Analysis of Turbulent Structure of an Impulsively Started Jet by Applying Image Processing

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An image processing method for the analysis of a turbulent structure was devised. It was applied to an impulsively started axisymmetrical jet growing into an otherwise quiescent fluid. The development of the statistical average contour and the growing behaviour of the jet were clarified. Simultaneous measurements of the concentration and the velocity at a fixed position were carried out and the transient phenomenon at the position was made clear. By comparing the results of the two methods, the average behaviour and the characteristics of the turbulence of the jet were revealed. It is confirmed that the image processing method developed here is effective for the analysis of the spatial structure of turbulence in an unsteady flow field.

Key Words: Unsteady Flow, Image Processing, Impulsively Started Jet, Turbulent Structure, Turbulence, Axisymmetrical Jet

1. Introduction

To investigate the structure of turbulence experimentally, simultaneous measurements of the velocity, the temperature or the concentration fluctuations at several fixed points in the flow field have been conducted so far. However, in this method, numerous sensors are inserted in the flow and enormous treatments of the obtained fluctuation signals are required to image out the instantaneous spatial structure of turbulence in the whole flow field. Moreover, the conventional method does not work effectively when the flow is unsteady in the sense of statistical average.

Although an image of a visualised flow contains an information on the whole flow field at an instant, image data have been so far used exclusively to understand the qualitative nature of the flow in order to aid the analysis of the point fluctuation signal [1]. However, the development of the techniques of the electronics computer system and the image processing have made it possible to obtain quantitative information directly from image data. In this work, these techniques were applied to investigate the behaviour of the contour of an impulsively started axisymmetrical jet growing into an otherwise quiescent infinite fluid [2,3] and the results were compared with those obtained by the conventional method.

2. Nomenclature

- Concentration of solute (NaCl)
- Diameter of the nozzle, 5.12 mm
- \( f(t) \): arbitrary fluctuation signal
- \( L \): length of a growing jet, see Fig.4
- \( L_1, L_2 \): see Fig.8
- \( L \): spatial scale of turbulence
- \( M \): number of test runs under a fixed condition
- \( R \): correlation coefficient
- \( t \): time from start of the jet
- \( t_0 \): time from start of the jet
- \( T_b \): rise time of a fluctuation signal
- \( T_b \): transient period of a fluctuation signal
- \( U \): velocity in z-direction
- \( U_{n0} \): velocity at \( t = T_b \), see Fig.2 or Fig.11(b)
- \( X \): spread angle of jet, see Fig.6
- \( \theta \): coordinate system, see Fig.1

Subscripts and superscripts
- \( C \): pertaining to the concentration
- \( \tilde{C} \): value obtained by image processing
- \( L \): sequential number in runs under a fixed condition
- \( m \): value on the jet axis
- \( N \): value at the nozzle
- \( \dot{u} \): pertaining to the velocity
- \( \bar{u} \): ensemble average defined by Eq. (4)
- \( \bar{u} \): conditional average defined Eq. (5)

3. Apparatus and Procedure

3.1 Apparatus

The experimental apparatus is shown in Fig. 1. The flow in the measuring section is vertically upward and its velocity is kept constant at a low enough value, 0.7 cm/s, to be effectively at rest by supplying tap water. At the beginning of every run, the cylinder is filled with
dye-added water or NaCl water solution. Then the piston is driven impulsively with a constant velocity to start a jet. Thus the nozzle velocity $U_n$ is raised from 0 to 1.3 m/s at time $t=0$ and the jet begins to grow into an otherwise almost quiescent water. Measurements on the steady jet formed in this apparatus show that the characteristics of the steady jet, such as the velocity distribution, the decay of the velocity along the jet axis, the spread angle of the jet and the turbulence intensity, are virtually equivalent to those of a jet in an infinite quiescent fluid ![4,5](image)

3.2 Techniques of visualization and image processing

Black dye was added in a weight fraction of 1/20 to the water in the cylinder to visualize the jet. A sequence of photographs were taken with an interval of 0.1 s after the start of the jet. 20 sequences of such photographs were taken under a fixed condition in order to obtain the statistical average characteristics of the jet.

The area of a photograph was divided into 256 x 256 meshes and the brightness of every mesh was converted into a 4-bit digital value for the treatment by a personal computer.

3.3 Measurement of velocity and concentration

In order to measure the velocity and the concentration fluctuation at a fixed point simultaneously, a hot film probe and an electric conductivity probe were coupled as shown in Fig. 1. In this case, 2 wt% sodium-chloride water solution was used as the nozzle fluid. Two such couples of probes were used, one of them being set at point A(x,0), a point on the jet axis, and the other at B(x,y), a point on the same cross section with A but a different radial position.

An example of the fluctuation signals at A is shown in Fig. 2, together with the signals of the start of the piston motion and its displacement. From this figure, it is confirmed that the start of the piston motion was sharp enough and its velocity afterwards was kept constant. Since the jet thus formed is a stochastic phenomenon, the velocity and the concentration signals at a point, $U$ and $C$, include chaotic variation among the runs under a fixed condition. To obtain the statistical mean property of the jet, 30 runs were conducted under a fixed condition. The analogue signals of $U$ and $C$ were converted into 12-bit digital values with a sampling time 2 ms and the sample number 1000 for a signal, the corresponding signal length 2 s being long enough to attain a quasi-steady state at every measuring point in this experiment.

![Fig. 1 Outline of apparatus](image)

![Fig. 2 Oscillogram of the velocity, concentration, and piston motion](image)

![Fig. 3 Photograph of a growing jet](image)

![Fig. 4 Jet contour obtained by image processing from photographs in Fig. 3](image)
For the details of the apparatus, readers may refer to [6].

4. Results and Discussions

4.1 Analysis of the behaviour of a jet by image processing

In Fig. 3, an example of the original image data, namely, the photographs of a growing jet, is reproduced. The contours of the jet in Fig. 3 were obtained from the photographs in Fig. 3 by the following procedure.

The 2-bit digital value of the brightness of every mesh of a photograph was converted into a binary value by using an appropriate threshold value, and the whole area of a photograph was divided into two regions, the jet region and the surrounding fluid region. The jet contours in Fig. 4 are the bounding lines of these two regions. \( \xi \)-axis was determined from the parallel straight lines of this contour near the nozzle, where the flow is laminar. The origin \( O \) was taken at the intersection of \( \xi \)-axis and the edge of the nozzle.

4.1.1 Jet length. As illustrated in Fig. 6, the length of a jet \( L(x) \) at time \( t \) for the \( k \)-th run was defined as the \( \xi \)-coordinate of the crossing point of the contour line and \( \xi \)-axis. The statistical mean of the jet length over runs of number \( K \) under a fixed condition was defined by Eq. (1).

\[
\bar{L}(t) = \frac{1}{K} \sum_{i=1}^{K} L_i(t) \quad \cdots \cdots \cdots \cdots \cdots (1)
\]

In Fig. 5, the nondimensionalized mean jet length \( L/d \) thus obtained is plotted against the nondimensionalized time \( \beta t \), with \( \gamma \) constant, together with the relation between the rising times of the fluctuation signals and \( \xi \)-coordinate of the point where they were measured. They are discussed later in detail.

5.1.2 Average contour. Since the contour line is different for each run, a normalization of the spatial dimension is required to define its average configuration. Here, \( \xi \)-coordinate was normalized by Eq. (2), and the distances \( d_{L(x)} \) and \( d_{R(x)} \) of a contour line from \( \xi \)-axis on each side of a jet were expressed as a function of \( \gamma \) as illustrated in Fig. 6.

\[
X = (\bar{L}/L_0)X \quad \cdots \cdots \cdots \cdots \cdots (2)
\]

The mean value of \( d_{L(x)} \) and \( d_{R(x)} \) with common \( X \) and \( t \) was calculated over \( K \) runs under a fixed condition by Eq. (3). Thus the ensemble average contour of the jet at time \( t \) was represented by \( \bar{d}(X) \).

\[
\bar{d}(X) = \frac{1}{2M} \sum_{l=1}^{M} (d_{L(x)} + d_{R(x)}) \quad \cdots \cdots \cdots \cdots \cdots (3)
\]

The growing process of the ensemble average contour thus obtained is shown in Fig. 7. The ridges and valleys seen in the contours of jets in Fig. 4 are almost smoothed out by averaging over 20 runs.

![Fig. 6 Definition of average jet contour and fluctuation component](image)

![Fig. 7 Average jet contour](image)
The average contour is roughly divided into two regions, a conical region where \( \delta \) increases linearly with \( x \) and a semi-spherical region on the head of the jet. When the velocity \( U \) and the concentration \( C \) are measured at a fixed point \( A(x,0) \), they rise from zero to a quasi-steady turbulent level after passing a transient period between them. The transient period is considered to correspond to the period from the time when \( U \) or \( C \) at \( A \) begins to be affected by the approach of the semi-spherical head of the jet to the time when the conical region reaches the point \( A \). The quasi-steady state of the signal \( U \) or \( C \) is obtained when the point \( A \) is surely included in the conical region of the jet. The characteristics of the signals \( U \) and \( C \) in the quasi-steady stage were shown to be equivalent to those of a steady turbulent jet. Then, the semi-spherical head region and the conical region are called the unsteady region and the steady region respectively in this paper.

Since it was difficult to distinguish definitely the two regions from the contour of a run shown in Fig. 4 or its average in Fig. 7, here they were bounded by the following procedure:

Since the region \( x/\delta-(1/2) \) was considered to be definitely steady, a straight line was determined from \( \delta(x) \) in this region by the least square method. The region of \( x \) where \( \delta_{\text{max}} \), the deviation of \( \delta(x) \) from the straight line, is less than or equal to 10% of the maximum value \( \delta_{\text{max}} \) of \( \delta(x) \) was defined as the steady region as shown in Fig. 8. The virtual origin \( 0' \) was defined as the crossing point of the straight line and \( x \)-axis. The distances from \( 0' \) to the edges of the steady region and the unsteady region, \( L_1 \) and \( L_2 \), were

![Fig. 8 Definition of boundary between steady and unsteady regions of a jet](image)

![Fig. 9 Spread angle and relative length of steady region of jet](image)

![Fig. 10 Velocity of jet front](image)

![Fig. 11 Averages of fluctuation signals \( \langle U' \rangle \) and \( \langle C' \rangle \) \((R_e=5000, x/\delta=14, N=29)\)](image)

![Fig. 12 Transient process at a fixed point](image)
obtained and the ratio \( \frac{L_1}{L_2} \), together
with the angle of the spread of the jet,
is plotted against time \( t \) in Fig. 9. The
result of \( \frac{L_1}{L_2} \) with \( \Delta d/\Delta x_{\text{max}} = 0.05 \) is
also plotted for comparison. \( \frac{L_1}{L_2} \) with
\( \Delta d/\Delta x_{\text{max}} = 0.05 \) does not differ
significantly from that with \( \Delta d/\Delta x_{\text{max}} = 0.1 \),
although the scatter of the data is large. This
shows that the method applied here to
define the boundary of the two regions is
reasonable.

In Fig. 9 both \( \frac{L_1}{L_2} \) and the spread
angle \( \theta \) approach constant values at time
\( t = 0.8 \tau \), meaning that the jet attains
self-similar stage at \( t \theta/\tau < 150 \).

4.2 Comparison of the results from
the fluctuation signal and the image
4.2.1 Behaviour of the jet front In
Fig. 5 the relation between \( z \)-coordinate of a point \( A(x, y, z) \) and the average rising
times of the fluctuation signals at \( A \) is
compared with that between the jet length
and time. Here, the rising time \( T_{\text{RU}} \) or
\( T_{\text{RC}} \) was defined as the time when the signal
\( \bar{u} \) or \( C \) attained \( 3\% \) of its time mean
value in the quasi-steady period.

From Fig. 5 it is confirmed that the relation
between \( T_{\text{RC}} \) and \( z \) agrees well with that
between \( z \) and \( \tau \), and that the rise of the
velocity precedes that of the concentration.
In Fig. 10, \( U_{\text{RU}} \), the average velocity at
\( t = T_{\text{RU}} \), is compared with the growing
velocity of the jet \( U_{\text{RU}} \) obtained from the
image data. If the surrounding fluid is
entrained into the jet at the front interface,
\( U_{\text{RU}} \) is larger than \( U_{\text{RC}} \). The result
in Fig. 10 shows that the velocity of the
jet growth \( U_{\text{RU}} \) and the velocity of the fluid
at the point where the front is just
passing are nearly equal, meaning that there
is no intense entrainment at the turbulent
interface of the jet front. The result that
\( U_{\text{RU}}>U_{\text{RC}} \) for small \( z \) in Fig. 10
may be due to the threshold of \( C \) in
determining \( T_{\text{RC}} \). Therefore, the motion of the
surrounding fluid near the approaching jet
front is considered to be similar to that

observed when a solid body is approaching.

4.2.2 Transient process of the fluctuation
signals The statistical mean of an
arbitrary fluctuating signal \( \bar{f}(t) \) is
defined by Eq. (4) applying data of number \( M \)
obtained from runs under a fixed condition.

\[
\bar{f}(t) = \frac{1}{M} \sum_{i=1}^{M} f_i(t)
\]  

However, in this case, the phenomenon ob-
served at a fixed point changes abruptly
at the instant of the arrival of the jet fluid,
\( t = T_{\text{RU}} \), and \( T_{\text{RC}} \) differs among the
runs. Then, in order to analyze the tran-
sient behaviour of the signals, it is more
rational to average \( f(t) \) after shifting the
time so as to make the times of the
arrival of the jet fluid coincide with
each other. For this reason, the condi-
tional average of \( f(t) \) was calculated by
Eq. (5), denoted as \( \bar{f}(t') \).

\[
\bar{f}(t') = \frac{1}{M} \sum_{i=1}^{M} f_i(t' = t - T_{\text{RU}})
\]  

In Fig. 11 conditional averages \( \bar{U}(t') \) and
\( \bar{C}(t') \) by Eq. (5) are compared with \( \bar{U}(t) \) and
\( \bar{C}(t) \). The rise of \( \bar{U}(t') \) in Fig. 11(b) is
abrupt, indicating that the smooth rising of
\( \bar{C}(t) \) in Fig. 11(a) is apparent and

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![Fig. 13 Velocity and concentration distributions on jet centerline and average jet contour (t=0.6 \( \tau \)).](image)

(a) Distributions of velocity and concentration, (b) Average contour.

![Fig. 14 Velocity and concentration correlation coefficients in radial direction](image)

![Fig. 15 Auto-correlation coefficient of concentration on jet axis](image)
caused by the scattering of \( T_{gr} \) among the runs. On the contrary, it is confirmed again that \( \dot{U}(t') \) rises smoothly preceding \( \dot{C}(t') \). After rising from zero, \( \dot{U} \) and \( \dot{C} \) increase to the quasi-steady levels, including large fluctuations. Since it was difficult to determine definitely the transient period from these signals, the transient period at a fixed point \( T_{gr} \) was determined from the image data as follows.

In Fig. 12, the point \( A(t_{l1}'\), 0) is the measuring point. \( T_{gr} \) at \( A \) is the period from the arrival time of the jet front to the time when the steady region of the jet reaches the point \( A \) through the growing of the jet. Therefore, we have

\[
\bar{T}_{gr} = \int_{x=t_{l1}'}^{x=t_{l1}''} \frac{dx}{U_l(x)}.
\]

For \( \bar{U}_l(t) \), the following expressions from the result of the image processing shown in Fig. 5 were applied.

\[
x/d < 20: \quad \bar{L}_d = K_c (t U_l / d)^{m_1}
\]

\[
x/d \geq 20: \quad \bar{L}_d = K_c (t U_l / d)^{m_2}.
\]  

Fig. 16 Comparison of spatial scales in radial direction between by image processing and by conventional method

The constants in Eq. (7) are, \( K_1 = 0.61 \), \( K_2 = 0.80 \), \( K_3 = 1.34 \) and \( K_4 = 0.62 \) under the condition of \( \dot{U}_l = 5.12 \text{ mm/s} \) and \( U_l = 1.3 \text{ m/s} \). \( T_{gr} \) in Fig. 11(b) was calculated by the procedure mentioned above.

4.2.3 Velocity and concentration on the jet axis

In Fig. 13 the average velocity \( \bar{U}_g \) and the concentration \( \bar{C}_g \) on \( z \)-axis at \( t = 0.6 \text{ s} \) are compared with the average contour of the jet. The solid and the broken lines in the figure represent the velocity and the concentration distributions in the steady turbulent jet respectively. The position where \( \bar{U}_g / U_l \) and \( \bar{C}_g / C_0 \) attain the lines for the steady jet roughly agrees with the boundary of the steady and the unsteady regions in the average contour.

4.3 Scales of turbulence in the steady region

The spatial scales of turbulence in the steady region are estimated from the image data by the following procedure. As illustrated in Fig. 6, the mean square of the scale of turbulence in \( z \)-direction is considered to correspond to the deviation of a contour from the straight line. Then, it is calculated by Eq. (8).

\[
\tilde{L}_t(X) = \frac{1}{2 M} \sum_{\tau=0}^{N} \left[ (d_{ox}(x) - \bar{d}(X))^2 + (d_{oz}(x) - \bar{d}(X))^2 \right].
\]

The scale of turbulence in \( z \)-direction corresponds to the wave length of the contour. Then, as shown in Fig. 6, the average of the distance between neighboring two crossing points of a curve \( d_{i}(x) \) with \( \bar{d}(X) \) is calculated by Eq. (9).

\[
\tilde{l}_u = \frac{1}{2 M} \sum_{\tau=0}^{N} \left[ l_u(x) + l_u(x) \right].
\]

\( l_{TP} \) and \( l_{TP} \) thus obtained were compared with the integral scales obtained from the quasi-steady fluctuation signal at fixed points using the conventional method. Namely, \( \dot{U} \) and \( \dot{C} \) were measured at two points, \( A(x, 0) \) and \( B(x, 0) \), on the same cross section simultaneously. Cross correlation coefficients between the velocities or the concentrations at \( A \) and \( B \) were calculated. From the measurements with various values of \( n \), the velocity and the concentration correlation coefficients, \( R_{Uo} \) and \( R_{Co} \), were obtained as a function of \( n \) as shown in Fig. 14. A spatial scale of turbulence in \( x \)-direction, \( l_{TP} \), was determined from the concentration correlation coefficient as the area of \( \Delta AOC \) in Fig. 14. The other scale \( l_{TP} \) was determined from the velocity correlation coefficient by a similar process. The auto-correlation coefficient was calculated from the concentration at the point \( A(x, 0) \) as shown in Fig. 15. A time scale \( r_{TP} \) was defined as the area of \( \Delta AOC \) in Fig. 15, and it was converted into a length scale \( l_{TP} \) in \( x \)-direction by \( l_{TP} \). The same procedure was applied to the velocity signal and another spatial scale \( l_{TP} \) in \( z \)-direction was determined. The scales of turbulence from the image data are compared with these inte-
eral scales from the fluctuation signals in Fig. 16 and Fig. 17. The solid lines in these figures represent those for a steady jet obtained by Becker et al. [7], from the concentration measurement using the light scattering method. In Fig. 16, $\frac{l_{om}}{l_{1/2}}$, $\frac{l_{oc}}{l_{1/2}}$ and $\frac{l_{oc}}{l_{om}}$ increase linearly with $\nu$, coinciding with the results of Becker et al., and the ratio $\frac{\nu}{l_{om}} = 1$. In Fig. 17, $\frac{l_{om}}{l_{1/2}}$, $\frac{l_{oc}}{l_{1/2}}$ and $\frac{l_{oc}}{l_{om}}$ also increase linearly with $\nu$ and the ratios $\frac{l_{oc}}{l_{om}}$ or $\frac{l_{oc}}{l_{om}}$ are constant and equal to each other, having a value of nearly four. These ratios are significantly large compared with those in the radial direction. This may be caused by the lack of the resolving ability in the image processing.

5. Conclusions

The spatial structure of turbulence in an impulsively started axisymmetrical jet was investigated experimentally by applying the image processing. The results were compared with those of conventional point measuring method. The conclusions are summarized as follows.

(1) The growing characteristics of the jet obtained from the ensemble average contour are shown to correspond to those from the conventional method, and the average contour is divided into the steady and the unsteady region.

(2) A new conditional averaging method was introduced to investigate the transient phenomenon from fluctuation signals at a fixed point during the passing of the unsteady region of the jet. Applying this method to the velocity and the concentration at a point on the jet axis, it is shown that the velocity at the point rises gradually preceding the concentration, and that the concentration rises abruptly in the conditional average. Moreover, the entrainment of the surrounding fluid is shown to be small at the front interface of the jet.

(3) The scale of turbulence in the steady region of the jet obtained from the image processing is shown to correspond well with that from the conventional method.

The results obtained in this work suggest that the image processing technique is an effective method to analyze the spatial structure of turbulence in an unsteady flow.

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