The Relation between Load and Strength in Bolted Joints Subjected to Fatigue Loading

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This study reports on a method of making the fatigue design of bolted joints. Two types of bolted joints are treated—one is the simplest cylindrical joint, and the other is a more practical T-flange joint. The following conclusions are obtained through stress analysis and fatigue tests of the above-mentioned bolted joints; (1) A method is proposed for expressing the load applied to the bolt in joints by plotting the dynamic stresses of the bolt on Haigh diagram, taking the maximum load and the preload as parameters; (2) A stress analysis of the T-flange bolts is performed. The analytical results show good agreement with the experimental results measured with strain gages; (3) It is verified by the fatigue test that fatigue design of the T-flange bolts can be done by comparing the above-mentioned load and the fatigue strength of the bolts.

Key Words: Fixing Element, Bolted Joint, Fatigue Strength, Haigh Diagram, Stress Analysis

1. Introduction

The demand on the limit design for bolted joints has increased with an increase in the variation or the complexity of machines or structures. The prevention of the fatigue failure in bolts is one of the most important problems in the fatigue design of bolted joints. Many studies have been performed about fatigue of the bolt-nut joints (1-3). However, the relation between the load on the bolt in the joints and the fatigue strength of the bolt-nut joints has not been fully clarified.

The authors investigated the possibility of a fatigue failure of the connecting-zod bolts by using the results of the stress measurements. In this study they propose a method on the basis of the previous study for the fatigue design by comparing the load (stress) on the bolt with the strength. Although the relationship between the external load and the stress on the bolt is usually very complicated in the practical bolted joints, such stress on the bolt should be taken as "load". In the case where repeated loads are acting, the results of the fatigue tests correspond to "strength".

2. Case of Cylindrical Joint

2.1 Bolted joint model

Figure 1 shows a bolted joint model —

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where $K_0$ is the stiffness constant of the bolt, $K_r$ and $K_H$ are the spring constants of the clamped parts subjected to compression and tension respectively [they can be calculated by the methods shown in Refs. (8) and (9)]. The behavior of the non-linear region can be approximated on the basis of the consideration in Refs. (7), as,

$$F_r = \phi \left[ (W - W_0)^{\frac{n}{2}} + \phi \cdot W + F_r \right]$$

$$\frac{1}{n} = (1 - \phi) (W_0 - W_f)^2$$

where $W_0$ is the preload. The bolted joint in Fig. 3 is shown in Fig. 3, where two cylinders (outer diam. 533 mm, height 200 mm) are clamped by a set of bolt (M10, property class 8.8) and nut and two washer plates (3 mm thick). The external load is applied on the joint through the threaded portion cut on the cylinders. The calculated values indicated by the broken lines and x marks show good agreement with the experimental results (bold lines). These results show that the additional tension on the bolt is extremely small unless the contact planes separate perfectly.

2.3 Estimation of the fatigue strength for cylindrical joint

![Diagram](image)

Fig. 3 Relationship between load and axial tension on bolt

where $W_r$ is the preload, $D_a$, $D_r$, $D_h$, $D_t$, $C_0$, and $C_1$ are the spring constants and the clamped parts subjected to compression and tension respectively [they can be calculated by the methods shown in Refs. (8) and (9)]. The behavior of the non-linear region can be approximated on the basis of the consideration in Refs. (7), as,

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3. Case of T-flange Joint

3.1 Bolted joint model

Since the bolt is not subjected to bending in the bolted joint model treated in Chapt. 2, the stress on the bolt can be estimated only by the axial tension on the bolt. However, the bolt in the practical joint is usually subjected to bending even when a pure tension load is applied. Figure 3 shows a bolted joint model - two T-flanges clamped by two sets of bolts and nut - corresponding to such a condition. The external load on the joint is tensile force $2W$ (load per bolt) as shown in Fig. 5.

The tests were done under the condition of $W_{\text{preload}} = 0.1$, but Haigh diagram hardly changes from Fig. 4 in this case.
Fig. 5 Geometry of T-flange joint

3.2 Relationship between external load and stress on bolt

3.2.1 Analysis

An analytical model is shown in Fig. 6, where two T-flanges and bolts are transformed into beams and the reactant forces from the bolts are assumed to distribute uniformly along the width \( a \) (the effects of the bearing area and the bolt hole are not considered) on the flanges. It is also assumed that the flanges A and B come in contact with each other only at a point \((l_b/2\) away from the centerline of the T-flange joint), where the reactant forces in \( z \)-direction and \( x \)-direction \( (R_x \) and \( R_z \) ) are acting. Eight unknown values - the forces and the bending moment on the bolt \( (F_x, F_z, M_x, M_z) \), the relative displacements between the bearing surfaces \( (\delta_x, \delta_z, \phi_x, \phi_z) \), the reactant force \( R_z \), and the location of the fulcrum \( l_b \) - are shown in Fig. 6. Their values can be determined by the following eight equations:

\[
F_x = F_x + \delta_x K_x
\]

\[
F_x = \frac{6F_x}{l_b^2} (3l_b - l_z, \phi_x)
\]

\[
M_x = \frac{2E_l}{l_z} (3l_b - 2l_z, \phi_x)
\]

\[
\delta_x = [\delta_x - \delta_z] x = l_z
\]

\[
\phi_x = [\phi_x - \phi_z] x = l_z
\]

\[
M_x = M_x - \frac{l_x}{2} \phi_x
\]

\[
\left( \frac{1}{2} - \frac{l_x}{2} \right) \phi_x, x = l_z
\]

\[
[\phi_x - \phi_z] x = l_z = 0
\]

\[
[\phi_x - \phi_z] x = l_z = 0
\]

\[
[\phi_x - \phi_z] x = l_z = 0
\]

values of the external load \( W \) are determined conversely from the values of \( l_z \) which are assumed to be known for simplification. The location of the fulcrum moves from the point on the axis of the bolt towards the outer edge \((x = l_x/2\) corresponding to an increase of \( W \). The relationship between the load \( W \) and the axial stress on the bolt becomes linear when the load is great enough to give \( l_x = l_z \) (eq (g) is deleted at this time).

3.2.2 Verification of analytical result

Figure 8 shows an example of the relationship between the load \( W \) and the axial stress \( \sigma_x \) on the bolt for the T-flange whose dimensions are shown in Table 1 clamped by two bolts shown as A in Fig.7. The bold lines and the broken lines show the experimental results and the analytical results respectively. Two lines for the certain preload \( F_x \), indicate the maximum stress \( [\sigma_{max}(tensile\ stress)] + \sigma_{min}(bending\ stress) \) and the minimum stress \( [\sigma_{min} + \sigma_{max}] \) at two gage locations shown in Fig.8(a) and (b) respectively. The equivalent force \( F_e \) in ordinate is the axial stress on the bolt multiplied by the sectional area at the gage location. It is clear from Fig.8 that the additional stress by the load \( W \) decreases with an increase of the preload \( F_x \), and that the transverse force \( F_z \) (see Fig. 6) is acting on the bolt since the magnitude of the bending stress \( \sigma_{max} \) is different.

Fig. 7 Geometry of bolts for stress measurement

Fig. 6 Analytical model for T-flange joint
between Fig.8 (a) and (b) — only the gage locations are different. The analytical results show good agreement with the experimental results. However, the transverse force $F_1$ does not exist in the analysis since the upper and lower flanges have the same geometry. The transverse force is thought to be generated in the experiments by the asymmetry of the bolt-nut joint in the axial direction — especially by the effect of the mating threads. The deformation of the threads should be considered for more accurate calculation.

3.3 Estimation of the fatigue strength for T-flange joint

3.3.1 Relation to the strength of bolt-nut joint

Figure 9 shows the relationship between the dynamic load on the bolt — the stress amplitude $\sigma_a$ and the mean stress $\sigma_m$ for the maximum stress $(\sigma_{max}+\sigma_{mean})$ in the T-flange joint shown in Table 1 subjected to the tension-tension loading $(W_{max}=0)$ and the preload by using eq.(5). If it could be assumed that the fatigue strength is not different between a bolt subjected to pure tension and one subjected to bending besides tension, the comparison between the load (stress) and the strength of the bolt-nut joint could be performed by Fig.9 in the same manner as in the case of Fig.4.

Otaki and Sasaki(13) have pointed out that the fatigue notch factor $B$ decreases corresponding to the increase of the ratio $\sigma_{mean}/\sigma_{nom}$ of the nominal bending stress to the nominal tensile stress at the first thread root of the bolt engaging the nut. However, this theory comes from the bending moment being considered an internal force. By converting the value of $B$ in Ref.(13) to that for $(\sigma_{nom}+\sigma_{mean})$, the ratio $B_0/B_1$ of the fatigue notch factor in pure tension to that in tension plus bending are in the range from 0.95 to 1.06 for the case where the nominal designation is $M_8$ and the value of $\sigma_{mean}/\sigma_{nom}$ is 0.25 to 1. These results show that the above-mentioned assumption about the fatigue strength is reasonable.

3.3.2 Experimental procedure for verification

The fatigue tests for the bolt-nut joint and the T-flange joint are performed to verify the relation between the load and the strength on the bolt shown in 3.2.1. Figure 10 shows the S-N curve for the bolt-nut joint under the conditions of $R_0(=\sigma_{mean}/\sigma_{max})=0.1$ and $\sigma_{max}=0.9\sigma_a$ ($\sigma_a$: standard proof stress). The plots marked $O$, $\bullet$, $\bigcirc$ and $\mathbb{M}$ show the results about the test specimens of the same lot (Lot1). The specimens of another lot (LotII) were also employed (plots of $\Delta$ and $\Delta\mathbb{M}$) since the specimens of Lot1 had a discontinuous portion in the thread root at the thread runout.

As for the fatigue test of the T-flange joint, the condition of the external load on the joint was determined to be uniform $(2W_{max}=22kN, 2W_{mean}=22kN)$ on the basis of the strength of the T-flange and of the capacity of the fatigue testing machine.
The relationship between the preload and the stresses \( \sigma_s \) and \( \sigma_m \) [stress amplitude and mean stress for the maximum stress \( \sigma_{max} + \sigma_{min} \)] on the bolt in the above-mentioned load condition is shown in Fig. 11. The plots in Fig. 11 are extrapolated values for the first thread from the results of the stress measurement described in 3.1.1 - the plots marked \( \Delta \) and \( \Delta \) are for Bolt A in Fig. 7 and \( \wedge \) and \( \Delta \) for Bolt B (the body of the bolt is converted to \( \phi B \)). The calculated values shown as the broken lines are higher than the estimated values from experiments on both \( \sigma_s \) and \( \sigma_m \) in the region where the preload \( F_T \) is relatively low. But the calculation show good agreement with the experiment in the region of higher preload. By using the experimental result for the condition of \( \sigma_{max} = 0.9 \sigma_s \) in Fig. 10 as the fatigue strength \( \sigma_f = 49 \text{ MPa} \) for Lot I and \( \sigma_f = 65 \text{ MPa} \) for Lot II, and the mean values of the experiments for Bolts A and B in Fig. 11 as the \( F_T - \sigma_s \) relationship, the preload \( F_T \) at the fatigue limit (the preload that the load on the bolt is just equal to the fatigue limit) becomes about 25 kN for Lot I and 22 kN for Lot II.

### Table 2

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Bolt</th>
<th>N</th>
<th>Failed at</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>L R</td>
<td>1.20x10^6</td>
<td>Idle thread</td>
</tr>
<tr>
<td>2</td>
<td>L R</td>
<td>6.32x10^6</td>
<td>Idle thread</td>
</tr>
<tr>
<td>3</td>
<td>L R</td>
<td>1.33x10^6</td>
<td>Idle thread</td>
</tr>
<tr>
<td>4</td>
<td>L R</td>
<td>6.77x10^6</td>
<td>First thread</td>
</tr>
<tr>
<td>5</td>
<td>L R</td>
<td>1.35x10^6</td>
<td>None</td>
</tr>
</tbody>
</table>

Remarks: \( 2M_{max} = 25 \text{ kN}, 2M_{max} = 22 \text{ kN}, T_f = 36 \text{ Nm}, (P_{mean} = 24 \text{ kN}) \)

#### 3.3.3 Comparison between experimental and predicted results

Table 2 shows the results of the fatigue tests for the T-flange joint with the bolt of Lot I. Tests are done under the condition of \( \sigma_f \) (estimated) -1.1 \( \sigma_f \) (\( \sigma_f \): fatigue limit of the bolt-nut joint) considering the estimation error on the bolt stress. From the result in Fig. 11, the aimed value of the preload \( F_T \) becomes 23.5 kN for the above-mentioned condition. However, the preload is thought to vary from 21 kN to 27 kN in the case of using the torque control method (aimed value: \( T_f = 36 \text{ Nm} \)) from the estimation using the dispersions of the torque coefficient in the tightening experiments. In these tests, the bolts in all of five sets of the T-flange joints failed - two bolts at the root of the first thread and five bolts at the root of an idle thread (two or three threads away from thread runout).

Figure 12 shows the relationship between the preload \( F_T \) and the stress amplitude \( \sigma_s \) for the maximum stress at the first thread, the idle thread (corresponding to the failed portion), and the underhead fillet. Each plot in Fig. 12 has been determined by averaging the estimated values.
Fig. 13 Results of fatigue test by staircase method (Lot II)

from the stress measurement using Bolts A and B in Fig. 7. It is clear from Fig. 12 that the stress amplitude is greater at the idle thread than at the first thread for the same level of the preload. Since the fatigue notch factors are thought to be nearly same at these locations considering the results of the fatigue test for the bolt-nut joint (Fig. 10), the failure probability is greater at the idle thread in the case of the bolt of Lot I. It should be also noted that the failure at the underhead fillet more likely occurs in the T-flange joint than in the bolt-nut joint since the stress amplitude acting is comparable to that on the first thread in the T-flange joint in spite of its greater sectional area than the threads.

Figure 13 shows the result of the fatigue test for the bolted joints with the bolt of Lot II. The test is performed by the staircase method to obtain the preload $F_w$ at the fatigue limit. The preload is controlled by using the pipe strain gage inserted into the body of each bolt. From Fig. 13, the preload $F_w$ is determined to be about 165N — fairly smaller than the estimation (228N).

Figure 14 shows Haigh diagram for the T-flange joint to compare the load on the bolt in the joint with the fatigue limit of the bolt-nut joint. The half dotted line indicates the load on the bolt estimated from the stress measurement, and the plots marked $\Delta A$ correspond to the fatigue limit of the T-flange joint. Both plots show good agreement with the fatigue limit of the bolt-nut joint (shown as bold line) in spite of the above-mentioned difference of $F_w$ between the estimation and the experiment for Lot II. Moreover the estimated value (plot of $\Delta A$) for Lot I shows good agreement also with the calculation.

From these results, it is concluded that the estimation can be performed concerning the fatigue strength of the bolt in the joint by plotting the load (stress) of the bolt on Haigh diagram for the given preload and external load. Furthermore the effects of the factors — such as the preload loss, dispersion of the preload in tightening or the variation of the external load — can be grasped intuitively by drawing up Haigh diagrams as shown in Fig. 4 or Fig. 9 for various bolted joints. For example, in Fig. 4 or Fig. 9, the above-mentioned factors on the preload become less effective when making the preload higher, i.e.

Fig. 14 Relationship between stress on bolt and fatigue limit in T-flange joint

when using the region where the gradient of the $\Delta A$-constant lines is smaller, and consequently the desirable condition for the strength design can be realized.

4. Conclusions

The main conclusions obtained in this study are as follows:

1. A method is proposed for expressing the load applied to the bolt in joints by plotting the dynamic stresses of the bolt on Haigh diagram, taking the maximum load and the preload as parameters.

2. A stress analysis of the T-flange bolts is performed. The analytical results show good agreement with the experimental results measured with strain gages.

3. It is verified by the fatigue test that fatigue design of the T-flange bolts can be done by comparing the above-mentioned load and the strength of the bolt.

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References