Notch Tensile Strength of Ductile Materials

by Tamatsu MAJIMA**, Masato ANZAI***, and Hajime NAKAZAMA†

Notch tensile tests are performed to evaluate the effects of stress triaxiality at the net section and uniaxial tensile properties on the notch strength ratio. Notched cylindrical specimens with a U-shaped circumferential notch are employed. These specimens are made of eight kinds of materials chosen for providing a wide variety of deformation characteristics. The notch strength ratio increases with stress triaxiality factor proportional to the ratio of hydrostatic to equivalent stress, and with the ratio of local to total elongation in an engineering stress-strain curve. The notch strength ratio is determined by the multiplication effect of these two factors. Notch strengthening occurs owing to the strong multiplication effect. The value of the notch strength ratio can be obtained by an empirical equation expressed in terms of these two factors. The notch strength ratio is independent of the absolute quantity of ductilities of a material.

Key Words: Strength, Notch Tensile Strength, Notch Strength Ratio, Notch Strengthening, Stress Triaxiality Factor

1. Introduction

Many investigations have been made on the notch tensile strength of ductile materials. However, the notch strength ratio or the ratio of notch tensile strength to ultimate tensile strength cannot be predicted because of the lack of knowledge on the mechanism of notch strengthening. From previous investigations main effects of notch shape on the notch tensile strength are summarized as follows: (1) In the range of notch radius larger than a certain value determined by ductility of a material, the notch strength ratio increases with decreasing notch radius. (2) The notch strength ratio increases with increasing notch depth up to approximately one-half of the net-to-gross diameter ratio. (3) The notch strength ratio of a circumferentially notched cylindrical specimen is greater than that of a rectangular specimen notched on opposite two sides with the same notch profile as the sectional shape of the circumferential notch. These facts indicate that the notch strength ratio increases with notch sharpness, or stress triaxiality in the vicinity of the net section.\(^{(1)-(5)}\)

One of the authors revealed that a difference in shape of the engineering stress-strain curve is a cause to produce different notch strength ratios even in specimens with an identical notch\(^{(2)}\). This is explained by the following reasons: Load carrying capacity of a smooth specimen becomes greater than the maximum load giving the ultimate tensile strength if the smooth specimen deforms without necking; that is, the extension of the stable deformation increases the load carrying capacity because the same stress-strain curve as that obtained prior to necking can be applied also to a strain larger than the strain at maximum load. This rate of increase in load carrying capacity, therefore, becomes high as the ratio of local to total elongation is large, where the local elongation \(\delta_l\) is the one from point of maximum load to fracture and total elongation \(\delta_r\) or percentage elongation, is the one from initial loading to fracture. It was also found by one of the authors that strain gradient in the vicinity of the notch root makes the stable deformation in its place larger than the uniform elongation\(^{(4)}\). The uniform elongation is the one from initial loading to point of maximum load in uniaxial tension and it indicates the magnitude of the stable deformation of a material under uniaxial stress and uniform strain distribution. An extension of the stable deformation is also recognized in bending, which is accompanied with strain gradient\(^{(5)}\). Consequently, if two notched specimens have an identical notch, the notch strength ratio of a notched specimen made of a material with a large value of \(\delta_l/\delta_r\) is greater than that of another notched specimen made of a material with a small value of \(\delta_l/\delta_r\), that is, the notch strength ratio increases as the ratio \(\delta_l/\delta_r\) increases. These considerations have been verified by experiments\(^{(6)}\).

From the above discussion it is obvious that for notch strengthening a notched specimen should have a notch accompanied with high stress triaxiality in the vicinity of the net section and should be made of a material having a large value of \(\delta_l/\delta_r\). This has been also verified by experiments\(^{(4)}\). No studies have been made on the notch

\(^{*}\) Received 13th June, 1985.
\(^{**}\) Associate Professor.
\(^{†}\) Professor, Faculty of Engineering, Chiba University (1-3 Yayoicho, Chiba, Japan).
\(^{***}\) EIM Japan Co., Ltd. (3-2 Roppongi, Minato-ku, Tokyo, Japan).
strength ratio under various combinations of stress triaxiality and ratio $\delta/\sigma$. Apogino$^6$ examined experimental results carried out by many other investigators and came to the conclusion that the reduction of net-sectional area of a notched specimen is approximately equal to the ratio of reduction of area of a smooth specimen to stress triaxiality factor proposed by Davis and Connelly$^9$. This conclusion implies the applicability of this stress triaxiality factor to the problems of notch strength. However, this relationship cannot give the value of the notch strength ratio.

In the present investigation the prediction of the notch strength ratio of ductile materials only from the stress triaxiality factor and uniaxial tensile properties is discussed.

2. Experiments

Notched cylindrical specimens with a circumferential U-notch, as shown in Fig.1, were used in experiment. The net diameter $d$ and the value of the net-to-gross diameter ratio $d/D$ are 10 mm and 0.6, respectively. Notch effect is varied by employing three notch radii $\rho=1.2, 3.3$ mm.

Eight kinds of materials listed in Table 1 were chosen to provide a wide variety of deformation characteristics mainly in terms of $\delta/\sigma$. These materials are high strength steel (HT-80), austenitic stainless steel (SUS304), Ni-Cr-Mo steel (SNCM), brass (C3602), duralumin (A2024), 0.2 and 0.4% carbon steels (S20C and S45C) and annealed S45C. Mechanical properties of these materials are listed in Table 1 and engineering stress-strain curves are shown in Fig.2.

Uniaxial tensile tests for obtaining mechanical properties were performed using two or three cylindrical smooth specimens for each material. The diameter of the smooth specimen is 10 mm, which is equal to the net diameter of the notched specimen. The longitudinal length of the constant diameter of 10 mm and the gauge length are 100 mm and 50 mm, respectively. The elongation measurement is made using a linear differential transformer. The total and uniform elongations are obtained directly, while the local elongation is obtained by subtracting the uniform elongation from the total elongation. Two or three specimens are employed for one kind of notch radius. Experimental values shown in this paper are average ones.

3. Stress triaxiality factor

High stress triaxiality in the vicinity of the net section is a principal factor for notch strengthening, as discussed in Introduction. It is important, therefore, to define the stress triaxiality factor for evaluating the degree of stress triaxiality both reasonably and quantitatively. The stress triaxiality factor should be useful as a standard measure in comparing notch strength ratios of notched specimens with a different notch in shape; that is, the effect of notch shape on the notch strength ratio is evaluated through the stress triaxiality factor if the stress triaxiality factor is obtained from the stress distribution determined by the initial notch shape. In the present investigation stress triaxiality is evaluated by the following equation proposed by Davis and Connelly,$^6$}

$$\eta = \frac{\sqrt{\delta(\sigma_1 + \sigma_2 + \sigma_3)}}{\tau_{yi}} - \frac{3\sigma_y}{\sigma_{pe}}$$

The value of $\eta$ is proportional to the ratio of hydrostatic to equivalent stress. This ratio indicates that larger incremental load is necessary for the same development of plastic deformation when the value of this ratio is larger. It follows that the contribution of stress triaxiality to the increase in the notch tensile strength becomes larger as the value of this ratio is larger.

The notch tensile strength depends strongly on the initial notch shape and then the elastic stress distribution, although maximum tensile load occurs under a large degree of development of plastic deformation in the vicinity of the net section. In addition, the notch tensile strength is determined by the stress distribution at the entire net section and not at a part of the net section. For these reasons the stress triaxiality factor $\eta$ is defined here...
by the average value obtained from the distribution of above ratio in the entire net section which is calculated from the elastic stress distribution. The elastic stress distribution at the net section is obtained by various methods. Analytical solution to an infinitely deep hyperbolic notch obtained by Neuber\textsuperscript{2} gives a good approximation to the real elastic stress distribution because the notch employed is deep, as indicated by the ratio \( d/D = 0.6 \), and the difference in the stress concentration factors between a U-shaped notch and a 60-deg V-shaped notch whose shape is similar to hyperbolic is negligible. Stress triaxiality factors calculated from Neuber's solution for the U-shaped notch of \( r = 1, 2, 3 \) mm are 3.93, 2.59, 2.10 \((\psi = 0.3)\), respectively.

4. Experimental results

Figs. 3 and 4 show the variations of notch strength ratio with total elongation and reduction of area \( \Phi_t \). Symbols in the figure are explained in Table 1. No good correlation can be seen between the notch strength ratio and absolute quantities of deformation characteristics, or ductilities of a material. Notch strength ratios of austenitic stainless steel, brass, and duralumin are, for instance, almost equal in spite of a large difference in deformation characteristics. Therefore, we conclude that absolute quantities of ductilities of a material such as \( \delta_t \) and \( \Phi_t \) are not a principal factor dominating the notch strength ratio; that is, a large ductility is not necessarily an essential factor for notch strengthening.

Figs. 5 and 6 show the variations of the notch strength ratio with \( \delta_t/\delta \) and \( \Phi_t/\Phi \), where \( \Phi_t \) denotes the reduction of area occurring in the deformation from point of maximum load to fracture, namely, in the local elongation. It is obvious from Figs. 5 and 6 that the notch strength ratio increases with increasing \( \delta_t/\delta \) or \( \Phi_t/\Phi \). There is a specific relation between \( \delta_t/\delta \) and \( \Phi_t/\Phi \) when the gauge length and the shape and size of smooth specimen are taken constant. Consequently, one of these two ratios is obtained from the other. The ratio \( \delta_t/\delta \) depends on the gauge length while \( \Phi_t/\Phi \) is independent of it. The ratio \( \Phi_t/\Phi \) is, therefore, a better measure than \( \delta_t/\delta \). In a smooth specimen other than the cylindrical cross-section specimen, however, it is hard to obtain the reduction of area so that experimental data are illustrated by both ratios in this paper.

Fig. 7 shows the variations of notch strength ratio with stress triaxiality factor. The notch strength ratio increases with increasing stress triaxiality factor, namely, decreasing notch radius.

![Fig. 3: Dependence of notch strength ratio on total elongation of smooth specimen](image3.png)

![Fig. 4: Dependence of notch strength ratio on reduction of area of smooth specimen](image4.png)

![Fig. 5: Variations of notch strength ratio with \( \delta_t/\delta \)](image5.png)

![Fig. 6: Variations of notch strength ratio with \( \Phi_t/\Phi \)](image6.png)
Agogino examined many experimental results on the notch tensile strength obtained by other investigators and pointed out that there is a good relation between $\phi_N$ and $\phi/\eta$, where $\phi_N$ is the reduction of net-sectional area of notched specimens. For a mild notch Agogino calculated the stress triaxiality factor from the plastic stress analysis to the neck in a cylindrical smooth specimen obtained by Bridgman and for a sharp notch from elastic solution by Neuber. The same illustration for the present investigation is shown in Fig. 8, where the stress triaxiality factor is calculated only from the solution by Neuber. Numerical values in the figure indicate notch strength ratios of some notched specimens. Rather good correlation can be also seen between $\phi_N$ and $\phi/\eta$. Conclusions obtained by Agogino are that the equation $\phi_N = \phi/\eta$ (dashed line in Fig. 8) holds and notch weakening occurs in the range of $\phi/\eta < 10\%$. Present results show that $\phi_N > \phi/\eta$ and notch strengthening occurs even in the range of $\phi/\eta < 10\%$. The same conclusions as those indicated by Fig. 8 are obtained when the stress triaxiality factor is calculated only from the solution by Bridgman. Therefore, conclusions obtained by Agogino are considered not to be valid. The subject of Agogino’s work is to predict notch weakening, and not to obtain the value of the notch strength ratio. Fig. 8 shows that there is no relation between notch strength ratio and $\phi/\eta$. As described earlier in this chapter, the notch strength ratio is not associated with absolute quantities of ductilities such as $\delta$ and $\phi$ so that we easily recognize that the value of the notch strength ratio cannot be predicted from the relation between $\phi_N$ and $\phi/\eta$. Error will appear in measurement of $\phi_N$ of a notched cylindrical specimen made of a material with small $\delta$, and even in measurement of $\phi$ of a rectangular smooth specimen. We finally point out that reliability of conclusions obtained by Agogino is low, especially in the range of small values of $\phi/\eta$.

Figs. 5, 6, and 7 show that the notch strength ratio is able to be expressed as a power function of $\delta/\delta_r$ or $\phi_r/\phi_r$ for the constant $\eta$ and as a power function of $\eta$ for the constant $\delta/\delta_r$ or $\phi_r/\phi_r$. These figures also show that the notch strength ratio is increased by the multiplication effect of these two principal factors $\delta/\delta_r$ or $\phi_r/\phi_r$ and $\eta$. Thus we conclude that the notch strength ratio is given by an equation composed of the product of above two power functions. Figs. 9 and 10 show that this conclusion is correct and the notch strength ratio depends strongly on the multiplication effect. Following empirical equations (solid lines in the figures) can be derived

\[
\text{NSR} = 0.60(\delta/\delta_r)^{0.5}(\eta-1)^{0.35} + 1
\]

\[
\text{NSR} = 0.58(\phi_r/\phi_r)^{1.4}(\eta-1)^{0.6} + 1
\]

NSR indicates the notch strength ratio. Further investigations are necessary to clarify whether these empirical equations can or cannot be applied to notched cylindrical specimens with a different notch depth from the present one and to notched rectangular specimens. Also in the notch depth of $d/D = 0.6$ the lower limit of $\delta/\delta_r$ and $\phi_r/\phi_r$ and the upper limit of $\eta$ should be determined.
From the above discussion we conclude that for notched cylindrical specimens with a circumferential notch of the same degrees of notch sharpness and notch depth as those employed in the present investigation the notch strength ratio can be obtained by the above empirical equations; that is, the value of the notch strength ratio can be predicted only by performing a uniaxial tensile test.

5. Conclusions

An attempt to predict the notch strength ratio from uniaxial tensile properties and stress triaxiality factor is made. Notched cylindrical specimens with a circumferential U-notch of \( D/D = 0.6 \) are employed in experiment. Results show that two factors are necessary for increasing the notch strength ratio; that is, for notch strengthening. These two factors are (1) a ductile material with large value of \( \delta_s/\delta_T \) or \( \Phi_s/\Phi_T \), and (2) a notch accompanied with high stress triaxiality in the vicinity of the net section. Large value of \( \delta_s/\delta_T \) or \( \Phi_s/\Phi_T \) indicates that the material has the capability of increasing the load carrying capacity through conversion of unstable deformation into stable deformation, namely, the extension of stable deformation. In the vicinity of the notch root the magnitude of strain under the stable deformation becomes much greater than the uniform strain which is the stable deformation under uniaxial tension. This difference between two stable strains increases with increasing \( \delta_s/\delta_T \) or \( \Phi_s/\Phi_T \). The notch strength ratio, therefore, is increased as the value of \( \delta_s/\delta_T \) or \( \Phi_s/\Phi_T \) is large. Higher stress triaxiality factor requires larger incremental load for the same development of plastic deformation in the vicinity of the net section, so that the notch strength ratio will be increased as the stress triaxiality is higher. Stress triaxiality is evaluated as the average of the ratio \( 10^a/Q_4 \) in the entire net section of the initial notch shape. Average value of this ratio, or the stress triaxiality factor, is able to be evaluated with sufficient accuracy from the elastic stress distribution at the net section. The multiplication effect of these two factors increases the notch strength ratio and as a result produces notch strengthening. For the same degrees of notch depth and notch sharpness as those employed in the present investigation the notch strength ratio will be predicted to a rather high degree of accuracy by the empirical equations. These equations are composed of the product of two power functions of \( \eta \) and either \( \delta_s/\delta_T \) or \( \Phi_s/\Phi_T \). Present investigation should be developed to clarify the lower limit of \( \delta_s/\delta_T \) and \( \Phi_s/\Phi_T \) and upper limit of \( \eta \) which are variables in these equations, and the effect of other factors e.g., anisotropy, etc., on the notch strength ratio.

Acknowledgement - the authors would like to express their sincere thanks to K. Matsuoka for doing experimental work.

References