A Study of a Load Combination Method Considering the Correlation among Dynamic Loads* 
(2nd Report, Load Combination for the Seismic Response Analysis of a Liquid Storage Tank—Part 2) 
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This report presents a load combination methodology for dynamic response analysis problems, based on a stochastic approach which has been practically introduced in the field of random vibration theory. The methodology proposed in this report has the advantageous point that the appropriate correlation effect among combined load components is taken into consideration and is evaluated in terms of the dynamic correlation factor ρ. The specific combination problem discussed here is the calculation of the maximum base shear coefficient of a large-scale cylindrical liquid storage tank which can be simulated by a simplified lumped mass model, whereby the sloshing effect of the liquid is neglected. It is concluded that the equations of combination constructed here are applicable for the case where the fundamental natural frequency of a tank is higher than half of the dominant frequency of input acceleration such as seismic ground excitation, while an equation proposed in a previous report is effective for other cases. The accuracy of this method is statistically examined by use of the random vibration technique; and its advantage over SRSS (square root of sum of squares) law and ABS (absolute value summation) law is examined with the aid of the numerical computation of practical examples.

Key Words : Vibration, Random Vibration Theory, Cylindrical Tank, Load Combination, Correlation Coefficient

1. Introduction

In the previous report (1), a new load combination method taking account of the correlation effect among dynamic loads in the structural response problems has been proposed in replacement of the square root of the sum of the squares (SRSS) method and the absolute value summation (ABS) method which have been commonly utilized in the structural problems. The method proposed was applied to the seismic response analysis problems of a large-scale rigid liquid storage tank with sloshing effect and the validity of the method was confirmed through a numerical simulation. In this combination problem, the effect which the quantitative relation between the dominant frequency of input motion and the fundamental sloshing frequency of tank gives on the evaluation of correlation coefficients between input motion and sloshing response and on the results of combination was numerically investigated. Through the examination it has been recognized that when the dominant frequency of input motion, sloshing frequency of a tank and dominant frequency of base shear coefficient (or overturning moment) given by their summation take different values from each other, the correlation coefficients between input and response do not match those obtained from the exact value through numerical analysis. Therefore, the method proposed in the previous report is not applicable to the above case.

In this report, the method taking account of the correlation among dynamic loads is further extended in order to apply to the common cases where the effect of dominant frequency components among input and responses are considerable, and the accuracy of the extended method is examined by use of the random vibration theory. First, in this report, we deal with the seismic load combination problem of a large-scale flexible storage tank with deformable wall. According to quantitative relation between the dominant frequency of input motion and fundamental frequency of fluid-shell coupled vibration system, the correlation coefficients between input and response are then calculated and investigated by use of a stochastic approach and through the response analysis. Furthermore, in order to evaluate the dominant frequencies of input and responses, their mean zero-crossing rates are estimated and the combination technique introducing this evaluation is also examined.

Finally, in a practical application, using the artificial time history ground
motions compatible to a design response spectrum, the base shear coefficient is calculated and the availability of the method is confirmed by comparing with other methods.

2. Fundamental Equations of Base Shear Coefficient and Overturning Moment of Cylindrical Tank

Figure 1 shows a mechanical model of a cylindrical storage tank with deformable wall. This model includes the effective mass corresponding to the "dynamic" liquid associated with shell deformation due to the ground motion and also the effective mass corresponding to the "fixed" liquid rigidly attached to the tank while the effect of sloshing liquid is negligible.

2.1 Base shear coefficient

In this model, the base shear coefficient of a tank subjected to the seismic ground motion is obtained as follows:

\[ C_B = \frac{m_p X_f + m_e \bar{U}_f}{m_S g} \ldots \ldots (1) \]

where \( m_p \) and \( m_e \) are the effective masses associated with the forces corresponding to fluid-shell coupled vibration response and input ground motion, respectively, and \( m_S \) is the total mass of liquid in the tank. \( \bar{U}_f \) is the input ground acceleration to the tank-base and \( X_f \) is the relative displacement which can be given as the solution of the following equation of the coupled vibration:

\[ \ddot{X}_f + 2\xi \omega_0 \dot{X}_f + \omega_0^2 X_f = -\bar{U}_f \ldots \ldots (2) \]

By use of non-dimensional parameters \( \alpha_f = m_f/m_p \) and \( \alpha_r = m_r/m_e \), \( C_B \) is rewritten as:

\[ C_B = \alpha_f X_f + \alpha_r \bar{U}_f \ldots \ldots (3) \]

\[ = \alpha_f X_f + (1 - \alpha_f) \bar{U}_f \ldots \ldots (4) \]

Therefore, the base shear coefficient \( C_B \) can be formulated as two types of linear summations:

(1) Summation in terms of absolute response \( X_f \) and the ground excitation \( \bar{U}_f \).

(2) Summation in terms of relative response \( \dot{X}_f \) and the ground excitation \( \dot{\bar{U}} \).

In the above equations (3) and (4), the acceleration of gravity is normalized as 1. Also, parameters \( \alpha_f \), \( \alpha_r \) and the fundamental frequency \( \omega_p \) of a fluid-shell coupled model are evaluated from the equations (2) introducing the mass density of liquid, the diameter \( D \) and height \( H_p \) of a tank, the material of shell, etc.

2.2 Overturning moment

For evaluating the overturning moment of a tank, two models can be considered; the one takes account of the dynamic pressure on the bottom plate and the other does not. In order to simplify the problem, the latter model is here introduced and we have

\[ M_f = m_r H_p \bar{U}_f \ldots \ldots (5) \]

where \( H_p \) and \( H_e \) are the vertical distances from the tank-bottom to \( m_r \) and \( m_e \), respectively. Dividing the above equation by \( H_p = m_r g H_p / 2 \), we have non-dimensional \( C_B \) as follows:

\[ C_B = \beta_f X_f + \beta_r \bar{U}_f \ldots \ldots (6) \]

\[ = \beta_f X_f + (\beta_r - \beta_f) \bar{U}_f \ldots \ldots (7) \]

where

\[ \beta_f = \frac{m_r H_p}{m_e H_e}, \quad \beta_r = \frac{m_r H_p}{m_e H_e} \ldots \ldots (8) \]

\( C_B \) in Eqs. (6), (7) are non-dimensionalized and normalized by \( g \) in the same manner as the calculation of the base shear coefficient \( C_B \) in Eqs. (3), (4). Since \( C_B \) is proportional to the value of the overturning moment, \( C_B \) can be used in place of \( H_p \) for this analysis. From the right hand sides of Eqs. (6), (7), \( C_B \) is expressed by two types of the linear algebraic summation in the same manner as \( C_B \) in Eqs. (3), (4).

\( \beta_f \) and \( \beta_r \) are also obtained referring to equations in Ref. (2). In this report, fundamental parameters of tank model in Fig. 1 for numerical simulation are selected through the investigations in Refs. (2)-(4); the diameter of tank, \( D = 15, 60 \) and \( 85 \) m, and three or five values are chosen for the ratio \( D/H_p \) among the values of 0.5 through 5.0. Using these values, the other parameters like \( \alpha_f \), \( \alpha_r \) etc. are given by the practical equations (2) as shown in Table 1, under the assumptions that the liquid in the tank is water, the material of shell is mild steel and the ratio \( t/R \) of the mean thickness of shell to the radius of tank is 0.001. Furthermore, the natural period \( T_p \) of fluid-shell coupled model is shown in Fig. 2.

![Fig.1 Mechanical model of flexible tank with deformable shell](image)

| Table 1 Specifications and non-dimensional parameters of the tank |
|-------------------------|-------------------------|-------------------------|-------------------------|
| \( D/H_p \) | \( \alpha_f \) | \( H_p/H_e \) | \( \alpha_r \) | \( H_p/H_e \) |
| 0.5 | 0.69 | 0.58 | 0.69 | 0.45 |
| 1.1 | 0.69 | 0.48 | 0.75 | 0.42 |
| 1.8 | 0.57 | 0.42 | 0.60 | 0.39 |
| 2.8 | 0.42 | 0.40 | 0.45 | 0.39 |
| 4.8 | 0.25 | 0.29 | 0.28 | 0.37 |
Fig. 2 Natural period of fluid-shell coupled vibration system of tank

For $\xi_t$, we can use a value of 0.05 given for a flexible cylindrical storage tank in Ref. (5).

3. Stochastic Calculation of Load Combination

Under the assumption that the strong phase of seismic ground motion may be regarded as a stationary random vibration motion, the stochastic approach (6) is applied to the combination problem of a cylindrical tank.

Figure 3 shows a flow chart for calculating the mean of the maximum value of base shear coefficient $C_B$. In the main part of this flow chart, Tajimi's filter transfer function (7) for simulating input ground acceleration as Gaussian stationary zero-mean process with a predominant frequency and Vanmarcke's spectral moment function (8) are introduced. The latter is used in order to evaluate the correlation coefficient between input excitation and fluid-shell coupled vibration response and their mean zero-crossing rates which are calculated based on the quantitative relationship between the natural frequency of coupled vibration tank model and dominant frequency of input motion. These characteristics are shown by comparison of the calculation results with the simulation results. Furthermore, using an expression of the mean of the maximum value of the random process over a duration shown by Havensport (1), the amplification factors are obtained. Finally, based on these results, the combination equations for the mean of the maximum value of $C_B$ can be constructed.

It is recognized from Eqs. (3), (6) or Eqs. (4), (7) that both $C_B$ and $C_Y$ can be similarly evaluated by the summations of an input excitation $U_R$ and response of tank for the model of flexible tank $X_F$ or $X_T$, and a difference among the terms appears just in the coefficients of the terms to be multiplied. Therefore, in the following section, combination concerning $C_B$ is discussed.

3.1 Spectral moment of base shear coefficient

The one-sided power spectral density function of the base shear coefficient $C_B$ in Eqs. (3), (4) is obtained as follows:

$$G_C(\omega) = [\sigma^2]H_C(\omega) + \sigma_1^2H_f(\omega) + \sigma_2^2H_B(\omega)$$

where

$$H_C(\omega) = \frac{\omega^2}{\omega^2 + i2\xi_C\omega}\omega$$

and

$$H_B(\omega) = \frac{\omega^2}{\omega^2 + i2\xi_B\omega}\omega$$

Fig. 3 Flow chart for calculating the mean of the maximum value of base shear coefficient
$G_D(\omega) = \frac{\omega^2 + 4 \xi_s^2 \omega \omega_0^2}{(\omega^2 - \omega_0^2)^2 + 4 \xi_s^2 \omega^2 \omega_0^2}$

where $\xi_s$ is the constant scale factor, and $\omega_0$ and $\omega$ are the variances of $G_D$ and its time derivative $G_{DB}$ respectively. This derivative $G_{DB}$ is obtained by using the term of jerk $X_{AF}$ (or $X_F$) which can be given as

$X_{AF} = -2\xi_s \omega_0^2 X_F - \omega_0^2 X_F$  

3.2 Correlation coefficient between input and response

For the calculation of the spectral moment $\lambda_{m,CG}$ of $G_D$ by substituting Eq.(9) into Eq.(12), the cross-spectral moments between input and response must be introduced. For example, in the case of summation of input $U_F$ and absolute acceleration response $X_{AF}$, the cross-spectral moments $\lambda_{m,au}$ concerning the cross-spectral density functions $H_{RD}(\omega)\cdot G_D(\omega)$ or $H_D(\omega)\cdot G_D(\omega)$ must be evaluated and these are obtained from only the real part as follows:

$\lambda_{m,au} = \int_0^\infty \Re[\omega^m H_{RD}(\omega) G_{D}(\omega)]d\omega$

$= G_D \left[ \frac{(\omega^2 + 4 \xi_s^2 \omega \omega_0^2)}{(\omega^2 - \omega_0^2)^2 + 4 \xi_s^2 \omega^2 \omega_0^2} - \frac{(\omega^2 + 4 \xi_s^2 \omega_0^2)}{(\omega^2 - \omega_0^2)^2 + 4 \xi_s^2 \omega^2 \omega_0^2} \right] d\omega$

$= G_D \left[ \frac{(\omega^2 + 4 \xi_s^2 \omega \omega_0^2)}{(\omega^2 - \omega_0^2)^2 + 4 \xi_s^2 \omega^2 \omega_0^2} - \frac{(\omega^2 + 4 \xi_s^2 \omega_0^2)}{(\omega^2 - \omega_0^2)^2 + 4 \xi_s^2 \omega^2 \omega_0^2} \right] d\omega$

...... (14)

The first term and the second term of the right hand side of above equation are the spectral moments $\lambda_{m,aa}$ of $X_{AF}$ and the product of spectral moment $\lambda_{m,uv}$ of $X_F$ and $\omega_0^2$, respectively. Also, by use of the spectral moments $\lambda_{m,uu}$ of input $U_F$, correlation coefficients $\rho_{m,au}$ between input and absolute response are obtained as follows:

$\rho_{m,au} = \frac{\lambda_{m,au}}{\sqrt{\lambda_{m,aa}\lambda_{m,uu}}} = \frac{(4 \xi_s^2 - 1 + \omega_0^2)}{4 \xi_s^2} \sqrt{\frac{\lambda_{m,uu}}{\lambda_{m,aa}}}$

...... (15)

where $k$ is the ratio of the spectral moments $\lambda_{m,dd}$ of $X_F$ to $\lambda_{m,aa}$. In the above equation, $\rho_{m,au}$ and $\rho_{2,au}$ are the correlation coefficients between $X_{AF}$ and $U_F$ and that between their time derivative, respectively. $\lambda_{m,uu}$ of $U_F$ are given by

$\lambda_{m,uu} = \int_0^\infty \Re[\omega^m H_{RD}(\omega) G_{D}(\omega)]d\omega$

...... (16)

On the other hand, the spectral moments $\lambda_{m,ru}$ of $X_F$ and the cross-spectral moments $\lambda_{m,ru}$ between $X_F$ and $U_F$ are essentially obtained in terms of the spectral moments $\lambda_{m,aa}$ of $X_{AF}$, $\lambda_{m,uu}$ of $U_F$ and the cross-spectral moments $\lambda_{m,au}$.

$\lambda_{m,ru} = \lambda_{m,au} + 2 \lambda_{m,aa}$

...... (17)

Based on these relations, the correlation coefficients $\rho_{m,ru}$ between $X_F$ and $U_F$ are given by

$\rho_{m,ru} = \frac{\lambda_{m,ru}}{\sqrt{\lambda_{m,ru} \lambda_{m,uu}}} = \frac{(4 \xi_s^2 - 1 + \omega_0^2)}{(\lambda_{m,ru} + 2 \lambda_{m,aa} - \lambda_{m,uu})^{3/2}}$

...... (18)

Accordingly, it can be shown from above equation that $\rho_{m,ru}$ and obtained in terms of the spectral moments of $\lambda_{m,aa}$, $\lambda_{m,uu}$ and their related correlation coefficients.

In order to clarify the characteristics of two correlation coefficients $\rho_{m,au}$ and $\rho_{m,ru}$ in Eqs. (15) and (18), the calculation results from these equations are compared with the numerical simulation results by use of a set of 10 ground motions with a
time duration $T=60.06$ s. Figures 5 and 6 show examples for the ground motions corresponding to the ground models with $f_g=1.0$ Hz, $\xi_g=0.4$ and $f_g=1.0$ Hz, $\xi_g=0.05$, respectively. In these figures, the solid and dashed curves correspond to the calculation results of $\nu_{0,au}$ and $\nu_{0,ru}$ for $n=0$ in Eqs.(15) and (18), respectively, and their simulation results are plotted with the symbols $\circ$ and $\triangle$, respectively. Close agreement between these results is obtained irrespective of the ratio of the natural frequency of a coupled vibration tank model to the dominant frequency $f_g$ of ground motion. From these figures, it can be observed that the relation between the two curves is symmetric with respect to the point of $f/f_g=1$ and $\nu_{0,ru}=0$, and the curve slope remarkably changes in the neighbourhood of the axis $f/f_g=1$, as input motion becomes a narrow-band motion. Generally, the values of $\nu_{0,au}$ for $f/f_g<f_g$ and for $f/f_g>f_g$ come close to 0 and 1, respectively, while the values of $\nu_{0,ru}$ for both the frequency range come close to 0 and 1, respectively.

3.3 Mean zero-crossing rate of input and response and amplification factor

From the spectral moment of input $\bar{u}_g$, absolute response $\bar{x}_af$, and relative response $\bar{x}_f$, mean zero-crossing rates $\nu_a$, $\nu_r$, and $\nu$ of their time histories are obtained as follows:

$$\nu_a = \frac{1}{\pi \sqrt{\lambda_{a,uu}}} \quad \nu_r = \frac{1}{\pi \sqrt{\lambda_{r,rr}}} \quad \nu = \frac{1}{\pi \sqrt{\lambda_{a,rr}}} \quad (19)$$

Using the approximate expression of the mean of the maximum value and the variance (spectral moment for $n=0$) of a random process, the acceleration amplification factor can be calculated. The mean $\bar{M}$ of the maximum value of sampled time histories over a duration $T$ is

$$\bar{M} = p_1 f_g \lambda_{a,ii} \quad (20)$$

where $\nu_i$ is the variance of the process and constant value $p_1$ is given by the following equation:

$$\nu_i = \frac{1}{\nu} \ln \frac{0.5772}{\nu} \quad (21)$$

In the above equation, $\nu_i$ is the mean zero-crossing rate of the process in Eq.(19). Using the relations of Eqs.(19) and (21), acceleration amplification factors $R_a$ and $R_r$ for absolute response $\bar{x}_af$ and relative response $\bar{x}_f$ are given by

$$R_a = \frac{p_a \lambda_{a,uu}}{p_a \lambda_{a,rr}} \quad R_r = \frac{p_r \lambda_{a,rr}}{p_r \lambda_{a,uu}} \quad (22)$$

Acceleration amplification factors $R_a$ and $R_r$ in Eq.(22) versus $f/f_g$ are shown in Fig. 7. This figure shows an example of the comparison of the calculation results with the simulation results for a relatively soft ground model with $f_g=1.0$ Hz, $\xi_g=0.05$ and $T=60.96$ s. In this figure, both curves $R_a$ and $R_r$ fairly correspond to their simulation results $O$ and $\Delta$, respectively. From this figure, it can be shown that the relation between $R_a$ and $R_r$ is nearly symmetric in the same manner as the cases of gain-function of frequency response function and correlation coefficients between input and response. These curves have a dominant peak because this motion is narrow-band motion.

Mean zero-crossing rates $\nu_a$, $\nu_r$ and $\nu$ of absolute response, relative response and input in Eq.(19) versus $f/f_g$ are shown in Figs. 8 and 9. Figures 8 and 9 are examples of the ground models with $f_g=1.0$ Hz, $\xi_g=0.4$ and $f_g=1.0$ Hz, $\xi_g=0.05$, respectively. In these figures, both solid and dashed curves $\nu_a$, $\nu_r$ corresponding to the calculation results agree well with their simulation results $O$ and $\Delta$, respectively and close agreement between the calculation and simulation results of $R_a$ is also obtained. From these figures, it can be shown that the value of $\nu_r$ is close to the value of $\nu_0$ in the region of $f/f_g$ irrespective of any frequency components involved in input motion. However, as the value of $f/f_g$ increases in the region

![Fig. 5 Relation between the correlation coefficient and ratio $f/f_g$ (for relatively firm ground model with $f_g=1.0$ Hz, $\xi_g=0.4$)](image)

![Fig. 6 Relation between the correlation coefficient and ratio $f/f_g$ (for relatively soft ground model with $f_g=1.0$ Hz, $\xi_g=0.05$)](image)
Fig. 7 Relation between the amplification factor and ratio $f_f/f_g$
(for relatively soft ground model with $f_g=1.0$ Hz, $\zeta_g=0.05$)

Fig. 8 Relation between the mean zero-crossing rate and ratio $f_f/f_g$
(for relatively firm ground model with $f_g=1.0$ Hz, $\zeta_g=0.4$)

Fig. 9 Relation between the mean zero-crossing rate and ratio $f_f/f_g$
(for relatively soft ground model with $f_g=1.0$ Hz, $\zeta_g=0.05$)

of $f_f \geq f_g$, the value of $\nu_r$ grows large compared with that of $\nu_a$. On the other hand, the value of $\nu_a$ becomes twice the natural frequency $f_n$ in the region of $f_f \leq f_g$, irrespective of any frequency com-
ponents involved in input motion. Furthermore, for the ground motion with $\zeta_g=0.05$ the values of $\nu_a$ and $\nu_0$ are almost equal in the region of $f_f \geq f_g$.

3.4 Mean zero-crossing rate and mean of the maximum value of base shear coefficient

Based on the results obtained from sections 3.1-3.3, the mean zero-crossing rate $V_{CB}$ of base shear coefficient $C_B$ and the combination equations for the mean $\bar{M}_{CB}$ of the maximum value of $C_B$ are here presented. Taking into consideration the previous results and the results of $V_{CB}$, reasonable and convenient combination equations can be proposed.

First introducing the correlation coefficients $\rho_{au}$ and $\rho_{ru}$ in Eqs. (15) and (18) into the spectral moments $\lambda_{m,C_B}$ in Eq. (12), we have

$$\lambda_{m,C_B} = \left[ \frac{a_j}{h_{m,B}} + \frac{2J_{\alpha}(\alpha_i, \alpha_j)}{h_{m,B}J_{\alpha}(\alpha_i, \alpha_j)} + \frac{2a_i a_j}{\lambda_{m,B} + \lambda_{m,B}} \right] \lambda_{m,B} + \frac{a_i a_j}{\lambda_{m,B}}$$

while the mean zero-crossing rate $V_{CB}$ of $C_B$ is obtained as follows.

$$V_{CB} = \sqrt{\frac{\lambda_{m,C_B}}{\rho_{CB}}} \frac{\bar{M}_{CB}}{\bar{M}_{CB}}$$

By applying the relation in Eq. (20) to this case, the combination equations for the mean of the maximum value of $C_B$ are obtained as follows;

(1) The case based on Eq. (13)

$$\bar{M}_{CB} = \rho_{CB} \left[ \frac{a_j^2}{p_s^2} + \frac{2a_i a_j}{p_s^2} \right] \frac{\lambda_{m,B}^2}{\rho_{CB}^2} + \frac{a_i a_j}{\lambda_{m,B}^2}$$

(2) The case based on Eq. (14)

$$\bar{M}_{CB} = \rho_{CB} \left[ \frac{a_j^2}{p_s^2} + \frac{a_i a_j}{\rho_{CB}^2} \right] \frac{\lambda_{m,B}^2}{\rho_{CB}^2}$$

where $\rho_{CB}$ is a constant value given by Eq. (21). On the other hand, the mean of the maximum value of overturning moment $M_B$ can be introduced in the same manner as $\bar{M}_{CB}$ in the above equations. In evaluating $\bar{M}_{CB}$ from Eqs. (25) or (26), $\rho_{CB}$ based on $V_{CB}$ must be always calculated. Therefore, in order to simplify these equations, $V_{CB}$ is estimated and compared with $\nu_a$, $\nu_r$ and $\nu_u$. Figure 10 shows an example of the tank with $D=15$ m and for the ground model with $f_g=1.0$ Hz, $\zeta_g=0.05$. In this figure, the simulation results denoted by the symbols $\Delta$ agree well with the calculation results denoted by the symbols $\circ$. For this ground model, it is made clear that the value of $V_{CB}$ is approximately equal to those of $\nu_a$ of $\nu_r$ and $\nu_u$ of $\nu_l$, in the region of $f_f \geq f_g$, irrespective of the ratio $f_f/f_g$. 
Fig. 10 Comparison of the mean zero-crossing rates
(in the case of tank with \( D=15 \) m, for ground model with \( f_g=1.0 \) Hz, \( C_g=0.05 \))

From the comparison among the mean zero-crossing rates of input, response and base shear coefficient based on the quantitative relation between the natural frequency of fluid-shell coupled vibration system and the dominant frequency of input motion, the combination equations of Eqs. (25) and (26) can be reduced to an approximately simplified equations as follows:

1. In the region of \( f_f \leq f_g \)
   
   \[
   M_C = \left[ a_2 M + 2 a_1 (a_r - a_f) \rho_{e,w} M_a + (a_r - a_f)^4 \rho_e M_a \right]^{1/4} \tag{27}
   \]

2. For relatively firm soil-rock ground motion
   
   \[
   M_C = \left[ a_2 M + 2 a_1 (a_r - a_f) \rho_{e,w} M_a + (a_r - a_f)^4 \rho_e M_a \right]^{1/4} \tag{28}
   \]

Consequently, \( M_C \) is obtained from the quantities of input and responses. On the other hand, since the relation of \( P_{e,w} \) is obtained in the region of \( f_f < f_g \), the equation proposed in the previous report \(^{1} \) can be effective. Equations from Eq. (27) to Eq. (29) are summarized in Table 2 for general application.

4. Results of Practical Application for Combination

The equations proposed in Chapter 3 are applied to the practical seismic response analysis problems. Exact solution of the mean of the maximum value of base shear coefficient \( C_0 \) obtained from the time history response analysis is compared with the simulation results obtained by the specific combination methods.

As an example of application of the proposed method, a set of 10 artificial time history (A.T.H.) ground motions with a duration \( T=30 \) s compatible to a design spectrum(s) and a set of 10 relatively narrow-band ground motions (F.W.N.) with \( T=40.96 \) s corresponding to the soft ground model with \( f_g=1.0 \) Hz, \( C_g=0.05 \) are used for numerical simulation. Also, the calculation results only by use of the equations introducing the parameters \( C_0, C_g \) of ground model are presented as a simplified one. For the case of A.T.H. excitations, the parameters of ground model estimated for Tajimi's filter transfer function by the least square method are used as \( C_0 = 19.4 (cm/s^2)^{-1}, f_g = 3.5 \) Hz and \( C_g = 0.5 \).

Tables 3 and 4 summarize the simula-
tion results and calculation results of the mean $H_0$ of the maximum value of base shear coefficient $C_B$. These tables are examples for the F.W.N. excitations corresponding to a relatively soft ground model and for the A.T.H. excitations, respectively. Furthermore, these tables are results of tanks with $D=15, 60$ and $85$ m for selected values of $D/H_0$. In these tables, the values estimated by the ABS and SRSS methods are based on the ordinary absolute response. The estimated values by the ABS method closely approximate the exact solution in the case of the F.W.N. excitations as shown in Table 4, namely the case where the correlation coefficient between input and absolute response is closer to unity, but this method gives substantially overestimated values compared with exact solution in the cases of tanks, $D=15$ m with $D/H_0=0.5$ and $1.1$ for the A.T.H. excitations because of weak correlation effect. Conversely, the estimated values by the SRSS method agree with the exact solution in the case of the A.T.H. as shown in Table 4, namely in the case where the correlation coefficient between input and absolute response is closer to 0. However, this method never gives a value on safe side in the case of the F.W.N. excitations as shown in Table 4 because of a strong correlation effect.

On the other hand, the simulation results obtained from the proposed method agree well with the exact solution irrespective of the tank size, the ratio of $D/H_0$ and the characteristic of ground model, namely input excitation. Also, calculation results do not match the exact solution in a few cases where the calculated response spectrum does not agree with a given design response spectrum. However, the other results agree well with the exact solution.

5. Conclusions and Acknowledgements

Based on a stochastic approach, a proposed combination technique introducing an evaluation of the correlation effect among dynamic loads is applied to the seismic response analysis problems of a large-scale liquid storage tank with deformable wall in order to calculate the mean of the maximum value of base shear coefficient. The results are summarized as follows;

(1) It has been made clear that the proposed method can be applied to the combination problem of a flexible tank.

(2) Based on the ground condition and the quantitative relation between the natural frequency of a fluid-shell coupled vibration system and the dominant frequency of input motion, the conventional combination law is presented for general application and the availability of the law is confirmed with the aid of the numerical computation of practical examples.

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References


