On the Characteristics of Bolted Joints Subjected to External Bending Moments*

(The case where clamped parts are T-flanges with gaskets or
filler plates)

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Few investigations have been carried out on the characteristics of bolted joints subjected to external bending moments. Furthermore, it seems that the characteristics of bolted joints with gaskets or filler plates subjected to external bending moments have not been studied. In this study, the characteristics of T-flanges with gaskets (filler plates) are examined. In an analysis, replacing T-flanges and a gasket (filler plate) with finite strips, the contact stress distribution, the force ratio for the external bending moment (the relationship between an increment (decrement) of bolt axial force and the external bending moment) and the maximum stress produced in bolts are analyzed by using a two-dimensional theory of elasticity. For verification, experiments are performed with respect to the force ratio for the external bending moment and the maximum stress. Analytical results are in a fairly good agreement with experimental ones.

Key Words: Fixing Element, Elasticity, Bolted Joint, Bending Moment,
Contact Stress, Force Ratio, Gasket, Filler Plate, T-flange

1 Introduction

In designing bolted joints, it is necessary to estimate the force ratio (the ratio of an increment of bolt axial force to an external load) more precisely. Up to this time, many investigations⁽¹⁾⁻⁽³⁾ have been done on the force ratio. Recently, the force ratio has come to be estimated with fairly high accuracy. But, these investigations treat only the cases where bolted joints are subjected to external loads in the axial direction. Investigations on the characteristics of bolted joints subjected to external bending moments are very few except some reported⁽⁴⁾⁻⁽⁶⁾ and they are not systematized in VDI 2230⁽⁹⁾ either. Pipe flange connections with gaskets and T-flange connections⁽¹⁰⁾ with filler plates in steel structures are subjected to external bending moments caused by an earthquake and a wind. However, it seems that a study has not been done on bolted joints with gaskets or filler plates in these cases.

In this paper, discussion is made on the distribution of contact stresses, the force ratio for the external bending moment, which is a factor representing the relation between the increment (decrement) of bolt axial force and the external bending moment, and the maximum stress

produced in bolts when the external bending moment is applied to the T-flange connections with gaskets or filler plates. Therefore, the purpose of this paper is to contribute to the optimal design of general bolted joints with gaskets subjected to external bending moments. The experiments are performed about the force ratio for the external bending moment and the maximum stress produced in bolts. The analytical results are compared with the experimental ones.

2 Theoretical Analysis

2-1 Force ratio for external bending moment (F.B.M.)

When an external bending moment M is applied around the center of two symmetrical T-flanges with a gasket or a filler plate fastened by two sets of bolts and nuts with an initial clamping force F_f as shown in Fig.1, an increment of bolt axial force F_t on the tension side and a decrement F_c on the compression side are produced. In order to obtain the value of F_t with an application of the external bending moment M , the calculation method of the force ratio for the external bending moment (we denote this force ratio by F.B.M.) which has been investigated by one of the authors is utilized. For the purpose of this study, the T-flanges and the gasket (filler plate) shown in Fig.1 are replaced with finite strips [I] and [II] shown in Fig.2. The external bending moment M is replaced with a linear load applied to a web having the width $2e$ as shown in Fig.2(a). The bending spring constant K_b is defined by the equation $K_b = (3M/4e)/(2\epsilon)$,

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using the mean displacement ε_1 at the part corresponding to the bearing surfaces 2a. The compressive spring constant K_c' is defined by the equation $K_c' = F_t / (2\varepsilon_1)$, using the mean displacement ε_1 at the bearing surfaces in the case where the compressive and the tensile forces are applied uniformly to the bearing surfaces as shown in Fig.2(b). The spring constant⁽¹¹⁾ for a bolt-nut system is defined as K_t , and the compressive one⁽¹²⁾ for clamped parts around a bolt hole is defined as K_c . By defining the spring constants as mentioned above, the value of F.B.M. ϕ_b , which is a factor to obtain the increment (decrement) of bolt axial force with an application of bending moment M , is obtained as follows⁽⁷⁾⁽⁸⁾;

$$\phi_b = \frac{F_t}{(3M/4e)} = \frac{K_t}{K_t + K_c} \left(\frac{K_c'}{K_b} \right) \dots\dots\dots (1)$$

Consequently, by analyzing K_c' and K_b in Eq. (1), the value of F_t is obtained when the bending moment M is applied. In the analysis of K_c' ⁽¹²⁾, it is necessary to determine the outer diameter of hollow cylinders. In this study, the width W of T-flange is used in similar way to the references (3) and (12) as one method to determine the outer diameter of hollow cylinders. When the external bending moment M is applied to the T-flange connection, the distribution of contact stresses

is estimated by superposing the analytical results of Fig.2(c) on those of Fig.2(a) and (b).

2-2 Analysis of bending moment produced in bolts

When an external bending moment M is applied to T-flanges, a stress due to a bending moment M_b produced in bolts is added to the stress due to the increment or the decrement of bolt axial force obtained from the analysis in Section 2-1. The value of M_b is obtained by analyzing the ratios γ_p , γ_c and γ_e of the bending moment produced in clamped parts to the inclinations of bearing surface and that of γ_b for a bolt as shown in Fig.2. Taking M_b into consideration, the maximum stress produced in bolts is expressed as follows;

$$\sigma_{\max} = \frac{F_t + F_c}{A} + \frac{M_b}{z} \dots\dots\dots (2)$$

where A is the cross sectional area and z is the modulus of cross section.

2-3 Method of analysis

By expanding the stresses acting on the boundary of the finite strips shown in Fig.2(a) and (b) into Fourier series, the boundary conditions are expressed as follows, where b_s ($s=1,2,3,\dots$) is Fourier coefficients.

Fig.1 T-flange connection subjected to external bending moment

$$\left. \begin{aligned} \text{For the finite strip [I] (T-flange)} \\ y_1 = h_1: \sigma_y^I = \sum_{s=1}^{\infty} b_s \sin \lambda_s x, \tau_{xy}^I = 0 \\ y_1 = -h_1: \tau_{xy}^I = 0 \\ x = \pm l: \sigma_x^I = \tau_{xy}^I = 0 \\ \text{For the finite strip [II] (gasket)} \\ x = \pm l: \sigma_x^{II} = \tau_{xy}^{II} = 0 \\ y_2 = \pm h_2: \tau_{xy}^{II} = 0 \\ \text{On the contact surface} \\ (\sigma_y^I)_{y_1=-h_1} = (\sigma_y^{II})_{y_2=h_2} \\ \left(\frac{\partial v^I}{\partial x} \right)_{y_1=-h_1} = \left(\frac{\partial v^{II}}{\partial x} \right)_{y_2=h_2} \end{aligned} \right\} \dots\dots\dots (3)$$

where v is the displacement in y_1 and y_2 directions, the postscripts of right shoulder I and II represent the strips [I] and [II], and the shear stresses on contact

(a) K_b

(b) K_c'

(c) State of initial clamping

Fig.2 Model for analysis

surface are neglected. In order to analyze the finite strips[I] and [II] under the boundary condition Eq.(3), Airy's stress functions χ^I and χ^{II} are taken as follows;

$$\begin{aligned} \chi^I = & \sum_{n=1}^{\infty} \frac{\bar{A}_n^I}{\alpha_n^2 \bar{D}_n} [(\cosh \alpha_n l + \alpha_n l \sinh \alpha_n l) \sinh \alpha_n x - \cosh \alpha_n l (\alpha_n x) \cosh \alpha_n x] \cos \alpha_n y \\ & + \sum_{s=1}^{\infty} \frac{\bar{B}_s^I}{\lambda_s^2 \bar{Q}_s} [(\sinh \lambda_s h_1 + \lambda_s h_1 \cosh \lambda_s h_1) \cosh \lambda_s y - \sinh \lambda_s h_1 (\lambda_s y) \sinh \lambda_s y] \sin \lambda_s x \\ & + \sum_{n=1}^{\infty} \frac{\bar{A}_n^I}{\alpha_n^2 \bar{D}_n} [(\cosh \alpha_n' l + \alpha_n' l \sinh \alpha_n' l) \sinh \alpha_n' x - \cosh \alpha_n' l (\alpha_n' x) \cosh \alpha_n' x] \sin \alpha_n' y \\ & + \sum_{s=1}^{\infty} \frac{\bar{B}_s^I}{\lambda_s^2 \bar{Q}_s} [(\cosh \lambda_s h_1 + \lambda_s h_1 \sinh \lambda_s h_1) \sinh \lambda_s y - \cosh \lambda_s h_1 (\lambda_s y) \cosh \lambda_s y] \sin \lambda_s x \dots\dots\dots (4) \end{aligned}$$

where

$$\bar{Q}_s = \sinh \lambda_s h_1 \cosh \lambda_s h_1 + \lambda_s h_1, \bar{Q}_s' = \cosh \lambda_s h_1 \sinh \lambda_s h_1 - \lambda_s h_1, \bar{D}_n = \cosh \alpha_n l \sinh \alpha_n l - \alpha_n l, \\ \bar{A}_n' = \cosh \alpha_n' l \sinh \alpha_n' l - \alpha_n' l, \alpha_n = n\pi/h_1, \alpha_n' = (2n-1)\pi/(2h_1), \lambda_s = (2s-1)\pi/(2l)$$

$$\begin{aligned} \chi^{II} = & \sum_{n=1}^{\infty} \frac{\bar{A}_n^{II}}{\bar{\alpha}_n^2 \bar{D}_n} [(\cosh \bar{\alpha}_n l + \bar{\alpha}_n l \sinh \bar{\alpha}_n l) \sinh \bar{\alpha}_n x - \cosh \bar{\alpha}_n l (\bar{\alpha}_n x) \cosh \bar{\alpha}_n x] \cos \bar{\alpha}_n y \\ & + \sum_{s=1}^{\infty} \frac{\bar{B}_s^{II}}{\bar{\lambda}_s^2 \bar{Q}_s} [(\sinh \bar{\lambda}_s h_2 + \bar{\lambda}_s h_2 \cosh \bar{\lambda}_s h_2) \cosh \bar{\lambda}_s y - \sinh \bar{\lambda}_s h_2 (\bar{\lambda}_s y) \sinh \bar{\lambda}_s y] \sin \bar{\lambda}_s x \dots\dots\dots (5) \end{aligned}$$

where

$$\bar{Q}_s = \sinh \bar{\lambda}_s h_2 \cosh \bar{\lambda}_s h_2 + \bar{\lambda}_s h_2, \bar{D}_n = \cosh \bar{\alpha}_n l \sinh \bar{\alpha}_n l - \bar{\alpha}_n l, \bar{\alpha}_n = n\pi/h_2, \\ \bar{\lambda}_s = (2s-1)\pi/(2l) \quad (n, s=1, 2, 3, \dots)$$

$\bar{A}_n, \bar{A}_n', \bar{B}_s, \bar{B}_s', \bar{A}_n^{II}$ and $\bar{B}_s^{II}$ ($n, s=1, 2, 3, \dots$) are unknown coefficients determined from the boundary condition. The stresses and the displacements are obtained by using Airy's stress functions χ^I and χ^{II} described in Eq.(4) and (5), and by equating the results to the boundary conditions, the unknown coefficients $\bar{A}_n, \bar{A}_n', \dots, \bar{A}_n^{II}$ and $\bar{B}_s^{II}$ are determined. Using the coefficients determined by the above mentioned method, the stresses and the displacements are obtained. The analysis of the case of Fig.2(c) is similar to the reference (13).

(a) T-flange

(b) gasket (filler plate)

Steel, AL		
C	2h ₂	W
40	5	30
40	10	30
40	10	48
475	10	30
Asbestos		
40	1.5	30

Fig.3 Dimensions of T-flanges and gasket used in experiments

3 Experimental Method

Figure 3 shows the T-flanges and the gasket (filler plate) used in experiments. As shown in Fig.3(a), two types of T-flanges, of which the distances $2c$ between the centers of bolts are 80 and 95 mm, are prepared. In order to examine the effect of thickness and Young's modulus on the value of F.B.M., several types of gaskets (filler plates) shown in Fig.3(b) are prepared. The size of the bolts⁽¹³⁾ is M16 (JIS) and two strain gauges are attached to the shank of bolt in 180° phase to each other. After fastening the two T-flanges with a gasket (filler plate) by two sets of bolts and nuts, a compressive load $\bar{W}$ is applied to the joint by a material testing machine using the experimental apparatus shown in Fig.4. As the result, an external bending moment is produced on the joint. Detecting the compressive load $\bar{W}$ with a load cell, the bending moment M is obtained from the relation $M = \bar{W}/2 \times 50$, and the bolt

Fig.4 Experimental setup

axial force is detected by the strain gauges attached to the shank of bolt. The output signals are recorded by an X-Y recorder.

4 Comparisons of Analytical Results with Experimental Ones

In order to compare the analytical results with the experimental ones, numerical computations are carried out using the dimensions of test specimens. The number of series is taken as 30 terms. Young's modulus E_1 of T-flange is 206 GPa, and Poisson's ratio ν_1 is 0.3. The material of gasket (filler plate) is mild steel for structural use, aluminium and asbestos (Asahi Asbestos Co., Ltd., Compressed Asbestos Sheet, No.4984, thickness 1.5mm), and their Young's modulus E_2 and Poisson's ratio ν_2 are 206 GPa, 0.3, 66.7 GPa, 0.3, 1400 MPa and 0.345, respectively. The distributions of contact stresses ($y_2=h_2$) corresponding to the cases of Fig.2(a) and (c) are shown in Fig.5. The ratio h_1/h_2

is 20.0 in the case when asbestos ($E_1/E_2=148$) is used. It seems that the distribution of contact stresses tends to be averaged with an increase of the ratio E_1/E_2 . Figure 6 shows the displacements at the upper surface of T-flange and the contact surface corresponding to the cases of Fig.2(a) and (b). Using these displacements, the spring constants K_b and K_c' are obtained. When the external bending moment is applied, the distribution of contact stresses is estimated by superposing the results of Fig.5(a) and (b) on the result of Fig.2(b). Figure 7 shows an example of comparisons of the numerical results with the experimental ones with respect to F.B.M. The abscissa is the external bending moment M , and the ordinate is the bolt axial force F_f+F_b . The initial clamping force F_f is 30 kN. With an increase of M , the bolt axial force on the tension side increases and it decreases on the compression side. The numerical result is in a fairly good agreement with the experimental one. Table 1 shows comparisons of numerical results with experimental ones with respect to F.B.M. in other cases. It

(a) case of Fig.2(a) ($e/l=0.214$)

(b) case of Fig.2(c) ($c/l=0.571$)

Fig.5 Analytical results of distribution of contact stresses ($h_1/h_2=3.0$)

(a) case of Fig.2(a) ($e/l=0.214$)

(b) case of Fig.2(b) ($c/l=0.571$)

Fig.6 Analytical results of displacement at upper surface ($y_1=h_1$) of T-flange and contact surface ($y_2=h_2$) ($h_1/h_2=3.0$)

(case of $E_1/E_2=3.0$, $2h_2=10$, $c=40$)

Fig.7 An example of comparison with experimental result with respect to the force ratio for the external bending moment

(case of $E_1/E_2=3.0$, $2h_2=10$, $c=40$)

Fig.8 Comparison of stress taking account of bending moment

is noticed that the value of F.B.M. becomes large with an increase of the ratio of Young's modulus of a T-flange to that of a gasket (filler plate) and in the thickness of gasket (filler plate). When the distance between the centers of bolt 2c and the width W of the T-flange becomes large, the value of F.B.M. becomes small.

Table 1 shows the experimental results of the F.B.M. for the bolts on the tension side and on the compression side, and it is seen that the values of F.B.M. on the compression side are pretty larger than those on the tension side. It is thought that this difference leads to the load distribution of the external bending moment M [Fig.2(a)]. From the results, it seems that the value of F.B.M. can be calculated with fairly high accuracy. Figure 8 shows comparison of the maximum stress produced in a bolt. The stress due to F_t obtained by F.B.M. is shown. It is seen that the numerical result is in a fairly good agreement with the experimental one

5 Conclusions

The purpose of the present paper is to contribute to the optimal design of bolted joints with gaskets or filler plates by analyzing the characteristics of joints subjected to an external bending moment. Discussion is made on the distribution of

Table 1 Comparisons of numerical values with experimental ones with respect to the force ratio for the external bending moment

	1	2	3	4	5	6
E_1/E_2	3.0	3.0	1.0	1.8	3.0	3.0
$2h_2$	10.0	5.0	5.0	1.5	10.0	10.0
c	40.0	40.0	40.0	40.0	47.5	40.0
W	30.0	30.0	30.0	30.0	30.0	48.0
$\Phi_b^{(Ten)}$ EXP	0.065	0.045	0.030	0.14	0.043	0.040
$\Phi_b^{(Com)}$ EXP	0.075	0.050	0.035	0.18	0.045	0.045
Φ_b NUM	0.0652	0.0481	0.0300	0.165	0.0558	0.0404

contact stresses, the force ratio for the external bending moment (relationship between an increment or a decrement of bolt axial force and an external bending moment) and the maximum stress produced in bolts. The following results are obtained.

- (1) A method of analysis of the contact stress distribution is demonstrated as a three body contact problem by using a two-dimensional theory of elasticity.
- (2) A method of analysis of the force ratio for the external bending moment and the maximum stress produced in bolts is demonstrated.
- (3) The effects of the thickness and the stiffness of gaskets (filler plates) on the distribution of contact stresses and on the force ratio for the external bending moment are made clear by numerical computations.
- (4) For verification of the results obtained from (3), experiments are performed. It is shown that the analytical results are in fairly good agreement with experimental ones.

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