Precise Measurement of Thermophysical Properties of Solids

by a Direct Electrical Heating Method *

(1st Report, Measurement and Theoretical Evaluation

of Accuracy for a Circular Rod Sample)

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An improvement of the principle for measuring thermophysical properties by the direct heating method has been done. In this report, an improved method for measuring thermal conductivity, k , and total hemispherical emittance, ϵ , is presented. Theoretical evaluations of accuracy in these measurements using a circular rod sample are also conducted. From the results of experiments on silver (99.99% pure), it becomes clear that this is an advantageous method for measuring precise thermophysical properties, such as k and ϵ , of an electrically conductive solid. The results of analysis have been verified by those of experiments. By using the theoretical evaluation of accuracy, the optimum sample size can be determined.

Key Words: Thermophysical Properties, Thermal Conductivity, Total Hemispherical Emittance, Direct Electrical Heating, Circular Rod Sample

1. Introduction

With the recent developments of material sciences, the working temperature-range of heat engines and thermal appliances becomes increasingly wide. Then, the thermophysical properties of the elements (solids) are required to be determined more precisely and extensively than those obtained previously to design the appliances. However, the reliable data on these properties are not so many. Moreover, considering for all concerned with this matter, there are some problems in the methods for both evaluation of the data such as thermal conductivity and measurement of these properties (1).

In general, a steady-state method has some disadvantages in comparison with a transient method; it takes very long time to measure these properties, and it is not easy to keep steady-state at high temperature level during the measurement.

A direct electrical heating method which has originally been developed for the steady-state measurement of thermal conductivity at temperatures higher than 500 K, is expected to be one of the most reliable methods without such disadvantages (2). This method has the following remarkable features: 1) easier heating of sample to a uniform high temperature; 2) simple measuring apparatus to satisfy the basic equation; 3) multi-purpose measure-

ment for obtaining many properties simultaneously, such as electrical resistivity, emissivity, thermal expansion, thermal conductivity, Thomson coefficient, and thermal diffusivity; and 4) high precision due to exact assessment of the radiative heat loss from a sample and the thermal expansion, which cause systematic errors in measuring these properties.

There are several ways to apply this method both in a sample configuration and in a measuring principle (3). Some of them, however, have a serious limitation to applicability due to the conditions of both linearization and simplification of the basic equation.

Recently, Taylor (4) conducted a measurement of thermal conductivity on tungsten at a high temperature region by using thin rod samples. The features of his techniques are to exclude the process of linearization, and to put the measured values of temperature-distribution directly into the basic equation; nevertheless the accuracy of obtaining thermal conductivity by his method tends to deteriorate on account of the errors included in the numerical derivatives of $T(z)$, especially in the second derivative (5). Moreover, the accuracy of measuring total hemispherical emittance can not be satisfactory enough unless the conductive heat loss in the axial direction is exactly evaluated.

In conclusion, the direct heating method needs to have these disadvantages eliminated so as to be a profitable one for measuring the above mentioned properties. Hence, the goal of this study is to improve the direct heating method, thereby to develop a multi-purpose measuring system which is precise and adaptable in a wide temperature-range. In this report, improved methods for measuring thermal conductivity and total hemispherical emit-

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tance are presented. Theoretical evaluation of errors in these measurements is also conducted.

Silver (99.99% pure) is selected as the sample material to confirm capability of the improved method in the temperature-range of 300 to 450 K, by comparing with literature values.

Nomenclature

- A : sectional area of a rod sample
 A_1, A_2, A_3 : surface areas of each constituent of the model in Fig. 9.
 a_{ij} : coefficient of the linear equation (6)
 b_i : constant term of the linear equation (6)
 E_k : relative error in thermal conductivity $= (\bar{k} - k)/k$
 E_R : relative error in estimation for radiative heat loss $= (\bar{q}_R - q_R)/q_R$
 E_ϵ : relative error in total hemispherical emittance $= (\bar{\epsilon} - \epsilon)/\epsilon$
 E_ρ : relative error in electrical resistivity $= (\bar{\rho} - \rho)/\rho$
 F_{21}, F_{22} : configuration factors
 I : current in a sample
 I_m : current in the sample heated in a uniform temperature-distribution
 I_0, I_1 : modified Bessel functions of the first kind of orders 0 and 1
 k : thermal conductivity
 $L = l/2$
 $L^* = L/R$
 l : length of a rod sample
 $m = 2\xi^{1/2}$
 n : number of the intervals
 P : perimeter of a rod sample
 $p = 1 - (T_s/T_m)^4$
 Q_j : Joule's heat on the interval
 Q_R : radiative heat loss on the interval
 q_R : radiative heat flow
 $\mathcal{R}(z, T_i)$: residual of Eq.(1)
 R : radius of a sample
 R_s : radius of an inner surface of enclosure
 r : coordinate of the radial direction
 T : temperature
 T_m : temperature of the sample heated in a uniform temperature-distribution, or temperature at $z=L$
 T_1, T_2 : surface temperatures of each constituent of the model in Fig.9
 T_0 : temperature at $z=0$
 T_s : surface temperature of the enclosure
 z : coordinate of the axial direction
 α_{ij} : coefficient of the polynomial expression (2)
 δ : factor of reflective effect of an enclosure, defined by Eq.(31)
 ϵ : total hemispherical emittance
 ϵ_s : total hemispherical emittance of the inner surface of enclosure
 $\theta_0 = (T_0 - T_m)/T_m$
 μ : Thomson coefficient
 $\xi = 2R\epsilon\sigma T_m^3/k$
 ρ : electrical resistivity
 σ : Stefan-Boltzmann constant
 Superscript * refers to dimensionless parameters, and the mark - such as in $\bar{k}$ implies the value has an error.

2. Principle of the Measuring Method

Governing equation for a steady-state heat flow through a homogeneous circular rod heated electrically in vacuum is given by

$$\frac{d}{dz} \left(k \frac{dT}{dz} \right) - \mu \frac{I}{A} \frac{dT}{dz} + \rho \frac{I^2}{A^2} - \frac{P}{A} \epsilon \sigma (T^4 - T_s^4) = 0 \quad (1)$$

Each term in the left-hand side of Eq. (1) expresses heat conduction, Thomson's heat, Joule's heat, or radiative heat. Then, it is assumed that both the temperature and the current density are uniform in a cross section of the sample, and that the surface of enclosure around the heated sample is an ideal black body at the temperature T_s .

Formulation of the equation for obtaining thermal conductivity and Thomson coefficient is given as follows:

Temperature-distribution measured in a section $[l_0, l_n]$ along the z -axis of the sample is divided into $n (\geq 3)$ small intervals as shown in Fig. 1.

Fig.1 Coordinate system and measuring interval of temperature-distribution.

On an interval $[l_{i-1}, l_i]$, the temperature-distribution is expressed as a function of distance z by finding a least-squares polynomial; that is

$$T_i = \sum_{j=0}^M \alpha_{ij} \cdot z^j \quad (2)$$

where M is more than two and chosen such as to minimize the mean squares of each difference in the values between the polynomial and the measured points.

From the requirement that the polynomial satisfies Eq.(1) in the mean on each interval, the residual of Eq.(1), $\mathcal{R}(z, T_i)$, should be such that

$$\int_{l_{i-1}}^{l_i} \mathcal{R}(z, T_i) dz = 0 \quad (3)$$

Taking account of the temperature dependence of thermal conductivity, we can represent the following equation on all intervals:

$$k = k_0 + k_1 \cdot T \quad (4)$$

Hence Eq.(5) is derived from Eqs.(3) and (4).

$$\begin{aligned} & k_0 \left[\frac{dT}{dz} \right]_{l_{i-1}}^{l_i} + k_1 \left[T \frac{dT}{dz} \right]_{l_{i-1}}^{l_i} - \mu \frac{I}{A} [T]_{l_{i-1}}^{l_i} \\ &= - \frac{I^2}{A^2} \int_{l_{i-1}}^{l_i} \rho dz + \frac{P\sigma}{A} \int_{l_{i-1}}^{l_i} \epsilon (T^4 - T_s^4) dz \end{aligned} \quad (5)$$

where both ρ and ϵ are expressed as a function of temperature by measuring them beforehand.

On all the intervals, Eq.(5) gives a linearly independent set of three unknown parameters, k_0 , k_1 , and μ . When $n > 3$, normal equations are also derived by the method of least-squares; that is

$$\begin{bmatrix} a_{ij} \end{bmatrix} \begin{Bmatrix} k_0 \\ k_1 \\ \mu \end{Bmatrix} = \begin{Bmatrix} b_i \end{Bmatrix} \dots\dots\dots (6)$$

By finding the solution to the equations (6), the thermal conductivity at temperature T can be obtained by Eq.(4).

The methods for measuring the electrical resistivity ρ and the total hemispherical emittance ϵ are as follows:

When the temperature-distribution in the sample becomes uniform in the axial direction owing to both resistive self heating and auxiliary heating of the chucks, it is right to assume that Joule's heat generated in the sample balances with the radiative heat loss at the surface. In this case, by measuring voltage gradient $\Delta V/\Delta l$ at T_m in the axial direction, ρ is obtained from

$$\rho = \frac{A}{I_m} \cdot \frac{\Delta V}{\Delta l} \dots\dots\dots (7)$$

Solving Eq.(1) for ϵ under the condition that $dT/dz=0$ for all z , we have

$$\epsilon = \frac{\rho I_m^2}{A P \sigma (T_m^4 - T_s^4)} \dots\dots\dots (8)$$

3. Experimental Apparatus

Apparatus of this electrical direct heating technique is shown schematically in Fig. 2.

Fig.2 Schematic diagram of experimental apparatus.

At the center of the vacuum enclosure ①, a rod sample ② with 3 mm in diameter and 200 mm in length is vertically clamped at each end by water-cooled chucks ③ which are equipped with auxiliary heaters in order to control the temperature. The bottom chuck can move in the vertical direction to release thermal expansion of the sample from the main frame. The sample is heated electrically by a regulated stable DC current, whose value can be determined by the use of a shunt ⑮.

Temperature-distribution is measured by a thermocouple (T-type in a wire diameter of 0.04 mm) in the probe which is a ceramic tube sheathed in a stainless steel tube ④. A top part of the sheath is heated separately by an electrical DC current to prevent the heat loss through the thermocouple wires. In order to obtain the axial temperature-distribution of the sample, the probe is traversed in both vertical and horizontal directions. That is performed by a servomotor ⑧, under remote control. The position of the probe which moves vertically can be known by detecting the revolution angle of screw shaft ⑥. A precision-potentiometer ⑤ is mounted at the upper end of shaft. Accuracy of determining the probe-position was found to be within 20 μ m per 100 mm by inspection with a microscope.

Two thermocouples ⑯ attached to each end of the sample, are used to determine the voltage drop ΔV and thermal expansion (to which this report does not refer).

The dimensions of vacuum enclosure are 400 mm in inner diameter and 550 mm in height. Its wall is a water-cooled double structure, with a black refractory-coating surface. The temperature of the wall, T_s , is obtained as the mean value of the outputs from the two thermocouples attached to the wall.

All measurements were performed under a vacuum of 5×10^{-4} Pa, using a computer controlled multi-meter ⑨.

4. Experimental Results

The thermal conductivity and the total hemispherical emittance of silver (99.99% pure) were measured in order to demonstrate the accuracy of this method by comparing with literature values. These measurements were also conducted to elucidate the validity of theoretical error-estimation.

Results of the measurements on the electrical resistivity and the total hemispherical emittance are presented in Fig.3 and Fig.4, respectively. The deviation of the electrical resistivity from the values by Mikryukov⁽⁶⁾ is less than 0.8 per cent. The values of total hemispherical emittance are considerably higher than those of Ramanathan and Yen⁽⁷⁾ using a transient calorimetric technique. The higher values may not be attributed to the difference in surface condition such as roughness or oxidation but rather to that in measuring condition.

A typical temperature-distribution measured along z-axis is shown in Fig. 5.

Fig.3 Electrical resistivity.

Fig.4 Total hemispherical emittance.

Fig.5 Measured temperature-distribution along z-axis of sample.

A solid curve in this figure is plotted by connecting with the least-squares polynomials on each interval, thereby the deviations of measured points from the curve being less than ± 0.05 K.

Results of thermal conductivity measurement are presented in Fig. 6. Obtained values are lower by 5 per cent than the recommended values of SRM(NBS) (6) shown by the broken curve.

Fig.6 Thermal conductivity.

5. Theoretical Evaluation of Accuracy

Main errors in this measurement are probably caused by the measuring condition that does not satisfy the requirements for the measuring principle. Those are,

- 1) error in obtaining k , due to neglect of the temperature-distribution in the radial direction,
- 2) error in the estimation of radiative heat loss, due to unsatisfactory condition of the surrounding walls,
- 3) errors in measuring both ρ and ϵ , arising from the fact that the condition required for the measuring principle is not fully satisfied, and
- 4) error in obtaining k , due to errors in measuring ρ and ϵ .

Theoretical evaluations of these errors have not been given, because it is not easy to solve the nonlinear equation for temperature and to consider exactly the radiative heat flow between sample and surrounding walls.

Hence, this study takes these causes of errors into consideration by means of some simplifications in order to obtain equations for evaluating these errors.

We assume that:

- a) temperature dependence of the thermophysical properties and Thomson's heat have little influence on the heat balance described by Eq.(1), therefore these can be neglected; and
- b) the term of radiative heat flow in Eq.(1) can be approximated by the following linearized expression:

$$\text{if } \theta \ll 1,$$

$$(1+\theta)^4 \approx 1+4\theta \dots \dots \dots (9)$$

Under the condition stated above, we solve Eq.(1). Then indicating some dimensionless quantities marked with *, we write

$$z^* = z/R, \quad r^* = r/R, \quad \theta = (T - T_m)/T_m, \quad \text{and}$$

$$L^* = L/R \dots \dots \dots (10)$$

Introducing these dimensionless quantities and the above mentioned condition into Eq.(1) yields

$$\frac{d^2\theta}{dz^{*2}} - 4\xi\theta + \zeta = 0 \dots \dots \dots (11)$$

where, $\xi = 2R\epsilon\sigma T_m^3/k$, and

$$\zeta = \rho(I^2 - I_m^2)/(\pi^2 R^2 k T_m) \dots \dots \dots (12)$$

The boundary conditions are,

for a symmetrical distribution about

$$z^* = 0, \quad d\theta/dz^* = 0 \dots \dots \dots (13)$$

for a constant temperature at

$$z^* = L^*, \quad \theta = 0 \dots \dots \dots (14)$$

Therefore, the temperature-distribution is obtained as follows:

$$\theta = \frac{\zeta}{4\xi} \left\{ 1 - \frac{\cosh(mz^*)}{\cosh(mL^*)} \right\} \dots \dots \dots (15)$$

where $m = 2\xi^{1/2}$.

On the other hand, the governing equation and the boundary conditions, considering the temperature-distribution in a cross section of the sample, are

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{\rho I^2}{kA^2} = 0 \dots \dots \dots (16)$$

$$\text{at } r=0, \quad \partial T/\partial r=0 \quad \dots\dots\dots(17)$$

$$\text{at } r=R, \quad -k\partial T/\partial r=\varepsilon\sigma(T^4-T_s^4) \quad \dots\dots\dots(18)$$

$$\text{at } z=0, \quad \partial T/\partial z=0 \quad \dots\dots\dots(19)$$

$$\text{at } z=L, \quad T=T_m \quad \dots\dots\dots(20)$$

Introducing the assumptions of both a) and b), Eq.(16) has the following solution:

$$\theta = \frac{\beta}{2}(L^{*2}-z^{*2}) - \frac{\xi}{L} \sum_{n=0}^{\infty} \frac{(-1)^n (4\beta/\lambda_n^2 + p) I_0(\lambda_n r^*)}{\lambda_n [2\xi I_0(\lambda_n) + \lambda_n I_1(\lambda_n)]} \cos(\lambda_n z^*) \quad \dots\dots\dots(21)$$

$$\text{where, } \beta = \rho l^2 / (\pi^2 R^2 k T_m), \quad p = 1 - (T_s/T_m)^4,$$

$$\text{and } \lambda_n = (2n+1)\pi / (2L^*) \quad \dots\dots\dots(22)$$

The temperature - distributions obtained theoretically are shown in Fig. 7. These are compared with the numerical solution of Eq.(1) which is calculated without the assumptions of both a) and b) by using Runge-Kutta's method. The difference of temperature is less than 0.2 K. The difference is mainly due to the assumption b). Hence, the smaller the radiative heat loss is, the more insignificant this difference is.

In this case, the temperature difference in the cross section at $z=0$, calculated from Eq.(21), does not exceed the maximum of 0.01 K. Consequently, the radial distribution of temperatures has little influence on the axial one, so that the curve by Eq.(21) almost agrees with that by Eq.(15).

Fig.7 Calculated temperature-distribution along z -axis of sample.

5.1 Error in obtaining k due to neglect of the temperature-distribution in the radial direction

The error in obtaining k , due to neglect of the temperature-distribution in a cross section of the sample, is inevitable for this measuring principle. In this paragraph, we estimate an extent of the error.

Thermal conductivity calculated from Eq.(11) is an apparent value with some errors, and so expressed by $\bar{k}$. A temperature-distribution in the axial direction is measured practically on the surface of the sample. Therefore, regarding the distribution calculated from Eq.(21) at $r^*=1$ as a measured one, we can derive the error

theoretically. From the condition that θ of Eq.(21) needs to satisfy Eq.(11) in the mean on the interval $[0, L^*]$, $\bar{k}$ can be represented as a function of k used in Eq.(21). Defining the following expression of relative error in the thermal conductivity:

$$E_k = (\bar{k} - k)/k \quad \dots\dots\dots(23)$$

we get:

$$E_k = \xi \{ 4(-L^{*2}/3 + S_3/L^{*2} + 4\xi S_3/L^{*2})(\theta_0 + \xi p S_2/L^*) - p(1 - S_1/L^{*2} - 4\xi S_3/L^{*2})(L^{*2}/2 - 4\xi S_4/L^*) / \{ (1 - 4\xi S_3/L^{*2})(\theta_0 + \xi p S_2/L^*) - \xi p S_1/L^{*2}(L^{*2}/2 - 4\xi S_4/L^*) \} \} \quad \dots\dots\dots(24)$$

where,

$$S_1 = \sum_{n=0}^{\infty} B_n, \quad S_2 = \sum_{n=0}^{\infty} B_n/\lambda_n, \quad S_3 = \sum_{n=0}^{\infty} B_n/\lambda_n^2, \quad S_4 = \sum_{n=0}^{\infty} B_n/\lambda_n^3, \quad S_5 = \sum_{n=0}^{\infty} B_n/\lambda_n^4 \quad \dots\dots\dots(25)$$

$$B_n = (-1)^n I_0(\lambda_n) / \{ 2\xi I_0(\lambda_n) + \lambda_n I_1(\lambda_n) \} \quad \dots\dots\dots(26)$$

Fig.8 Relation between E_k and ξ .

In Fig.8 the plot of E_k is given as a function of ξ under the conditions of $\theta_0=0.04$, and $p=0.75$. This figure shows that E_k is always negative, and that E_k is estimated at -0.0038 by giving the parameters of $L^*=33.3$ and $\xi=2.76 \times 10^{-6}$ in this experiment.

5.2 Error in the estimation of radiative heat loss

A radiative heat loss, described as the fourth term in Eq.(1), may be estimated inaccurately owing to an error in measuring the emittance of the sample and to the difference in the surrounding condition between practice and theory. Then it is not easy to evaluate the influence of the latter exactly, because of the complicated configurations of surrounding bodies and of their nonblack surfaces.

We make the evaluation on the following assumptions:

The surrounding is a cylindrical enclosure as shown in Fig. 9; both the sample and the enclosure have gray surfaces, each of which has uniform temperature. Then, the assumption of uniform temperature is acceptable by the fact that the variation in temperature of a sample is not more than 10 K and the wall of enclosure is kept at a constant temperature during the measurement. In this model, the net radiative heat flow from the sample 1 is given as

$$q_R = \varepsilon_1 \sigma T_1^4 - \varepsilon_1 (G_2 F_{21} + 2 G_3 F_{13}) \quad \dots\dots\dots(27)$$

where G_2 and G_3 are radiosities at the surfaces 2 and 3, respectively. Configuration-factor, for example F_{21} , denotes the factor from surface 2 to 1.

On the other hand, considering no surface reflection, the heat flow is represented as follows:

$$\bar{q}_R = \varepsilon_1 \sigma (T_1^4 - T_2^4) \quad (28)$$

Then, the error in estimating by Eq.(28) is defined as

$$E_R = (\bar{q}_R - q_R) / q_R \quad (29)$$

Using the net radiation method for this problem, we get

$$E_R = \delta \cdot \varepsilon_1 \quad (30)$$

where δ is a factor associated with reflective effect of enclosure on the radiative heat flow, and expressed in terms of configuration factors, surface areas, and the emittance of enclosure: that is

$$\delta = \frac{C_1 - (C_1 + C_2)\varepsilon_s + C_2\varepsilon_s^2}{(1 + C_3)\varepsilon_s - C_3\varepsilon_s^2} \quad (31)$$

where,

$$\begin{aligned} C_1 &= A_2/A_1 \cdot F_{21}^2 + A_1/(2A_3) \\ &\quad \times (1 - A_2/A_1 \cdot F_{21})^2 + C_2 \\ C_2 &= A_2/A_3 \cdot F_{21} - A_1/(2A_3) \cdot F_{22} - A_2/A_1 \\ &\quad \times \{A_2/(2A_3) + 1\} \cdot F_{21}^2 - A_2/(2A_3) \cdot F_{21}^2 \\ C_3 &= A_2/(2A_3) - \{1 + A_2/(2A_3)\} \\ &\quad \times F_{22} - A_2/A_1 \cdot \{A_2/(2A_3) + 1\} \cdot F_{21}^2 \end{aligned} \quad (32)$$

The configuration factors such as F_{21} and F_{22} can be given as functions of l/R and R_s/R ⁽⁸⁾. As a result, the estimation error included in the fourth term of Eq.(1) can be found by Eq.(30). The factor δ is shown in Fig.9, and plotted versus the rate of wall recession, R_s/R . It is evident that δ is required to be as small as possible in the case of large emissivity of a sample.

In this measurement, given the parameters of $\varepsilon_s = 0.8$, $\varepsilon_1 = 0.02$, $R_s/R = 130$ and $l/R = 80$, we get $E_R = 2 \times 10^{-4}$. Thus, it is obvious that this error is small enough to be neglected.

Fig.9 Relation between δ and R_s/R .

5.3 Measuring accuracies of ρ and ε

The measurements of ρ and ε are performed under the condition that a tempera-

ture-distribution is uniform in the z -direction of a sample. Therefore, unless this condition is fully satisfied, the errors in measuring these properties can not be deduced.

Equation (7) does not explicitly contain the value of temperature. Therefore, when the temperature-distribution is not necessarily uniform through the measuring section Δl , we regard the obtained ρ as the value with an error in indicating temperature of the sample.

In this case, ε has a large error caused by heat conduction in the z -direction. So we must rewrite Eq.(8) to calculate the correct ε as follows:

$$\varepsilon = \frac{kA d^2 T / dz^2 + \rho l^2 / A}{P \sigma (T^4 - T_s^4)} \quad (33)$$

When the value of ε calculated from Eq.(8) has an error, it is represented as $\bar{\varepsilon}$. We define the relative error E_ε as

$$E_\varepsilon = (\bar{\varepsilon} - \varepsilon) / \varepsilon \quad (34)$$

By substituting the value of $[d^2 T / dz^2]_{z=0}$ by Eq.(15) into Eq.(33), we get the following result:

$$E_\varepsilon = \frac{4\theta_0}{(\rho + 4\theta_0)(\cosh mL^* - 1)} \quad (35)$$

The error E_ε is plotted against θ_0 which is a parameter of temperature-nonuniformity, and shown in Fig.10. It is evident from this figure that the smaller the value of m is, the smaller the absolute value of θ_0 needs to be for determining the emittance with high accuracy.

In this experiment, the value of mL^* is about 0.2 and $\theta_0 \leq 1 \times 10^{-3}$, then E_ε is evaluated by Eq.(35) to be less than 0.5. As a result, it is found that the higher values of measured ε as compared with those by Ramanathan and Yen⁽⁷⁾ are a consequence of insufficient uniformity of temperature in the z -direction, and that a value of L^* over 80 is required for determining ε of silver more accurately.

Fig.10 Relation between E_ε and θ_0 .

5.4 Error in obtaining k due to errors in ρ and ε

In order to derive a simple equation for estimating the influence of errors in ρ and ε on the accuracy of obtaining k , it is assumed that temperature dependence of thermophysical properties and Thomson's heat can be neglected. Then solving Eq.(5) for k , we can get the following equation;

$$k = \left\{ \frac{P \sigma \varepsilon}{A} \int_{l_{i-1}}^{l_i} (T^4 - T_s^4) dz - \rho \frac{l^2}{A^2} (l_i - l_{i-1}) \right\} / \left[\frac{dT}{dz} \right]_{l_{i-1}} \quad (36)$$

The value of k calculated by using ρ with an error is,

$$\bar{k} = \left\{ \frac{P\sigma\epsilon}{A} \int_{l_{i-1}}^{l_i} (T^4 - T_s^4) dz - \bar{\rho} \frac{I^2}{A^2} (l_i - l_{i-1}) \right\} / \left[\frac{dT}{dz} \right]_{l_{i-1}}^{l_i} \quad (37)$$

Therefore, the error of k obtained by Eq.(37) is reduced to:

$$E_k = E_\rho \cdot \frac{Q_J}{Q_J - Q_R} \quad (38)$$

where E_ρ is defined as

$$E_\rho = (\bar{\rho} - \rho) / \rho \quad (39)$$

Q_J is Joule's heat on the interval $[l_{i-1}, l_i]$, and given by

$$Q_J = \rho I^2 / A^2 \cdot (l_i - l_{i-1}) \quad (40)$$

Q_R is the radiative heat loss on that interval, and given by

$$Q_R = \frac{P\epsilon\sigma}{A} \int_{l_{i-1}}^{l_i} (T^4 - T_s^4) dz \quad (41)$$

A procedure similar to the above leads to the result:

$$E_k = -E_\epsilon \cdot \frac{Q_R}{Q_J - Q_R} \quad (42)$$

On account of the fact that $Q_J > Q_R$, it is obvious that the effect of error in ρ on the accuracy of obtaining k is more sensible than that in ϵ . If Q_R is nearly equal to Q_J , we have

$$Q_J / (Q_J - Q_R) \gg 1 \quad (43)$$

Consequently, this case causes very large errors in obtaining k . It should be emphasized that the measurement of k has to be made under the condition that $Q_J \gg Q_R$ on all intervals.

An extent of error in measuring ϵ , caused by the error in ρ , corresponds to that in ρ as long as ϵ is calculated by using Eq.(8). Hence, it is a remarkable advantage for obtaining k to counteract the effects of both E_ρ and E_ϵ because the signs of Eqs. (38) and (42) are opposite to each other.

The relations between E_k and E_ρ or E_ϵ are shown as the solid lines in Fig.11. Errors in k calculated by using the measured temperature-distribution and by substituting either ρ or ϵ with some artificial errors, are plotted with the marks $\odot$ and $\triangle$, in Fig.11. It is known from the relation in Fig.11 that E_k is no more than -0.02 , even in the case such as $E_\epsilon \leq 0.5$.

6. Summary and Conclusions

An improvement of the principle for measuring thermophysical properties of a circular rod sample by the direct electrical heating method has been done. In this report, the improved method for measuring thermal conductivity and total hemispherical emittance is presented. Theoretical evaluations of errors in these measurements are also conducted. Moreover, the method has been verified to be accurate enough by the results of experiments on silver (99.99% pure).

The following conclusions are obtained:

Fig.11 Relations between E_k and E_ρ or E_ϵ .

- (1) This method allows the measurement of k to be performed with high precision.
- (2) Theoretical error in obtaining k , due to neglect of the radial temperature-distribution in a cross section of the sample is evaluated by Eq.(24).
- (3) Error in estimating a radiative heat loss, due to practical surrounding condition, is roughly deduced from Eq.(30).
- (4) The extent of error in measuring ϵ , due to the nonuniformity of axial temperature-distribution, is estimated by the use of Eq.(35).

It is one of the advantages of this method that it is easy to ascertain that the required condition for measuring ϵ is satisfied.

- (5) Effects of the errors in ρ and ϵ on the accuracy of obtaining k are evaluated by Eqs.(38) and (42), respectively.
- (6) Comprehensive evaluation of the above mentioned errors can be useful to determine the optimum sample size.

References

- (1) Sugawara, A. and Minges, M. L., J. of Jpn. Soc. Mech. Engrs. (in Japanese) Vol.84, No.751 (1981), p.523.
- (2) Fitzer, E., AGARD, Advisory Rep., No. 12 (1967), p.8.
- (3) Powell, R. W., et al., AFML-TR-67, No. 241 (1968)
- (4) Taylor, R. E., Trans. ASME, J. of Heat Transfer, Vol.100, No.5 (1978), p.330.
- (5) Sugawara, A. and Takahashi, I., J. of Jpn. Appl. Phys. (in Japanese), Vol.52, No.6 (1983), p.503.
- (6) Powell, P. W., et al., NSRDS-NBS, No.8 (1966), p.12.
- (7) Ramanathan, K. G. and Yen, S. H., J. of Opt. Soc. Am., Vol.67, No.1 (1977), p.32.
- (8) Siegel, R. and Howell, J. R., Thermal Radiation Heat Transfer, (1972), p.789 McGraw-Hill.