Forming Limit of Sheet Metal Considering Surface Roughness

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Experimental results concerning the forming limit of in-plane stretching of brass, copper and aluminum sheet alloys are presented. Theoretical curves of limit strains are calculated on the basis of the modified M-K (Marciniak and Kuczynski) analysis in which M-K analysis is applied to the post-heat-treatment (post-instability) condition. The imperfection is represented by the apparent initial inhomogeneity factor, \( f \), as estimated from the ratio of the minimum to maximum thickness of the sheet at the instability condition, which is equal to unity at plane-strain condition and then decreases with an increase of strain ratio. It can be concluded that the predicted limit strains depend on the work hardening law equations applied and, when best fit work hardening law equations for the materials, namely Voce, Swift, and Holomon law equations for brass, copper, and aluminum, respectively, are applied, the predicted limit strains are in good agreement with the experimental data plots. These results are explained in terms of the work hardening behavior.

Key Words: Plasticity, Forming, Sheet Metal, Modified M-K Analysis, Surface Roughness, Inhomogeneity Factor, Hardening Law

1. Introduction

There have been proposed various theoretical forming limits of in-plane stretching of sheet alloys[1]-[5]. Marciniak and Kuczyński[2] proposed the hypothesis that localized necking initiates from material imperfection (referred to hereafter as M-K analysis). It seems that the M-K analysis underestimates the limit strain in plane-strain tension region and overestimates it in equibiaxial tension region, compared with experimental data. To eliminate the above drawback and predict the proper value of instability condition at plane-strain, Tadros and Mellor proposed a modified M-K analysis[6] applying the M-K analysis to the post-instability condition. The value of inhomogeneity factor in modified M-K analysis by which reasonable results are obtained seems to be unknown and should be chosen properly[7], and it is still indefinite how to evaluate the inhomogeneity factor.

Inhomogeneity of sheet thickness may be associated with the growth of surface roughness. The experimental results[9][10] reveal that the growth of surface roughness is represented as a function of effective strain. Although the effects of the growth of surface roughness are taken into account in M-K analysis[11][12], quantitative discussion on the relation between inhomogeneity factor in modified M-K analysis and the growth of surface roughness has not been made yet.

Agreement between theoretical and experimental limit strains in plane-strain condition can be achieved by considering the work hardening behavior precisely[13]. Furthermore, if the above treatment is expanded to the forming limit calculation of in-plane stretching, disagreement between experimental and theoretical forming limits reported for brass, i.e., the experimental fracture comes before fulfillment of the instability condition[6], can be solved.

In the present paper, the forming limit is calculated by using the modified M-K analysis, in which the growth of surface roughness is taken into account and it is verified experimentally using three kinds of sheet metals. A method of calculating the initial inhomogeneity factor reversely from the ratio of the minimum to maximum thickness of the sheet at the instability condition is proposed. The influence of the variation of surface roughness and work hardening behavior upon the forming limit of in-plane stretching is carefully examined.

2. Analytical procedure

2.1 M-K analysis

In the M-K analysis[2], it is postulated that an eventual failure-region during in-plane stretching grows by means of local weakness induced by the initial surface roughness. The M-K analysis supposes an incipient groove already normal to the direction of the major strain, \( \varepsilon_1 \), as shown schematically in Fig.1. The intermediate strain, \( \varepsilon_2 \), along the groove axis is required to be

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the same both in- and outside the groove. Therefore, thinning (ε₁) and extension (ε₂) are necessarily more rapid in the groove than outside of it. The compatibility equation is
\[ \text{de}_1\tau = \text{de}_0 \varepsilon \]  \hspace{1cm} (1)
and the equilibrium equation under in-plane stretching (ε₂>0) is
\[ t\varepsilon_2 = \lambda \sigma \varepsilon_0 \]  \hspace{1cm} (2)
where \( t \) is the thickness of the sheet. Subscripts F and G imply that the quantities are the values at outside of and in the groove, respectively. While \( \text{de}_0 \) grows steadily as the rate becomes larger than \( \varepsilon \), when \( \varepsilon \) increases, the value of strain increment ratio in the groove, \( u = \text{de}_0/\text{de}_1 \), approaches zero from that value outside of the groove, \( u = \text{de}_0/\text{de}_1 \). Thus plane-strain condition is eventually produced in the groove, and the \( \varepsilon_2 \) at outside is the limiting strain. Using the quadratic Hill yield function[16] for plane-stress and the relevant flow rule in the M-K analysis, a set of the differential equations to obtain the forming limit strain for a specific constant strain ratio outside of the groove are formulated, which are composed of Eqs.(1),(2) and the equations written below, Eqs.(3)-(7).

\[ \text{de}_0 = \frac{1 + 2r}{1 + r} \frac{(1 + u)^2}{1 + r + 2ru + (1 + r)a^2} \text{de}_1 \]  \hspace{1cm} (3)

\[ \text{de}_1 = \frac{u}{a} \frac{1 + 2r}{1 + r} \frac{(1 + u)^2}{1 + r + 2ru + (1 + r)a^2} \text{de}_0 \]  \hspace{1cm} (4)

\[ \text{de}_4 = \frac{u}{a} \frac{1 + 2r}{1 + r + 2ru + (1 + r)a^2} \text{de}_0 \]  \hspace{1cm} (5)

\[ \text{de}_5 = \frac{1}{4} \frac{1 + 2r}{1 + r} \frac{(1 + r + ra)^2}{1 + r + 2ru + (1 + r)a^2} q(\varepsilon_0) \]  \hspace{1cm} (6)

\[ \text{de}_6 = \frac{1}{4} \frac{1 + 2r}{1 + r + 2ru + (1 + r)a^2} q(\varepsilon_0) \]  \hspace{1cm} (7)

where \( \varepsilon \) is effective strain, \( \text{de} \) increment.

![Fig.1 A schematic view of M-K analysis.](image)

Fig.2 Comparison of fit inhomogeneity factor by Tadros and Mellor[6] with apparent inhomogeneity factor calculated.
the instability condition, the inhomogeneity factor, $f$, is also defined as the ratio of the initial sheet thicknesses in and outside of the groove, $t_{0c}/t_{0g}$. The initial sheet thickness in the groove, $t_{0c}$, is an apparent one because it must satisfy Eq.(13) at the instability condition. Referring to the relations, $t_{0c}=1/\exp(\varepsilon_{gc})$, $t_{0g}=1/\exp(\varepsilon_{gr})$, and Eq.(2), the inhomogeneity factor, that is an apparent one, is given by

$$f = \frac{t_{0c}}{t_{0g}} = \frac{\sigma_{fr}}{\sigma_{gc}} \exp(\varepsilon_{fr} - \varepsilon_{gc}) \ldots \ldots \ldots \ldots \ldots \ldots (15)$$

Here, the value of $\varepsilon_{gc}$ expressed by Eq.(3) is assumed to be equal to the value by Eq.(14) at the first incremental step after post-diffuse necking. This assumption gives the initial value of strain increment ratio in the groove, $u$. Accordingly, $\varepsilon_{gc}$ and $\sigma_{fr}$ in Eq.(15) are rewritten, by substituting $\varepsilon = \varepsilon_{fr} + \varepsilon_{gc}$ into Eq.(6) and $r = r_{fr} + r_{gc}$ into Eq.(7), respectively, as $\varepsilon_{fr} = \varepsilon_{fr} + \varepsilon_{gc}$ and $\sigma_{fr} = \sigma_{fr} + \sigma_{gc}$, where subscript d designates the diffuse necking condition (i.e., instability condition). After the substitution, Eq.(15) can be expressed in terms of $a$, $u$, $\varepsilon_{fr}$, $\varepsilon_{gc}$, $\sigma_{fr}$, $\sigma_{gc}$, and other material constants.

2.3 Evaluation of inhomogeneity factor
The growth of surface roughness for rimming steel and soft aluminum reported by Tadros and Mellor[6] can be rearranged into the form of Eq.(12) by means of the least square method as follows.

$$R = 1.74 + 14.0 \cdot \varepsilon$$

for rimming steel and

$$R = 5.41 + 4.0 \cdot \varepsilon$$

for soft aluminum. Substituting this relation and other material constants into Eq.(15), the apparent initial inhomogeneity factor is obtained as a function of strain ratio and shown in Fig.2. The open and solid circles, given by Tadros and Mellor, are the $f$-values which give the limit strains in good agreement with the experimental results. The values calculated by the two different methods almost agree with each other. Then, the method proposed here seems to be an appropriate one and the values of forming limits of rimming steel and soft aluminum calculated by using $f$-values shown in Fig.2 are in good agreement with experiment. However, the calculated forming limit of brass disagrees with the experiment because of the overestimation of the instability strain and the discrepancy is explained later as an improper application of the work hardening law equations.

A comparison of $f$-value in cases of $a=20$ and $40\mu m$ and $n=0.2$ and 0.4 is made in Fig.3 where $R_S=0\mu m$ is assumed. The calculated $f$-value is equal to unity at plane-strain tension and then decreases with an increase of $a$-value. Smaller apparent inhomogeneity factor is obtained corresponding to the increase of $a$, $n$. The stress given by work hardening law equation, $\sigma = \varepsilon(\varepsilon)$, and the term $\exp(\varepsilon_{fr} - \varepsilon_{gc})$ change the apparent inhomogeneity factor little because of the relations $\Delta \varepsilon_{fr} \ll \Delta \varepsilon_{gc}$, $\Delta \varepsilon_{gc} \ll \Delta \varepsilon_{fr}$, $|\Delta u| < 1$, and $|\Delta \sigma| < 1$. Then, it is considered that the relative decrease of the strain increment ratio in the groove, $u$, from that outside of the groove, $a$, affects mainly the inhomogeneity factor. This $u$-value, which can be calculated from Eqs.(1), (3), and (14), decreases with an increase of $R_S$, $a$, or $\varepsilon_{fr}$, and the inhomogeneity factor also decreases.

The forming limit curves calculated by the modified M-K analysis with $f$-values shown in Fig.3 are depicted in Fig.4. The forming limit curves calculated by using the Hollomon law equation are not proportional to $n$-value, while the instability curves are proportional to $n$-value. The forming limits in equi-biaxial tension region at smaller $n$- or $u$-values are rela-

![Fig.3 Effect of $n$- and $u$-values on apparent inhomogeneity factor.](image)

![Fig.4 Effect of $n$- and $a$-values on forming limit curves.](image)
tively high compared to the values at larger n- or α-values. Additional results showing that the relative profile of forming limits seems to be affected by the n-value even though f-values are approximately the same are obtained from the comparison of forming limit for the case of curve 2: n=0.2 and d=40μm, with that of curve 3: n=0.4 and d=20μm.

The decrease of initial sheet thickness seems to match the increase of n-value. Therefore, small f-value and lower forming limit near the equi-biaxial tension region are expected from a thin sheet when a change in the mechanical properties due to initial thickness is neglected. This expectation does not contradict the observation[18].

3. Experimental procedure

The materials tested in the present study are commercial 70/30 brass, OFHC copper, and 1100-0 aluminum sheets of 1mm in thickness whose mechanical properties are summarized in Table 1. The best-fit work hardening law equations for brass, copper, and aluminum are Voce, Swift, and Hollomon law equations, respectively. The effect of work hardening law equation introduced in the analysis for the forming limit is studied by using three equations mentioned before, but, the discussion using Swift law equation for aluminum is omitted because Swift and Hollomon law equations agree precisely with each other.

The method of in-plane stretching test in the present study is known as Marcinik biaxial stretching test using a double action hydraulic press machine. The dimensions of specimens used have been presented in more detail elsewhere[7]. Plane-strain tests are carried out by using the notched tensile specimens[13].

Surface roughness represented by maximum roughness height, R, are investigated for various strain ratios including uniaxial and plane-strain tension and are measured along the major strain, c1, direction by the roughness tester.

4. Results and Discussion

4.1 Surface Roughness

Figure 5 shows the variation of the surface roughness, R, with regard to the effective strain, ε, which is based on Hill quadratic yield function. The plots are experimental surface roughness readings and the straight lines represent least square fit lines using Eq.(12). Surface roughness increases in proportion to effective strain and no strain ratio dependence is observed.

The effective strain is affected by the yield function used to describe the in-plane stretching behavior of a thin
sheet. The n-value also varies essentially depending on the choice of yield function, but the strain ratio dependence on the relation between surface roughness and effective strain is not observed when using other yield functions, e.g., Hill non-quadratic one[19] or Gotoh biquadratic one[20].

4.2 Inhomogeneity factor
The apparent inhomogeneity factors as a function of strain ratio which are based on three work hardening law equations are compared mutually in Fig.6 for the materials tested. The material parameters used are shown in Table 1 and the apparent inhomogeneity factors are smaller in the order of the Voce, Hollomon, Swift law equations for all materials. Figure 7 illustrates the relation between logarithmic work hardening rates and effective strain obtained directly from the Hollomon, Swift, and Voce law equations. Additional solid lines which are plots of the variation of the logarithmic work hardening rate with regard to limit effective strain for the materials assumed as isotropic, i.e., r=1, are also shown in Fig.7 for some strain ratios. The true r-values of the materials tested are nearly equal to unity as shown in Table 1, so that the intersecting points of the solid
line with other curves give rough values of the instability strain, $\epsilon_i$. These instability strains are greater in the order of Voce, Hollomon, Swift law equations in the $\varepsilon_p0$ region; then the apparent inhomogeneity factors are predicted in the reverse order.

4.3 Forging limit

Figure 8 shows comparison of the calculated forming limit curves based on three work hardening law equations with the experimental data plots represented by solid circles. The forming limit curves of a small $a$-value sheet material are relatively elongated from instability strain in equi-biaxial tension region. The larger instability strain gives the smaller apparent inhomogeneity factor, then the difference of predicted forming limit curve due to work hardening law equation is inconclusive because of difficulties of separating the effects of instability strain from those of $f$-value. As shown in Fig.8, it should be noted that the calculated forming limit curves by applying the best fit work hardening law equations which are the Voce, Swift, and Hollomon law equations corresponding to brass, copper, and aluminum, respectively [13], are in good agreement with the experimental data plots.

Previously, Tadros and Mellor[6] have shown that the theoretical forming limit curve for 70/30 brass calculated by using the modified M-K analysis and applying Hollomon law equation is unreasonable because the experimental limit curve was lower than the instability curve[6]. Such discrepancy between the theory and experiment can be explained in terms of an improper work hardening law equation. It has been shown statistically that the best fit work hardening law equation for brass can be represented by Voce law equation. The instability strain calculated by Voce law equation is remarkably lower than that by Hollomon law equation as shown in Fig.7, and Hollomon law equation leads to overprediction of the forming limit curve but Voce law equation gives the forming limit curves in good agreement with the experimental data as shown in Fig.8(a). It is worthwhile to note that the work hardening law equation of the sheet plays an important role in the forming limit prediction as well as in the work hardening behavior description, as evidenced in Fig.8.

5. Conclusions

(1) The apparent inhomogeneity factor for a modified M-K analysis, which is proposed here, depends on work hardening behavior, initial sheet thickness, initial surface roughness of the sheet, and incremental factor of surface roughness. The apparent inhomogeneity factor is equal to unity at plane-strain and then decreases with an increase of strain ratio.

(2) The importance of the usage of best fit work hardening law equation, which takes into consideration the difference among work hardening behaviors, comes from the facts that the instability condition is closely connected with the work hardening behavior and it affects the apparent inhomogeneity factor. Lack of this consideration leads to an unreasonable conclusion that a fracture comes before the instability condition for brass is fulfilled.

(3) The modified M-K analysis in conjunction with the best fit work hardening law equation, i.e., Voce, Swift, and Hollomon law equations for brass, copper, and aluminum, respectively, gives good forming limit predictions for those sheets.

Acknowledgments

Authors would like to thank many graduates of Kagoshima University for performing the experiment to obtain the results presented here.

References