximum pressure rise at \( t = \tau \), the profiles approach asymptotically with time \( t \) to Eq. (17), that is, the solid line in Fig. 3.

\[
\begin{align*}
\overline{P}(t,0) &= e^{-K_d t} \left[ I_0 \left( \frac{K_d t}{2} \right) + I_1 \left( \frac{K_d t}{2} \right) \right]
\end{align*}
\]

\( \tau \) in Fig. 3 is shown by a dotted line in Fig. 4 (this corresponds to Eq. (19) or a broken line in Fig. 3). This phenomenon differs from that in two-component two-phase flows where the maximum pressure rise is constant independently of \( \tau \). After the maximum pressure rise at \( t = \tau \), the profiles approach asymptotically with time \( t \) to Eq. (17), that is, the solid line in Fig. 3.

4.2.3 The effect of friction loss

The effect of the friction loss on the pressure responses is clarified. Substituting the term for the friction loss \( \Delta \overline{P}_L / \Delta x = \Delta \overline{K}_0 \) \( |W| \overline{D} \) into Eq. (4), the pressure response \( \overline{P}(t,0) \) is derived for a sudden valve closure similarly to the preceding procedure.
\[ \dot{\rho}(s, 0) = \frac{-\alpha_0 \rho \tau \dot{W}_0}{\tau_0} \left\{ \frac{2(s + \rho_0)\alpha_0}{(s + \rho_0)(s + \rho_0) + (W_0/\alpha_0)^2} \right\} \cdot \dot{\omega}(s, 0) \]

\[ = \frac{-\alpha_0 \rho \tau \dot{W}_0}{\tau_0} \sqrt{4(s + \rho_0)(s + \rho_0) + (W_0/\alpha_0)^2} \left\{ \frac{(s + \rho_0)}{(s + \rho_0)(s + \rho_0) + (W_0/\alpha_0)^2} \right\} \cdot \dot{\omega}(s, 0) \]

where \( \tau_0 = \alpha_0 \rho_0 \tau_0 \) (unit is \( 1/\text{sec.} \)).

For the initial condition of \( \dot{\omega}(s, 0) = -1/\alpha_0 \), assuming \( W_0/\alpha_0 = 0 \), Eq. (20) is expressed by

\[ \dot{\rho}(s, 0) = \frac{-\alpha_0 \rho \tau \dot{W}_0}{\tau_0} \left\{ \frac{2(s + \rho_0)\alpha_0}{(s + \rho_0)(s + \rho_0) + (W_0/\alpha_0)^2} \right\} \cdot \dot{\omega}(s, 0) \]

The calculated result is shown in Fig. 5 for \( \rho_0/\rho = 0, 0.05, 0.1, 0.2 \approx 1.0 \). The case of \( \rho_0/\rho = 0 \) corresponds to no friction loss and is equal to Eq. (17) or the solid line in Fig. 3. As the value of the friction loss \( \rho_0/\rho \) becomes greater, the decay of the pressure just after the initial pressure rise becomes smaller. The effect of \( W_0/\alpha_0 \) is shown in Fig. 6. Similarly, with an increase of \( W_0/\alpha_0 \), the pressure decay just after the initial pressure rise becomes smaller.

4.3 The pressure transient at any location

The profile of the transient pressure \( \dot{\rho}(s, -x) \) at any location \( x = -x \) is obtained by substituting Eq. (14) into Eq. (12).

\[ \dot{\rho}(s, -x) = -\frac{2\alpha_0 W_0 \dot{\omega}(s, 0)}{\alpha_0 \rho_0 (-\rho_0 W_0 + \sqrt{G_0})} \cdot \dot{\omega}(s, 0) \]

\[ x = \rho_0 W_0 + \sqrt{G_0} \cdot \frac{e^{s(s) - e^{s(s) - i\pi + k_1)}}{1 + \frac{K_0 W_0 + \sqrt{G_0}}{-K_0 W_0 + \sqrt{G_0}} e^{s(s) - i\pi}} \]

Accordingly, the pressure rise \( \dot{\rho}(s, -x) \) at \( x = -x \), until the rarefaction wave returns to that location, can be expressed as follows.

\[ \dot{\rho}(s, -x) = -\frac{2\alpha_0 W_0 \dot{\omega}(s, 0)}{\alpha_0 \rho_0 (-\rho_0 W_0 + \sqrt{G_0})} \cdot \dot{\omega}(s, 0) \]

\[ \dot{\omega}(s, 0) \]

For the initial condition of \( \dot{\omega}(s, 0) = -1/\alpha_0 \), assuming \( W_0/\alpha_0 = 0 \), the inverse Laplace transformation of Eq. (22) with respect to time \( t \) is given as

\[ \frac{\dot{\rho}(t, -x)}{\alpha_0 \rho_0 (-\rho_0 W_0 + \sqrt{G_0})} = \frac{1}{\sqrt{s(s + \rho_0)}} e^{-\kappa_0 t} \left\{ \frac{K_0}{2} \sqrt{1 - \left( \frac{1}{\alpha_0} \right)^2} \right\} \]

\[ \dot{\omega}(s, 0) \]

This becomes equal to Eq. (17) in the case of \( x = 0 \). The numerical result of Eq. (23) for the propagation delay of the pressure wave, that is, \( \kappa_0 \), is shown in Fig. 7. A broken line corresponds to the case of \( x = 0 \) in Eq. (23), that is, Eq. (17). The initial pressure rise at any location is marked by the symbol \( O \) in the figure. That is, it is expressed as \( e^{\nu_0 t/2} \) on account of \( t = s/\alpha_0 \).

It approaches asymptotically with time \( t \) to Eq. (17) (a broken line in Fig. 7). It is seen that the initial pressure rise at any location, \( -x \), decreases further as the pressure wave proceeds upstream and the locus is expressed as \( e^{-\kappa_0 t/2} \).

5. Conclusions

An analytical approach to water hammer phenomena in one-component two-phase flows involving interfacial mass transfer was made using a linearization method of Laplace transformation based on a homogeneous flow model with distributed-parameter. The profile of the transient pressure and the effects of the valve closing time and friction loss on transient pressure have
been clarified analytically. The results are summarized as follows:

(1) The pressure decreases exponentially just after a sudden initial pressure rise. This can be correlated with the relaxation time $\tau_0$ of the interphase mass transfer.

(2) The initial maximum pressure rise becomes smaller with an increase in the valve closing time $T$. The pressure takes a maximum value at $t = T$ and is expressed by Eq. (19).

(3) As the friction loss becomes greater, the pressure decay just after an initial pressure rise becomes smaller.

(4) The transient pressure at any location becomes smaller than that at the downstream closing valve and is expressed by Eq. (23).

References