Forced Torsional Vibration of a Two Degrees of Freedom System with a Clearance

(2nd Report: Analytical Solution)

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An analytical solution of the forced torsional vibration of a two degrees of freedom system with a clearance between two rotating bodies is derived. The analytical solution is obtained by connecting the solutions for the motion of each body caused by successive collisions of the bodies, without making assumptions on the number of times of collisions that occur during a cycle of the forced motion and on the periodicity of the collisions. The analytical solution shows the existence of a phenomenon of multi-time collisions at one side of the clearance during a half cycle of the forced motion. Effects of the dimension of the clearance and the average angular velocity on the strength of the impact force are investigated. The results of the analytical solution show good quantitative agreements with those of the simulation.

Key Words: Theory of Vibration, Shock, Analytical Solution, Two Degrees of Freedom System, Forced Vibration, Backlash

1. Introduction

The authors discussed in Part 1 the rattle vibration of a manual transmission gear as the forced torsional vibration of a two degrees of freedom system involving an input gear and a counter gear, and analyzed it by experiment and simulation. It was pointed out regarding the rattling gear phenomenon occurring in this system that the front face of teeth of the input gear collided many times with the rear face of teeth of the counter gear during the period in which the input gear was increasing the speed and that the rear face of teeth of the input gear repeatedly collided with the front face of teeth of the counter gear many times during the speed decreasing period that followed. An attempt was made to evaluate the magnitude of angular acceleration of gears.

Part 2 deals with this impact phenomenon using an analytical solution. Because the number of collisions that would occur and their periodicity were unknown, a solution method based on the assumption of a periodic solution could not be used as an analytical method. Therefore, a method of obtaining a regresstional formula was employed because each collision was considered a non-periodic exciting function. Comparison of the solution obtained with the results of the experiment and simulation presented in Report 1 disclosed that this analytical solution method was effective.

2. Mechanical Model and Symbols

Figure 1 shows the mechanical model of the system used for the experiment. The symbols are defined as follows:

- $J_1$: Mass moment of inertia of the input gear;
- $J_2$: Mass moment of inertia of the counter gear;
- $k_1$: Torsional spring constant;
- $k_2$: Torsional spring constant equivalent to the bending stiffness of a pair of gears in mesh;
- $z$: Exciting displacement;
- $\omega'$: Average rotational angular velocity ($=\omega/2$);
- $\omega$: Circular frequency of the varying part of exciting displacement;
- $E$: Amplitude of the varying part of exciting displacement;
- $c$: 1/2 of the clearance existing in the pair of gears;
- $c_1, c_2$: Viscous damping coefficients;
- $X$: Angular displacement of the input gear;
Y: Angular displacement of the counter gear;
X: Angular displacement of the varying part of the input gear;
y: Angular displacement of the varying part of the counter gear;
t, t': Times
G: Non-dimensional time (=ω t).

3. Equation of Motion and its Solution

3.1 Equation of motion

The equation of motion for mass moments of inertia $J_1$ and $J_2$ of the system shown in Figure 1 is:

$$
J_1 \frac{d^2 X}{dt^2} + c_1 \frac{dX}{dt} + k_1 (X-Z) = T(X-Y) \tag{1}
$$

$$
J_2 \frac{d^2 Y}{dt^2} + c_2 \frac{dY}{dt} - T(X-Y) = 0 \tag{2}
$$

It is assumed here that the time origin is at a point where exciting displacement has zero value, and that $T(X-Y)$ is the non-linear torque caused by the clearance. If $X = x + ω't$ and $Y = y + ω' t$, equation (1) can be rewritten in variables $x$ and $y$ that express only the varying parts as follows:

$$
J_1 \frac{d^2 x}{dt^2} + c_1 \frac{dx}{dt} + k_1 x = k_1 E \sin ωt + T(x-y) - c_1 \omega' \tag{3}
$$

It is assumed that the origin of $y$ is at the center of the clearance. $T(x-y)$ represents the exciting torque that acts on both gears upon collision of their tooth faces, and it is an impulsive function that takes a large value only when the two gears contact each other and a zero value when they do not. Since $x$ and $y$ are time functions, $(x-y)$ is also a time function. The exciting torque expressed as a time function is written as $G(t')$. The impact vibration dealt with in this study is such that neither the number of collisions that occur during one period of angular velocity variation nor the periodicity of collisions is known so that a solution based on the assumption of periodicity cannot be used for solving equations (2) and (3). Therefore, the motions $J_1$ and $J_2$ in cases where a general exciting torque acts will be obtained, and using the results, a solution in cases where a torque of impulse acts will be obtained. The motion of the system at time $t$ is dependent on the exciting torque of the time preceding it, and therefore, symbol $t'$ will be used in expressing the acting time of exciting torque. That is, exciting torque will be expressed as $G(t')$.

3.2 Motion of the vibration system when an arbitrary exciting torque acts

If the exciting torque acting between $J_1$ and $J_2$ shown in Figure 1 is $G(t')$, Figure 1 can be given in two single degree of freedom systems (a) and (b) as in Figure 2. Here, exciting torque per unit moment of inertia expressed in non-dimensional time $g' = \omega / ω'$ is expressed as $g(θ') = G(θ'/ω)/J_1$. This exciting torque causes each unit moment of inertia to generate a momentary angular velocity increment equivalent to

$$
dθ' = g(θ') dθ' / ω' \tag{4}
$$

where $g = d/dθ$.

If initial angular displacements and initial angular velocities are $x_0$, $x_0'$, $y_0$ and $y_0'$ respectively, solutions $x_1$ and $y_1$ of the equation of motion for $J_1$ and $J_2$ in a case where exciting torque $G(t')$ is not acting on the single degree of freedom system shown in Figure 2 will be, as is known,

$$
x_1 = e^{-ω't} \left( x_0 - B \sin θ + \lambda \cos θ \right) + L(x_0 - B \sin θ + \lambda \cos θ) / \sqrt{1 - y'^2} \times \sin ωt + B \sin(θ + δ) - \lambda \cos θ \tag{5}
$$

$$
y_1 = (1 - e^{-ω't}) (2y_0 + 1/2) + y_0 - θ'/2 \tag{6}
$$

where

$$
T = c_1 / 2 \sqrt{J_1}, \omega_0 = \sqrt{K_1 / J_1}, \lambda = \omega / \omega_0 = 1 / K, q = K / \sqrt{1 - y'^2}
$$

$$
c = c_1 / J_1, \delta = \tan^{-1} \left( \frac{2y_0}{1 - A_t} \right)
$$

$$
B = E \sqrt{(1 - A_t)^2 + (2y_0)^2}
$$

Fig. 2 Breakdown of System
Thus, the motion of the system when exciting torque \( g(\theta') \) acts can be obtained as follows. That is, assuming the incremental angular velocity at time \( \theta' \) and using equation (5), the incremental displacement of \( J_2 \) at an arbitrary time \( \theta' \) after \( \theta' \) will be,

\[
\Delta \theta = \int_{\theta'}^{\theta} \frac{g(\theta')e^{-\xi(t-s)}}{\omega^2(1-\xi^2)} \sin q(\theta - \theta') \, d\theta' \quad \cdots \cdots \cdots (7)
\]

Similarly, the incremental displacement of \( J_2 \) will be,

\[
\Delta \theta = \frac{g(\theta')}{\omega^2} \int_{\theta'}^{\theta} e^{-\xi(t-s)} \, d\theta' \quad \cdots \cdots \cdots (8)
\]

Thus, the incremental displacements by the exciting torque acting between time \( \theta' = 0 \) and time \( \theta' = \theta \) will be given by equations (7) and (8) integrated for the time from \( \theta' = 0 \) to \( \theta' = \theta \).

### 3.3 Exciting torque by collision

The exciting torque acting on the system during a very short contact time of collision is expressed as a symmetrical triangular wave. For this, a non-dimensional triangular wave \( f(\theta') \) with time \( \theta_1 \) in the center is introduced as shown in Figure 3. That is,

\[ f(\theta') = \begin{cases} 0 & : \theta' \leq \theta_1 - 1/n \\ \frac{n(n+1)(\theta' - \theta_1)}{\theta_1} & : \theta_1 - 1/n \leq \theta' \leq \theta_1 \\ \frac{n(n+1)(\theta' - \theta_1)}{\theta_1} & : \theta_1 \leq \theta' \leq \theta_1 + 1/n \\ 0 & : \theta_1 + 1/n \leq \theta' \end{cases} \quad \cdots \cdots \cdots (9)
\]

#### 3.4 Motions of \( J_1 \) and \( J_2 \) after first collision

It is assumed for convenience that a first collision occurs between the rear faces of the clearance of \( J_1 \) and \( J_2 \). Of course, the result will be the same if it occurs between the front face of the clearance of \( J_1 \) and \( J_2 \).

If the first collision occurs at time \( \theta_1 \), exciting torque \( g(\theta') \) that acts between \( J_1 \) and \( J_2 \) can be expressed by the following equation, using the triangular wave expressed by equation (9),

\[ g(\theta') = I_1 \omega f(\theta') / J_1 \quad \cdots \cdots \cdots \cdots (10) \]

where \( I_1 \) and \( \theta_1 \) are undecided as yet, and will be determined later. \( I_1 \) has a torque \( X \) time dimension, and corresponds to an impulse in a linear motion. It is called a torque impulse here.

By substituting equation (10) for \( g(\theta') \) of equation (7) and integrating the time from \( \theta' = 0 \) to \( \theta' = \theta \), the incremental displacement of \( J_1 \) by the collision will be,

\[ x = e^{-\xi(\theta)} \left( A \sin \theta + D \cos \theta \right) + B \sin(\theta - \theta_1) + \gamma \]

\[ + \frac{Hn^4e^{-\xi(\theta - \theta_1)}}{(\pi K)^2 - q^2} \left[ F_1 \times \left[ (\pi K)^2 - q^2 \right] \times q(\theta - \theta_1) \times (\pi K)^2 \sin q(\theta - \theta_1) + 2\pi Kq \cos q(\theta - \theta_1) \right] \quad \cdots \cdots \cdots (12) \]

Thus, the displacement subsequent to the first collision will be, by adding equation (11) to equation (5),

\[ x = e^{-\xi(\theta)} \left( A \sin \theta + D \cos \theta \right) + B \sin(\theta - \theta_1) - \gamma \]

\[ + \frac{Hn^4e^{-\xi(\theta - \theta_1)}}{(\pi K)^2 - q^2} \left[ F_1 \times \left[ (\pi K)^2 - q^2 \right] \times q(\theta - \theta_1) \times (\pi K)^2 \sin q(\theta - \theta_1) + 2\pi Kq \cos q(\theta - \theta_1) \right] \quad \cdots \cdots \cdots (12) \]

Similarly, from equations (6), (8), and (10), the displacement of \( J \) will be,

\[ y = \left( 1 - e^{-\xi} \right) \left( \frac{\mu_2 + 1/2}{c} \right) + \mu_2 - \theta_2 - \frac{1}{c} \frac{n^2}{\omega^2} e^{-\xi(\theta - \theta_1)} \times (e^{-\xi(\theta - \theta_1)} + e^{\xi(\theta - \theta_1)} - 2) \quad \cdots \cdots \cdots (13) \]
$$A = \frac{\gamma D + \lambda (x_e - B \cos \delta)}{\sqrt{1 - \gamma^2}} \quad \quad \quad H = \frac{\lambda (\gamma \alpha_0 \sqrt{1 - \gamma^2})}{\mu}$$
$$D = x_e - B \sin \delta + \gamma \alpha$$

$$F_1 = (e^{-\gamma \alpha n} + e^{\gamma \alpha n}) \cos q/n - 2$$
$$F_2 = (e^{-\gamma \alpha n} - e^{\gamma \alpha n}) \sin q/n$$

As it is assumed that the first collision occurs on the rear face of the clearance, the following equation holds from the relationship of displacements of J and K immediately preceding the collision,

$$\epsilon + x(\delta_1, \epsilon) = y(\delta_1, \epsilon) \quad \quad \quad (14)$$

By obtaining displacements x and y immediately preceding the collisions from equations (5) and (6) and substituting them in equation (14), we have

$$\epsilon + e^{-\gamma \alpha \delta}(A \sin \delta_1 + D \cos \delta_1) + B \sin(\delta_1 + \delta) - \gamma \alpha (1 - e^{-\gamma \alpha \delta}) \left( \frac{\delta_1 + 1/2}{\epsilon} \right) + p_0 - \delta_1/2 \quad \quad \quad (15)$$

Collision time is determined by solving the above equation. Then, by using rebounding coefficient , we have

$$-\epsilon = \frac{[\epsilon, \gamma_0 + 1 - x_0]}{[\epsilon_0, \gamma_0 - 1 - x_0]} \quad \quad \quad \quad \quad \quad \quad \quad (16)$$

By differentiating equations (5), (6), (12), and (13) by once, and substituting the results in equation (16), we have

$$f_1 = \frac{(1 + \epsilon_0)(\epsilon_k \sqrt{1/2} - e^{-\gamma \alpha \delta}(\gamma A - \gamma K D) \cos \delta_1 - (\gamma KD + q D \sin \delta_1) - B \cos(\delta_1 + \delta))}{\sqrt{1 - \gamma^2} + B \sin(\delta_1 + \delta) - \gamma \alpha (1 - e^{-\gamma \alpha \delta}) \left( \frac{\delta_1 + 1/2}{\epsilon} \right) + p_0 - \delta_1/2} \quad \quad \quad (17)$$

Thus, by substituting and obtaining from equations (15) and (17) in equations (12) and (13), the motion after the first collision till immediately before the occurrence of a second collision will be determined.

### 3.6 Determining second collision occurrence position

It is then necessary to determine on which face of the clearance, front or rear, a second collision will occur. According to the results of the experiment and simulation analysis presented in Part 1, the boundary at which a collision changes from one side of the clearance to the other was near the point where the gradient of varying velocity changes, that is, in the vicinity of \( \dot{x} = 0 \). Thus, equation (12) is differentiated twice using \( \dot{\theta} \), the time where \( \dot{x} = 0 \) for the front time after time \( \delta_1 \), is obtained, and if it is expressed by

Rear face of clearance collides if

\[
\begin{align*}
\dot{y}(\delta_1) &\geq x(\delta_1) \\
\ddot{y}(\delta_1) &\leq -\epsilon + x(\delta_1)
\end{align*}
\]

--- (18)

If

\[-\epsilon + x(\delta_1) < y(\delta_1) < \epsilon + x(\delta_1) \quad \quad \quad (19)\]

\( J_2 \) is half way in the clearance at time \( \delta_1 \). Therefore, it is necessary to compare positions after the passage of a specific time. Thus, as shown in Figure 4, the displacement of \( J_2 \) at \( \delta_1 \) is expressed as \( F_1 = y(\delta_1) \), straight line \( Y_{P_2} \) that passes \( P_1 \) and contacts the rear face, that is, is tangent to the \( x(\theta) + t \) curve, is obtained, contact point is assumed as \( y_2 \) and its displacement at \( \delta_2 \) is obtained as \( Y_{P_2} \) for comparison. If the displacement of \( J_2 \) at this time is \( P_2 \), its tangential equation will be

\[ Y_{P_2} = x(\delta_2) + y(\delta_1) \quad \quad \quad (20) \]

Tangential time \( \delta_{P_2} \) will be obtained by solving

\[ x(\delta_{P_2}) = \{x(\delta_0) - y(\delta_0) + x(\delta_1)\} / (\delta_2 - \delta_1) \]

--- (21)

Rear face of clearance collides if

\[ y(\delta_2) \leq x(\delta_1) + x(\delta_2) \quad \quad \quad (22) \]

Front face of clearance collides if

\[ y(\delta_2) \geq x(\delta_1) + x(\delta_2) \quad \quad \quad (23) \]

--- (24)
If
\[-\varepsilon + x(\theta_0) < y(\theta_0) < \varepsilon + x(\theta_0) \quad (23)\]

the same sequence as before may be repeated, that is, obtaining the straight line that passes \(P_2\) and is tangent to \(x(0) + \varepsilon\) and determining the reference time for comparison.

If the gradient of the tangent changes in this process, a collision occurs on the side of \(x(\theta) - \varepsilon\), that is, on the front face of the clearance.

Fig. 4 Collision Occurrence Position

1.7 Motion after \(S\) times of collisions

The above process is applied to the motion subsequent to repetition of \(S\) times of collisions of \(J\) with \(J_{\text{..}}\).

Putting \(i = 1, 2, 3, \ldots, S\) as \(1\), and assuming that \(-1\) when a collision occurs with the front face of the clearance and \(1\) when a collision occurs with the front face of the clearance, the motion after \(S\) times of collisions will be the sum of incremental displacements, expressed by equations (12) and (13), due to \(S\) times of collisions. Namely, we have

\[
x = e^{-\kappa t}(A \sin qd + D \cos qd) + B \sin(\theta + \delta) - \gamma d \]
\[
y = (1 - e^{-\kappa t}) \left( \frac{d_\text{in} + 1}{c} \right) + y_0 - \theta d' - \frac{1}{\cos \beta} \sum \rho_i \left( 1 - \frac{n^2}{c^2} e^{-c \theta_0} \right) \times (e^{-c \delta} + e^{c \delta} - 2) \quad (24)
\]

By differentiating equations (24) and (25), \(x, y, \) and \(x\) will be given as follows:

\[
x = e^{-\kappa t}((A - \gamma KD \cos qd - (\gamma KA + D) \sin qd) + B \cos(\theta + \delta)) \]
\[
y = (y_0 + 1/2) e^{-\kappa t} - 1/2 - \frac{n^2}{c} \sum \rho_i e^{-c \theta_0} \times (e^{-c \delta} + e^{c \delta} - 2) \quad (27)
\]
\[
x = e^{-\kappa t}((A + qd) \sin(q0 - q \cos(q0 - q0)) - (qA - qKD) \sin(q0 - q0)) - B \sin(\theta + \delta)
\]
\[
+ Hn^2 \sum \rho_i e^{-c \theta_0} \times (F_1 \sin(q0 - q0) + F_2 \cos(q0 - q0)) \quad (28)
\]

3.8 Determining \(\theta_0\) and \(I_n\)

From equations (24) and (25),

\[
\epsilon \theta_0 + \kappa \delta \theta_0 (A \sin \theta_0 + D \cos \theta_0) + B \sin(\theta_0 + \delta) - \gamma d \]
\[
+ \frac{Hn^2}{(K_0 + qd)^2} \sum \rho_i e^{-c \theta_0} \times \left[ F_1 \sin(q0 - q0) + F_2 \cos(q0 - q0) \right] \quad (29)
\]

As in solving equation (15), time \(t\) at which the \(S\)-th collision occurs can be obtained by solving the above equation about \(\theta_0\). Similarly to solving equation (17), torque impulse \(I_n\) for the \(S\)-th collision can also be obtained from equations (26) and (27).

\[
I_n = \frac{(y_0 + 1/2)e^{-\kappa t} - 1/2 - \frac{n^2}{c} \sum \rho_i e^{-c \theta_0} \times (e^{-c \delta} + e^{c \delta} - 2) \left[ F_1 \sin(q0 - q0) + F_2 \cos(q0 - q0) \right]}{(y_0 + 1/2) e^{-\kappa t} - 1/2 - \frac{n^2}{c} \sum \rho_i e^{-c \theta_0} \times (e^{-c \delta} + e^{c \delta} - 2)} \quad (30)
\]
3.9 Determining $S +$ first collision position

It is necessary to determine on which face of the clearance, front or rear, an $S +$ first collision occurs. First, time $t_{0*}$ at which $x = 0$ for the first time after $S$ times of collisions, that is, after $U_0$, is obtained from equation (28), and a similar sequence to that described in 3.6 may be followed to determine $\delta_0 + 1$ positive or negative. By substituting $U_0$ for $U_0$, $x$ in equation (19), time $t_{P2*}$ at which $x(\theta) + \delta_0 c$ contacts the tangent that passes $P_1$ will be obtained by replacing $c$ with $\delta_0 c$ in equation (21).

Therefore, it may be determined from equation (22). If equation (23) holds, the sequence of obtaining the time for comparison from the tangent may be followed. Thus, the motions of $J_1$ and $J_2$ can be determined from regressive formulas (24) through (30).

4. Calculation Results

4.1 Specification of vibration system

The numerical values used for calculation are the same as those in the simulation discussed in Part II.

\[ J_1 = 5.31 \times 10^{-6} \text{ kgm}^2; \ D_0 = 3.67 \times 10^{-6} \text{ kg}; \ m' = 59.33 \text{ N.m/ rad}; \ W' = 62.83 \text{ rad/s}; \ E = 0.0256 \text{ rad}; \ C_0 = 0.91; \ n = 70. \]

4.2 Varying displacement waveform and relative displacement waveform

Figure 5 shows the relationship of varying displacement $x$ and $y$ of $J_1$ and $J_2$ with relative displacement $X - Y (= x - y)$. Here, $x - y = \xi$ signifies a collision with the rear face of the clearance; $x - y = \eta$ a collision with the front face of the clearance; $\xi < x - y < \eta$ means that the face of gear teeth are not out of contact. It can be definitely confirmed from the figure of relative displacement $X = Y$, therefore, that $J_2$ collides many times with the front face of the clearance of $J_1$ during a half cycle of the speed increasing period of $J_1$, and with the rear face of the clearance during a half cycle of the speed decreasing period that follows.

4.3 Varying velocity waveform

Figures 6 and 8 show varying waveforms for $J_1$ and $J_2$ where $\xi = 0.8 \times 10^{-3}$ rad and $\xi = 3.5 \times 10^{-3}$ rad, corresponding to the results of the experiment and simulation discussed in Part I, shown in Figures 7 and 9. The analytical solution well represents the characteristics of the waveforms for the experiment and simulation solutions. It is also clear that the number of collisions decreases as the clearance increases.

4.4 Relationship of clearance dimension with torque impulse

Figure 10 shows collisions that occur when the ratio of exciting displacement amplitude to clearance dimension is changed, evaluated as the sum of torque impulses per cycle. The sum is $\int_0^T l \cdot l_d$ by the analytical solution, and $\int_0^T l_j^2 d\gamma(t)/dt dt$ by the experiment and simulation. Both are the same in physical quantity as is clear from equation (19).

$T$ is assumed to be equal to $6\pi / \omega$.

In Figure 10, the analytical solution is shown in good agreement with the simulation solution. In Part I, in which the experiment and simulation were compared in RMS value, the values of the experiment were about 10 dB lower than those of the simulation. But comparison in terms of the sum of torque impulses shows that both are about the same in value. The experimental values for angular acceleration waveforms are broad and low because of the time constant of the measuring equipment, and this seems to account for the large difference between the two in RMS values.

4.5 Relationship of average angular velocity with torque impulse

Figure 11 shows the sum of torque impulses when average angular velocity of exciting displacement is changed. The analytical solution is shown to agree well with the simulation solution.
Fig. 6 Varying Velocity Waveform by Analytical Solution

Fig. 7 Varying Velocity Waveforms by Experiment

Fig. 8 Varying Velocity Waveform by Analytical Solution

Fig. 9 Varying Velocity Waveforms by Simulation

Fig. 10 Clearance Dimension and Torque Impulse

Fig. 11 $\omega'$ and Torque Impulse
5. Conclusions

The phenomenon of multi-time collisions occurring in a forced torsional vibration system with a clearance and two degrees of freedom was studied by obtaining its analytical solution. Because the number of collisions and the existence of periodicity were unknown, each collision was regarded as an exciting function without periodicity, and regresional formulas were obtained. The obtained results were compared with the experiment and simulation solutions.

The following conclusions are drawn as a result.
(1) According to the analytical solution, the counter gear develops multi-time collisions with the front face of teeth of the input gear during the speed increasing period of the input gear, and multi-time collisions with the rear face of teeth of the input gear during the speed decreasing period that follows.

(2) The angular velocity waveforms obtained by the analytical solution show characteristics in good agreement with the characteristics of the angular velocity waveforms obtained by the simulation and experiment.

(3) Evaluation of impact torque in terms of the sum of torque impulses shows good agreement among the analytical solution with the simulation solution and experiment values.

(4) The agreement of the analytical solution with the simulation solution shows that the simulation solution provides correct values quantitatively. If the rattle vibration of a manual transmission gear is determined from a simulation solution as a system of increased degrees of freedom, the results will be quantitatively reliable.

(5) This method is found to be one of the effective methods in analyzing impact vibration in cases where the clearance dimension is so small as compared with the exciting amplitude and the number of collisions and its periodicity are unknown.

References


2) Timoshenko et al (translated by Taniguchi and Tamura), Industrial Vibration (1977), 81, published by Corona

3) S. Dubowsky and F. Freudenstein, Trans. ASME, Ser. B, 93-1 (February 1971), 305

4) Dodo, Irie; Study Report, Department of Engineering, University of Hokkaido, No.57 (1970)