Vibration Control of Beams by Ball Screw Type Dampers with a Governor*

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The forced nonlinear vibrations of simple beams supported with two kinds of ball screw type dampers are discussed both numerically and experimentally. The dampers are composed of a ball screw, a flywheel and a flyball or perpendicular type governor, and they possess nonlinear characteristics. The beam is replaced by a lumped mass system and the Continuous System Simulation Language is used to simulate the motion of the masses and the damper on a digital computer. The results may be summarized as follows: (1) Two types of dampers are effective for suppressing the amplitude of the beam at the point of attachment of the damper at the first resonance. (2) The linear solution which is obtained by regarding the governor as a flywheel differs from the nonlinear solution in the vicinity of the first resonance frequency of the beam. (3) Each damper approaches the snubber for larger values of moment of inertia J of the flywheel, and acts like a dynamic vibration absorber for smaller values of J .

Key words: Vibration of Continuous System, Vibration Control of Beam, Nonlinear Vibration, Ball Screw Type Damper, Governor.

1. Introduction

The mechanical snubber was devised as an earthquakeproof device for piping systems under the particular circumstances such as high temperature and radioactivity, where the oil damper is not good for use. The mechanical snubber which is used in practice at thermal and atomic power stations is composed of a ball screw, a flywheel and a brake disk. It has such characteristic that it moves without any resistance in response to a slow movement of the pipe due to thermal expansion while it restrains a rapid deformation of the pipe caused by earthquake motion. However, unlike an oil damper, it cannot be utilized as a general absorber.

The authors have devised ball screw type dampers using a flyball or perpendicular type governor instead of the brake disk of the conventional mechanical snubber in order to obtain both a mechanical snubber and a mechanical vibration absorber which have large effects of vibration isolation, and have investigated forced nonlinear vibrations, their stabilities and the effects of vibration isolation when they were attached to a single-degree-of-freedom system⁽¹⁾⁽²⁾. They also investigated a ball screw type damper using a magnetic damper instead of the brake disk⁽³⁾.

In this report, the forced nonlinear vibrations of beams and the effects of vibration isolation of the ball screw type dampers with two types of governors when each damper is attached to a simple beam

are discussed both numerically and experimentally. It will be very difficult to analyse such nonlinear vibrations of beams and it is expected that the programs for calculating the responses of the beams by means of the finite element method are very complicated. In this report, the beam is replaced by a lumped masses system, the equations of motion of the system when it is subjected to an exciting force or displacement are set up with use of influence coefficients, and the Continuous System Simulation Language is used to solve the simultaneous nonlinear ordinary differential equations.

2. Structure of Dampers and Resisting Forces

Figure 1 shows a conceptional sketch of the ball screw type damper with a flyball type governor (we call it the flyball type for short), and Fig.2 shows that of

Fig.1 Conceptual sketch of the flyball type

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the damper with a perpendicular type governor (we call it the perpendicular type for short). In the case of the flyball type, the equations of motion vary depending on whether the mounting ear A is attached to the beam or to the foundation. We call the former case A-type and the latter case B-type. In both cases of the flyball type and the perpendicular type, when a relative linear motion is caused between the top and bottom mounting ear, the screw shaft as well as the flywheel and the governor attached to the shaft end rotate by the ball nut. When the beam supported with the damper is going to resonate, the flyballs or the oscillators of the governor move to a larger radius, so that the natural frequency of the system changes and the system will fall into an antiresonant condition.

Let us first consider a case in which the A-type damper is attached to a single-degree-of-freedom system of mass m_0 and stiffness k_0 and the vibration system is excited by a sinusoidal displacement $u = a \cos \omega t$, as shown in Fig.3. Designating the absolute displacement of the main mass by y and the angle of rotation of the pendulum from its equilibrium position by φ and assuming small values of φ , the equations of motion for the main mass and the pendulum are given by ⁽¹⁾

$$\begin{aligned} m_0 \ddot{y} + (b_1 + b_2 \varphi)(\dot{y} - \dot{u}) + c_e(\dot{y} - \dot{u}) \\ + k_0(y - u) + (b_3 + b_4 \varphi)\dot{\varphi}(\dot{y} - \dot{u}) = 0 \end{aligned} \quad (1)$$

$$\begin{aligned} (b_5 + b_6 \varphi)\ddot{\varphi} + (b_7 + b_8 \varphi)\dot{\varphi} + (b_9 + b_{10} \varphi)\varphi \\ + (b_{11} + b_{12} \varphi)\ddot{u} + (b_{13} + b_{14} \varphi)(\dot{y} - \dot{u})^2 \\ + (b_{15} + b_{16} \varphi)\dot{\varphi}^2 = 0 \end{aligned} \quad (2)$$

in which

$$\left. \begin{aligned} b_1 &= \beta \{J + m_1(r + q \sin \varphi_0)^2\} \\ b_2 &= 2\beta m_1 q \cos \varphi_0 (r + q \sin \varphi_0), \quad b_3 = b_2 \\ b_4 &= -2\beta m_1 q \{r \sin \varphi_0 + q(\sin^2 \varphi_0 - \cos^2 \varphi_0)\} \\ b_5 &= q^2(m_1 + 4m_2 \sin^2 \varphi_0) \\ b_6 &= 8m_2 q^2 \sin \varphi_0 \cos \varphi_0, \quad b_7 = 4c_1 q^2 \sin^2 \varphi_0 \\ b_8 &= 8c_1 q^2 \sin \varphi_0 \cos \varphi_0, \quad b_9 = 4k_1 q^2 \sin^2 \varphi_0 \\ b_{10} &= 4k_1 q^2 \sin \varphi_0 \cos \varphi_0 \\ b_{11} &= q(m_1 + 2m_2) \sin \varphi_0 \\ b_{12} &= q(m_1 + 2m_2) \cos \varphi_0, \quad b_{13} = -b_2/2 \\ b_{14} &= -b_4/2, \quad b_{15} = b_6/2 \\ b_{16} &= 4m_2 q^2 (\cos^2 \varphi_0 - \sin^2 \varphi_0), \quad \beta = 4\pi^2/L^2 \end{aligned} \right\} \quad (3)$$

where c_e is the equivalent viscous damping coefficient corresponding to the frictional loss in the ball screw and the mounting portions, m_1 , m_2 , k_1 and c_1 are the mass of the flyballs, the mass of the sleeve, the stiffness and the equivalent viscous damping coefficient of the governor respectively, r the distance from the center of the shaft to the pivoted point of the link, q the length of the link, φ_0 the initial angle, J the summation of the moments of inertia of the flywheel and the screw shaft, and L the lead of the screw shaft. It will be seen from Eq.(1) that the force exerted on the beam by the damp-

er is given by the following expression:

$$\begin{aligned} f &= -(b_1 + b_2 \varphi)(\dot{y} - \dot{u}) - (b_3 + b_4 \varphi)\dot{\varphi}(\dot{y} - \dot{u}) \\ &\quad - c_e(\dot{y} - \dot{u}) \end{aligned} \quad (4)$$

Next, let us consider a case in which the damper in Fig.3 is the B-type. The expression for f and the equation of motion for the pendulum in this case are given by

$$\begin{aligned} f &= -(b_1 + b_2 \varphi)(\dot{y} - \dot{u}) - (b_3 + b_4 \varphi)\dot{\varphi}(\dot{y} - \dot{u}) \\ &\quad - (b_{11} + b_{12} \varphi)\dot{\varphi} - (b_{12} - b_{11} \varphi)\dot{\varphi}^2 - (m_1 + m_2)\ddot{y} \\ &\quad - c_e(\dot{y} - \dot{u}) \end{aligned} \quad (5)$$

$$\begin{aligned} (b_5 + b_6 \varphi)\ddot{\varphi} + (b_7 + b_8 \varphi)\dot{\varphi} + (b_9 + b_{10} \varphi)\varphi \\ + (b_{11} + b_{12} \varphi)\ddot{y} + (b_{13} + b_{14} \varphi)(\dot{y} - \dot{u})^2 \\ + (b_{15} + b_{16} \varphi)\dot{\varphi}^2 = 0 \end{aligned} \quad (6)$$

In the case of the perpendicular type damper, the expression for f and the equation of motion for the oscillator are given by ⁽²⁾

$$\begin{aligned} f &= -\beta J(\dot{y} - \dot{u}) - \beta m_1(d + z)^2(\dot{y} - \dot{u}) \\ &\quad - 2\beta m_1(d + z)\dot{z}(\dot{y} - \dot{u}) - c_e(\dot{y} - \dot{u}) \end{aligned} \quad (7)$$

$$m_1 \ddot{z} + c_1 \dot{z} + (k_1 + k_2)z - \beta m_1(d + z)(\dot{y} - \dot{u})^2 = 0 \quad (8)$$

3. Case in which a Damper Is Attached to a Simply Supported Beam

Let us next consider a case in which the A-type damper is attached to a simply supported beam and the foundation is moved

Fig.2 Conceptual sketch of the perpendicular type

Fig.3 Vibration system

in a sinusoidally varying displacement $u = a \cos \omega t$. Replacing the beam by n equal masses system which are placed at equal intervals as shown in Fig.4 and expressing the equations of motion in matrix notation using influence coefficients when the damper is attached to the i th mass, we have

$$\{ \ddot{y} \} = [A] \{ -[M] \{ \ddot{y} \} + \{ F \} \} + \{ \ddot{u} \} \quad (9)$$

where $\{ y \}$ is the absolute displacement vector, $\{ u \}$ the exciting displacement vector, $[A]$ the influence coefficient matrix, $[M]$ the mass matrix and $\{ F \}$ the resisting force vector, and they are given by

$$\begin{aligned} \{ y \} &= \begin{Bmatrix} y_1 \\ \vdots \\ y_n \end{Bmatrix} \quad \{ u \} = \begin{Bmatrix} a \cos \omega t \\ \vdots \\ a \cos \omega t \end{Bmatrix} \\ [A] &= \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \\ [M] &= \begin{bmatrix} m & & \\ & (m + b_1 + b_2 \varphi) & \\ & & m \end{bmatrix} \quad \{ F \} = \begin{Bmatrix} 0 \\ \vdots \\ f' \\ \vdots \\ 0 \end{Bmatrix} \end{aligned} \quad (10)$$

In this equation, f' is obtained by omitting the term of $\ddot{y}$ and replacing $\ddot{y}$ with $\ddot{y}_i$ in Eq.(4). Rearranging Eq.(9) we obtain

$$\{ \ddot{y} \} = [M]^{-1} \{ [A]^{-1} (\{ u \} - \{ y \}) + \{ F \} \} \quad (11)$$

From Eq.(2) we obtain

$$\begin{aligned} \ddot{\varphi} &= -[(b_7 + b_8 \varphi) \dot{\varphi} + (b_9 + b_{10} \varphi) \varphi \\ &\quad + (b_{11} + b_{12} \varphi) \ddot{u} + (b_{13} + b_{14} \varphi)(\dot{y}_i - \dot{u})^2 \\ &\quad + (b_{15} + b_{16} \varphi) \dot{\varphi}^2] / (b_5 + b_6 \varphi) \quad (2') \end{aligned}$$

Solving n equations given by Eq.(11) and Eq.(2)' simultaneously, we obtain the solution for this case. The solutions for the cases of the B-type and the perpendicular type can be obtained in a similar manner. Putting $u=0$ in above equations and adding $P_0 \cos \omega t$ to the i th row of the resisting force vector $\{ F \}$, we obtain the expressions for the case in which an excitation $P = P_0 \cos \omega t$ is applied to the i th mass of the beam shown in Fig.3.

4. Numerical Examples

The use of an electronic analog computer for solving Eqs.(11) and (2)' is

Fig.4 Simply supported beam

basically possible even if $\{ F \}$ has nonlinear terms. However, analog simulation for such a multi-degree-of-freedom nonlinear system tends to be very much complicated practically. Continuous System Simulation Languages(CSSL) are suitable for the digital simulations of complicated nonlinear vibration problems of multi-degree-of-freedom systems. Here, Eqs.(11) and (2)' for the case in which the beam is excited by a sinusoidal displacement are programmed using FACOM SLCS-4, one of the CSSL, and a digital simulation is carried out. In the simulation, the beam is replaced by a seven-masses system and it is assumed that a damper is attached to the central mass. The numerical integration method used here is double precision integration by the fourth-order Runge-Kutta method.

The numerical conditions of the beam and the dampers are given in Table 1 and an example of simulation results is shown in Fig.5. For the cases of the A-type damper and the perpendicular type damper, the amplitude responses at the middle of the beam in the vicinity of the first beam resonance are shown in Figs.6 and 7

Table 1 Numerical conditions of the beam and the dampers

Beam	Length l	2500 mm	
	Width	245 mm	
	Thickness	16 mm	
	Young's modulus	204 GPa	
Ball screw	Lead L	10 mm	
	Equivalent damping coefficient c_e	2.450×10^2 N·s/m	
Pendulum type	Governor	m_1	0.196 kg
		m_2	0.098 kg
		k_1	1.137×10^2 N/m
		c_1	1.960 N·s/m
		q	70 mm
		r	20 mm
	Moment of inertia of flywheel J	ϕ_0	0.681 rad
			8.996×10^{-5} kg·m ²
Perpendicular type	Oscillator	m_1	0.392 kg
		k_1	1.108×10^2 N/m
		k_2	0.475×10^2 N/m
		c_1	1.960×10^{-3} N·s/m
	Moment of inertia of flywheel J	d	29 mm
			2.940×10^{-4} kg·m ²
Excitation	Amplitude a	10 mm

Fig.5 An example of simulation results (1.3 Hz)

respectively. The linear analytical solutions⁽⁴⁾ which are obtained by regarding the governor as a flywheel and the finite element solutions which are obtained by replacing the beam by a seven-masses system and regarding the governor as a flywheel are also shown in Figs.6 and 7. The amplitude response curve for the case of the B-type agrees well with that for the case of the A-type except that the maximum amplitude ratio at the first beam resonance increases a few percentage points. Referring to the analytical results⁽¹⁾ for the case in which the A-type damper is attached to a single-degree-of-freedom system, the natural frequency of each governor in the cases of Figs.6 and 7 is adjusted to about twice as much as the first resonance frequency of the vibration system so that the damper might be most effective.

From Figs.6 and 7, the case of the B-type and the results of further calculations for various values of the numerical conditions of the dampers, the following statements can be made:

(1) In each case of the three types of dampers, the maximum amplitude ratio of the beam at the point of attachment of the damper at the first beam resonance decreases remarkably due to the working of the governor.

(2) There is little difference in the effect of vibration isolation between the A-type damper and the B-type damper.

(3) The linear analytical solutions which are obtained by regarding the governors as flywheels extremely differ from the results of digital simulations which consider the nonlinearities of the governors in the vicinity of the first beam resonance.

(4) The finite element solutions which are obtained by replacing the beam with a seven-masses system agree well with the linear analytical solutions.

(5) Each damper approaches a snubber for larger values of J (the amplitude ratio becomes approximately 1 over a wide frequency range), and it acts as a general absorber for smaller values of J .

5. Experiment

In order to confirm whether digital simulations described in Chapters 3 and 4 are accurate or not, we built a perpendicular type damper and attached it to the middle of a simply supported leaf spring. An excitation force was applied to the middle of the leaf spring by a rectilinear reaction-type vibration machine, and the displacement of that point was measured. The experiments were performed for both cases in which the oscillators of the governor were set free and in which they were fixed. The physical properties of the experimental apparatus are given in Table 2 and the experimental results are shown in Fig.8.

From Fig.8 the effect of the governor described in Chapter 4 is confirmed. Moreover, the experimental results when the oscillators are set free agree fairly well with the results of the digital simulation, so that the validity of the

Fig.6 Amplitude of the beam at the middle (in the case of the A-type)

Fig.7 Amplitude of the beam at the middle (in the case of the perpendicular type)

Table 2 Experimental conditions

Beam	Material	SUP 9
	Length l	2500 mm
	Width	245 mm
Ball screw	Thickness	16 mm
	Lead L	10 mm
	Equivalent damping coefficient c_e	3.283×10^2 N·s/m
Flywheel	Moment of inertia J	1.744×10^{-4} kg·m ²
	Mass m_1	0.218 kg
	Spring constant k_1	1.108×10^2 N/m
Oscillator	Spring constant k_2	0
	Viscous damping c_1	2.185×10^{-3} N·s/m
	Distance d	20 mm
Vibration exciter	Unbalanced mass type	
	Unbalanced mass	14.12 kg
	Eccentricity	33.1 mm

digital simulation is substantiated.

6. Conclusions

In this report, the effects of vibration isolation of simple beams supported with two types of mechanical dampers which consist of a ball screw, a flywheel and two kinds of governors are discussed numerically and experimentally. The results may be summarized as follows:

(1) In each case of the flyball type and the perpendicular type, the maximum amplitude ratio of the beam at the point of attachment of the damper at the first beam resonance decreases remarkably due to the working of the governor.

(2) In the case of the flyball type, the equations of motion vary depending on whether the ball nut is attached to the beam or to the foundation, but there is little difference in the effects of vibration isolation between these two cases.

(3) The linear analytical solutions which are obtained by regarding the governors as flywheels extremely differ from the correct nonlinear solutions in the vicinity of the first beam resonance.

(4) The digital simulation method in which a beam is replaced by a lumped masses system and the equations of motion are programmed using CSSL is suitable for solving vibration problems of beams supported with a damper possessing nonlinear characteristics.

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Fig.8 Amplitude of the beam at the middle (in the case of the perpendicular type)

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