Theory of Vibration Isolation of a System with
Two Degrees of Freedom*
(2nd Report, The Case Where a Harmonic Force Acts on
the Mass Located Far from the Supporting Foundation)

by Toshihiko ASAMI**, Hisayoshi SEKIGUCHI***
and Ryuji NISHIUCHI†

This paper presents expressions and computational graphs to determine the optimum tuning and damping parameters for a linear system with two degrees of freedom. The mass m_1 is connected to an immovable foundation through a spring k_1 and a dashpot c_1 . An impressed harmonic force acts on the mass m_2 . Either of the masses (named main mass) should be isolated from the vibration originating from the impressed force; whereas, the other mass functions as a dynamic absorber. The optimization criterion is minimizing the maximum displacement of the main mass. The optimum design parameters can be formulated when $c_1=0$. If $c_1 \neq 0$, optimization is carried out using numerical analysis, such as the Newton-Raphson method. The optimum conditions obtained theoretically are summarized in graphs, and tested on a vibratory model. The experiment makes it clear that the theory is very useful for the isolation of actual mechanical vibrations.

Key Words: Theory of Vibration, Vibration Isolation, Dynamic Absorber, Optimum Tuning, Optimum Damping, Damped Primary System, Numerical Analysis

1. Introduction

As possible means to reduce the mechanical vibrations, a dynamic absorber was proposed originally by Frahm in 1909⁽¹⁾. The dynamic absorber has wide applications where the vibratory motion is objectionable. The theory of dynamic absorber has long been studied by Ormondroyd⁽²⁾, Den Hartog⁽³⁾, Hahnkamm⁽⁴⁾ and Brock⁽⁵⁾ et al. using the mathematical model shown in Fig.1(b). In this figure, they have considered that m_1 was the main mass to be prevented from vibration and m_2 was the absorber mass. Their work was limited to a particular case of $c_1=0$, in other words, the primary system had no damping. Recently, the theory was extended to the damped primary system⁽⁶⁾⁻⁽¹⁰⁾.

The theory of dynamic absorber is one of the theories of suppression or isolation of vibration in linear systems with two degrees of freedom. The systems are grouped into three classes (a), (b) and (c) according to the location of the exciting part, as shown in Fig.1. The theory of dynamic absorber gives the optimum condition for suppression of vibration of mass m_1 in class

(b) and also gives the optimum isolation between the mass m_1 and the foundation in classes (b) and (c). However, this theory is not applicable to the case that the main mass is not m_1 but m_2 .

At least, three kinds of theories are needed for good suppression or isolation of vibration in all the classes shown in Fig.1. In the 1st report of this work⁽¹¹⁾, we have proposed the second theory, which is applicable to isolate the mass m_2 from the foundation in classes (b) and (c). Following the 1st report, this paper takes up the problem of how to prevent each mass (m_1 or m_2) in class (a) from vibrating, where the mass m_2 (not m_1) is subject to a harmonic exciting force. We propose the third theory in this report.

Two kinds of theories of the suppression of vibration are needed for the class (a), and these will be discussed in turn. One of the two is the theory proposed in the 1st report. It is used to determine the optimum suppression of vibration of the mass m_1 . The other is a new theory of suppression of vibration. It is used to determine the optimum condition for preventing the mass m_2 from vibrating. The theories are first discussed in the condition of $c_1=0$, and then expanded to the case of $c_1 \neq 0$. The optimization criterion is minimizing the maximum displacement of the main mass, which is the same criterion as in the earlier work. If $c_1=0$, optimization is carried out using numerical analysis, such as the Newton-Raphson method. The optimum design parameters obtained theoretically are represented in graphs.

* Received 27th May, 1985.

** Research Associate, Faculty of Engineering, Himeji Institute of Technology, 2167, Shosha, Himeji, Hyogo, 671-22 Japan.

*** Professor, Faculty of Engineering, Himeji Institute of Technology.

† Graduate Student, Graduate School; Himeji Institute of Technology.

A concrete example of the vibratory system (a) is found in a car and a seismic mounting of machinery. If the system (a) is a model of a car, then the masses m_1 and m_2 correspond to the body of the car and the engine, respectively. Further, the system (a) can also be considered a mechanical model of the seismic mounting of a machine⁽¹²⁾ (Fig.2), in which C_1 and k_1 are an equivalent damping coefficient and a spring constant of the ground. The theory of vibration isolation must be realized in those actual mechanical vibrations. However, the systems shown in Fig.1 are approximate models of the actual mechanical systems, so the optimum condition of isolation obtained theoretically is merely approximate no matter how accurate it is. Therefore, in this report we made a model of the seismic mounting and examined the theory of suppression of vibration obtained here. The previous research⁽¹³⁾ on the oil damper enabled us to verify the theory experimentally.

2. Fundamental Equation and Optimization Criterion

The equations of motion of the system (a) shown in Fig.1 can be written as

$$\left. \begin{aligned} m_1 \ddot{x}_1 + C_1 \dot{x}_1 + k_1 x_1 \\ + C_2 (\dot{x}_1 - \dot{x}_2) + k_2 (x_1 - x_2) = 0 \\ m_2 \ddot{x}_2 + C_2 (\dot{x}_2 - \dot{x}_1) + k_2 (x_2 - x_1) \\ = P_0 \sin \omega t \end{aligned} \right\} \dots\dots\dots (1)$$

Applying the complex notation to the displacements x_1 , x_2 and the exciting force, and deriving the steady state solutions, we have

$$|x_1/x_{st}| = \sqrt{(A_1^2 + B_1^2)/(C^2 + D^2)} \dots\dots\dots (2)$$

$$|x_2/x_{st}| = \sqrt{(A_2^2 + B_2^2)/(C^2 + D^2)} \dots\dots\dots (3)$$

Fig.1 Three classes of the two-degree-of-freedom system with harmonic excitation

Fig.2 A model of seismic mounting

$$\left. \begin{aligned} A_1 &= \beta^2, & A_2 &= \beta^2 + (1 - \alpha^2)/\mu \\ B_1 &= 2\alpha\beta\zeta_2, & B_2 &= 2\alpha(\beta\zeta_2 + \zeta_1/\mu) \\ C &= \alpha^4 - \alpha^2[(1 + \mu)\beta^2 + 1 + 4\beta\zeta_1\zeta_2] + \beta^2 \\ D &= -2\alpha(\alpha^2 - \beta^2)\zeta_1 + 2\alpha\beta[1 - (1 + \mu)\alpha^2]\zeta_2 \end{aligned} \right\} \dots\dots (4)$$

where the dimensionless parameters are defined as follows:

$$\left. \begin{aligned} x_{st} &= P_0/k_1, & \alpha &= \omega/\omega_1, & \beta &= \omega_2/\omega_1 \\ \mu &= m_2/m_1, & \omega_1 &= \sqrt{k_1/m_1}, & \omega_2 &= \sqrt{k_2/m_2} \\ \zeta_1 &= C_1/(2m_1\omega_1), & \zeta_2 &= C_2/(2m_2\omega_2) \end{aligned} \right\} \dots\dots (5)$$

Equations (2) and (3) are plotted in Fig.3(a) and (b), respectively. These figures show that the displacement response has one or two resonant peaks according to the magnitude of damping ζ_2 . We define the optimum suppression condition of vibration as the one that minimizes the maximum displacement response of the main mass in the frequency domain. If m_2 is the main mass, two parameters β , ζ_2 have optimum values. We designate the optimum values of β and ζ_2 as β_{opt} and ζ_{2opt} , respectively. If m_1 is the main mass, then only the parameter ζ_2 has an optimum value.

(a) Resonance curves of m_1 for various values of damping ζ_2

(b) Resonance curves of m_2 for various values of damping ζ_2

Fig.3 Amplitudes of the masses m_1 and m_2 versus frequency ratio α ; system shown in Fig.1(a)

It should be noted that the resonance curve of the mass m_2 shown in Fig.3(b) does not take a value of unity at $\alpha=0$ (vertical axis). In Fig.3(b), we divide the amplitude $|x_2|$ of m_2 by the static deflection $x_{st}=P_0/k_2$ of the other mass m_1 . However, even if $|x_2|$ is nondimensionalized by the static deflection P_0/k_2 of the main mass m_2 , the curves of $|x_2|/(P_0/k_2)$ and the vertical axis do not intersect at unity. The optimum suppression condition of vibration does not change whichever static deflection we choose. Thus, we shall divide both amplitudes $|x_1|$ and $|x_2|$ by the static deflection of the mass m_1 , because the comparison between the resonance curves of m_1 and m_2 becomes simple.

3. Optimum Suppression Condition of Vibration of m_2

In this chapter, m_2 is supposed to be the main mass and m_1 is the absorber mass.

3.1 Case of $\zeta_1 = 0$

If $\zeta_1 = 0$, then the resonance curves of mass m_2 pass through four fixed points O, P, S and Q independent of the amount of damping ζ_2 , as shown in Fig.3(b). When $\zeta_2 = 0$, there are two resonant peaks on the curves. However, the right-hand peak vanishes rapidly with an increasing ζ_2 , so we pay attention to only the left-hand peak. The left-hand peak moves from left to right and passes through the point P with an increasing ζ_2 . ζ_2 becomes the most favorable value where the minimum resonant peak takes place in the point P.

Next, we shall explain the method of determining the optimum tuning condition of β , using Fig.4(a). This figure illustrates how the height of the fixed points O, P, Q changes with the frequency ratio β for various values of μ . The height of another fixed point S is independent of β and equal

to $1/\mu$, which is the same height of point Q in $\beta = 0$. It may be observed in Fig.4(a) that point P is always higher than points Q and S and becomes higher with an increasing β ; and that the height of point O decreases from infinity to unity as β increases. Therefore, for all values of the mass ratio μ the curves of P always intersect the curves of O and the two points of P and O are equal in height at the point of intersection. In Fig.4(a) the intersections are designated as open circles and connected by a broken line. The optimum tuning condition of β is given by the abscissa of the intersection.

Thus we have concluded that the method of determining the optimum condition is first tuning the frequency ratio β so as to equalize the heights of points O and P, and then setting the damping ζ_2 in order that the resonance curve may have a horizontal tangent at the point P. The optimum tuning condition β_{opt} and damping condition ζ_{2opt} are obtained from Eq.(3), and represented as follows:

$$\beta_{opt} = \sqrt{g_1/\mu} \quad \dots\dots\dots (6)$$

$$\zeta_{2opt} = \sqrt{\frac{g_2[(1+\mu)g_2-2\mu]}{4\mu(1+\mu)^2g_1(g_1+1)}} \quad \dots\dots\dots (7)$$

$$\left. \begin{aligned} g_1 &= \cos(\gamma/3) - 1/2 \\ g_2 &= (1+\mu)(g_1+1)^2 - 1 \\ \gamma &= \cos^{-1}[2\mu^2/(1+\mu)^2 - 1] \end{aligned} \right\} \quad \dots\dots\dots (8)$$

In the optimally tuned and damped resonance curve, the peak value $|x_2/x_{st}|_{max}$ of the resonance curve and the corresponding frequency α_p are given by the equations

$$|x_2/x_{st}|_{max} = 1 + 1/(\mu\beta_{opt}^2) \quad \dots\dots\dots (9)$$

$$\alpha_p = 1/\sqrt{(1+\mu)(1+\mu\beta_{opt}^2)} \quad \dots\dots\dots (10)$$

which are equal to the ordinate and abscissa of the point P, respectively.

(a) Curves of the heights of points O, P and Q versus natural frequency ratio β , and locus of their intersections

(b) Curves of the optimum tuning β_{opt} and damping ζ_{2opt} and the position of resonant peak point P versus mass ratio μ

Fig.4 Optimum suppression condition of vibration of mass m_2 when $\zeta_1 = 0$

Fig.5 Optimum suppression condition of vibration of mass m_1 when $\zeta_1 = 0$; the upper limit of tuning condition β and the optimum damping ζ_{2opt}

Figure 4(b) is a graphical representation of Eqs. (6), (7), (9) and (10). It is apparent from this figure that the resonant peak becomes lower as μ increases. The resonant peak is less than 4 at $\mu = 2$, and takes a minimum value of 3 at $\mu = \infty$. Therefore a fairly large value of mass ratio μ (greater than about 2) has little effect on the resonant amplitude. It is interesting to know that the amplitude of an optimally tuned and damped primary system is less than the static deflection of the system.

3.2 Case of $\zeta_1 \neq 0$

If $\zeta_1 \neq 0$, i.e., damping is introduced in the secondary system, then the points P, Q and S no longer exist. Therefore, we use a numerical method combining the Newton-Raphson method and the bisection method.

We express the amplitude $|x_2/x_{st}|$ in $F(\alpha, \zeta_2)$; this symbol means that $|x_2/x_{st}|$ is a function of α and ζ_2 . First, we search for a particular value of ζ_2 which minimizes the left-hand peak. This can be found by solving the simultaneous equations

$$\left. \begin{aligned} F_\alpha &\equiv \partial F / \partial \alpha = 0 \\ F_{\zeta_2} &\equiv \partial F / \partial \zeta_2 = 0 \end{aligned} \right\} \dots\dots\dots (11)$$

If $\zeta_1 = 0$, the coordinates (α, F) where Eq. (11) is satisfied correspond precisely to the point P. Therefore, we name the coordinates the point P, though there is no fixed point (except 0) in the case of $\zeta_1 \neq 0$. However, Eq. (11) cannot be solved analytically, so we shall use the Newton-Raphson method for solving the two simultaneous equations; its iterative formulas are as follows:

$$\begin{pmatrix} \alpha^{(\nu+1)} \\ \zeta_2^{(\nu+1)} \end{pmatrix} = \begin{pmatrix} \alpha^{(\nu)} \\ \zeta_2^{(\nu)} \end{pmatrix} - J^{-1}(\alpha^{(\nu)}, \zeta_2^{(\nu)}) \begin{pmatrix} F_\alpha(\alpha^{(\nu)}, \zeta_2^{(\nu)}) \\ F_{\zeta_2}(\alpha^{(\nu)}, \zeta_2^{(\nu)}) \end{pmatrix} \dots\dots\dots (12)$$

where

$$J(\alpha, \zeta_2) = \begin{pmatrix} F_{\alpha\alpha} & F_{\alpha\zeta_2} \\ F_{\zeta_2\alpha} & F_{\zeta_2\zeta_2} \end{pmatrix} \dots\dots\dots (13)$$

The ordinate $|x_2/x_{st}|_{\max}$ of point P and the particular value of ζ_2 obtained from the Newton-Raphson method are drawn with solid lines and dashed lines, respectively, for various values of ζ_1 , in Fig. 6. The ordinate of P becomes higher with an increasing β as in the case of $\zeta_1 = 0$. It is the optimum tuning condition when the height of P is just equal to that of the point O shown as a dash-dotted line. Hence the optimum tuning condition β_{opt} is obtained as the abscissas of the intersections (designated as open circles) of the solid line and the dash-dotted line. Further, the optimum damping ζ_{2opt} is obtained as the ordinates of the intersections (designated as closed circles) of the dashed lines and the vertical straight lines passing each open circle. In the numerical calculations, this procedure is carried out by the method of successive division such as the bisection method.

Figures 7, 8 and 9 show the optimum tuning condition β_{opt} , optimum damping ζ_{2opt} and minimized resonance peak $|x_2/x_{st}|_{\max}$, respectively, in the range of $\zeta_1 = 0 \sim 0.3$, $\mu = 0.1 \sim 10$. If $\zeta_1 = 0$, the optimum values β_{opt} , ζ_{2opt} obtained from the numerical method

Fig. 6 Graphical representation of a method of searching for the optimum values β_{opt} and ζ_{2opt}

Fig. 7 Optimum tuning condition β_{opt} to prevent the mass m_2 from vibration

Fig. 8 Optimum damping ratio ζ_{2opt} to prevent the mass m_2 from vibration

Fig. 9 Maximum resonant amplitude of the mass m_2 in the optimally tuned and damped system

agree precisely with the calculated values from Eqs.(6) and (7). It will be seen from Figs.7 and 8 that the optimum values β_{opt} , ζ_{2opt} become large as ζ_1 increases. In particular, when the mass ratio μ is small, the change becomes more rapid. However β_{opt} cannot exceed unity, because ζ_{2opt} becomes infinity before β_{opt} reaches unity. On the other hand, ζ_{2opt} is usually greater than unity.

3.3 Favorable condition of design parameters for suppression of vibration of mass m_2

(1) Condition of ζ_1 (see Figs.8 and 9)

It is desirable to increase the damping ζ_1 . The effect of damping ζ_1 is remarkable when μ is small. However, the optimum damping ratio ζ_{2opt} increases rapidly as ζ_1 increases. Thus, if a larger value of ζ_1 were used, it would be difficult to realize the optimum damping ζ_{2opt} .

(2) Condition of μ (see Fig.9)

When ζ_1 is less than about 0.25, greater values of μ are favorable.

(3) The optimum conditions of the parameters β and ζ_2 can be obtained using Figs.7 and 8 for given values of ζ_1 and μ . The smaller the mass ratio μ becomes, the more susceptible the optimums β_{opt} and ζ_{2opt} are to the damping ζ_1 .

4. Optimum Suppression Condition of Vibration of m_1

In this chapter, m_1 is supposed to be the main mass and m_2 is the absorber mass.

There is good resemblance between the suppression conditions of vibration of mass m_1 in class (a) and mass m_2 in class (c) shown in Fig.1. These two conditions agree with each other at $\zeta_1 = 0$, but disagree at $\zeta_1 \neq 0$. Thus the optimum condition of the mass m_1 in class (a) is obtained from the theory proposed in the 1st report of this work. In this chapter, after explaining the theory briefly, we discuss the suppression condition of vibration of the mass m_1 .

4.1 Case of $\zeta_1 = 0$

By inspection of Fig.3(a), the optimum suppression condition of vibration is achieved when ① right-hand peak occurs at point S ($1/\mu$ in height) and ② left-hand

peak is lower than S. First condition ① gives the optimum damping ratio

$$\zeta_{2opt} = (1/2)\sqrt{1/(1+\mu) - \beta^2} \dots\dots\dots (14)$$

Second condition ② gives the upper limit of the natural frequency ratio β . The upper limit of β is obtained by the numerical method. After determining β , the optimum damping ζ_{2opt} is given by Eq.(14). The β and ζ_{2opt} obtained are plotted in Fig.5. For example, when $\mu = 0.3$, the upper limit of β and the optimum damping ζ_{2opt} become 0.600 and 0.320, respectively. Under this condition, both the left- and right-hand peaks of resonance curve are $1/\mu$ in height, as shown in Fig.3(a).

4.2 Case of $\zeta_1 \neq 0$

If $\zeta_1 \neq 0$, the smaller the value of β we set, the more reduced will be the maximum amplitude of the mass m_1 . Therefore β has no optimum value, so we search for the optimum damping ζ_{2opt} in the region of $\beta = 0 \sim 1$. ζ_{2opt} is restricted to either of two particular values of ζ_2 ; one symbolized by ζ_L (or ζ_R), which minimizes the left-hand (or right-hand) peak and the other symbolized by ζ_T , which equalizes the left- and right-hand peaks. Putting $|x_1/x_{st}| = F(\alpha, \zeta_2)$, we searched for ζ_L and ζ_R by the Newton-Raphson method; the iterative formulas are the same as Eq. (12). Further, ζ_T was obtained by the Newton-Raphson method for one variable (α) and the bisection method. An example of the search is shown in Fig.10. In this example, the optimum damping ζ_{2opt} changes from ζ_L to ζ_T at $\beta = 0.488$.

Similarly, for various values of ζ_1 and μ we searched for the optimum damping ζ_{2opt} and the minimized resonant amplitude $|x_1/x_{st}|_{max}$; the results are summarized in Figs.11~14. In each figure, (a) and (b) show plots of ζ_{2opt} and $|x_1/x_{st}|_{max}$ against the natural frequency ratio β , respectively.

4.3 Favorable condition of design parameters for suppression of vibration of mass m_1

(1) Condition of ζ_1 (see Figs.11 and 12)

Greater values of the damping ζ_1 is advantageous for suppression of vibration. However, as ζ_1 increases, the optimum damping ζ_{2opt} increases rapidly, so it will be difficult to realize the optimum suppression condition of vibration.

(2) Condition of μ (see Figs.13 and 14)

There is an optimum value of the mass ratio μ ; the value varies complicatedly depending on the values of β and ζ_1 . However, if we set $\mu (= m_2/m_1)$ large, then $\beta (= \sqrt{m_1 k_2 / (m_2 k_1)})$ becomes small inevitably. In this sense, to make μ large is desirable.

(3) Condition of β (see Figs.11~14)

It is desirable to make the natural frequency ratio β small. However, as the ratio β approaches zero, the optimum damping ζ_{2opt} increases more rapidly, and thus it will be difficult to realize the optimum suppression condition.

5. Model Experiment

In order to verify the theory of suppression of vibration discussed in former chapters, we produced experimental equipment

Fig.10 Graphical representation of a method of searching for the optimum damping ratio ζ_{2opt}

(a) Optimum damping ratio $\zeta_{2\text{opt}}$ (b) Maximum resonant amplitude of the mass m_2 in the optimally damped systemFig.11 Design curves in the condition of $\mu=0.1$ (a) Optimum damping ratio $\zeta_{2\text{opt}}$ (b) Maximum resonant amplitude of the mass m_2 in the optimally damped systemFig.12 Design curves in the condition of $\mu=0.5$ (a) Optimum damping ratio $\zeta_{2\text{opt}}$ (b) Maximum resonant amplitude of the mass m_2 in the optimally damped systemFig.13 Design curves in the condition of $\zeta_1=0.05$ (a) Optimum damping ratio $\zeta_{2\text{opt}}$ (b) Maximum resonant amplitude of the mass m_2 in the optimally damped systemFig.14 Design curves in the condition of $\zeta_1=0.2$

whose schematic diagram is shown in Fig.15. The equipment consists of two disks of masses m_1 and m_2 ; the disk m_2 is suspended from a beam on three ring-type springs arranged in parallel around an oil damper; the disk m_2 is coupled with the disk m_1 in the same manner. The disk m_2 is subjected to a harmonic exciting force, which is caused by a vibration generator and transmitted by means of a spring and a rod to the disk. This is an inverted (but equivalent mathematically) model of the class (a) in Fig.1.

Figure 16 shows the equation of motion and its solution of the equipment. This equation contains the mass effect of the springs and the oil. (The meaning of the symbols used in this equation is shown in Fig.15.) The damping coefficients c_1, c_2 and the added masses m_{A1}, m_{A2} of the oil dampers depend on the forced frequency.

The equations to be used in the theoretical calculation of C_1, C_2 and m_{A1}, m_{A2} are also shown in Fig.16; these are discussed in detail in Ref.(13).

The experimental results are summarized in Figs.17 and 18. In these figures, open circles and closed circles represent the magnification ratios of the masses m_1 and m_2 to the static deflection of m_1 , respectively. The solid lines represent the theoretical values of them; these are calculated by the solutions shown in Fig.16. There are two solid lines ③ and ④ corresponding to the masses m_1 and m_2 . The numerical values to be used in the calculation are shown in the lower portion of Fig.15 and in Figs.17 and 18. Of these values the damping coefficients C_1 and C_2 of the oil dampers are theoretical values at $\alpha=1$, and the others are measured values. The broken lines represent other

Fig.15 Schematic sketch of experimental equipment and explanation of the symbols

Equations of motion

$$m_1 \ddot{x}_1 + (c_1 + c_2) \dot{x}_1 + (k_1 + k_2) x_1 - (m_{A1} \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1) = 0$$

$$- (m_{A2} \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2) + m_2 \ddot{x}_2 + (c_2 + c_1) \dot{x}_2 + (k_2 + k_1) x_2 = - (m_{so}/B) \ddot{x}_0 + c_{so} \dot{x}_0 + k_{so} x_0 = P_{eq} \sin(\omega t + \gamma)$$

$$P_{eq} = a_0 \sqrt{(k_0 + \frac{m_{so}^2 \omega^2}{B})^2 + c_{so}^2 \omega^2}, \quad \gamma = \tan^{-1} \frac{c_{so} \omega}{k_0 + \frac{m_{so}^2 \omega^2}{B}}$$

$$m_1^* = m_1 + (3/8)(m_{s1} + m_{s2}) + m_{A1} + m_{A2} + (2A_1 + 1)m_{oi}$$

$$m_2^* = m_2 + (3/8)(m_{s1} + m_{so}) + m_{A2} + (2A_2 + 1)m_{oi}$$

$$m_{A1}^* = m_{A1} + A_1 m_{oi} - m_{s1}/B$$

$$c_1^* = c_1 + c_{s1}, \quad c_2^* = c_2 + c_{s2}$$

$$c_i = \mu l (b_i/h_i) \lambda^3 \{ (2A_i + 1) P_{2i} + S_{2i} / (2A_i c_{2i}) \}$$

$$m_{Ai} = (m_{oi}/2) \{ (2A_i + 1) P_{2i} - A_i + S_{2i} / (2A_i c_{2i}) \}$$

Solutions

$$|x_1/x_{st}| = k_1 \sqrt{(H_1^2 + H_2^2) / (H_3^2 + H_4^2)}$$

$$|x_2/x_{st}| = k_2 \sqrt{(H_1^2 + H_2^2) / (H_3^2 + H_4^2)}$$

$$H_1 = k_1 - m_1^* \omega^2, \quad H_2 = k_1 + k_2 - m_1^* \omega^2$$

$$H_3 = c_1^* \omega, \quad H_4 = (c_1^* + c_2^*) \omega$$

$$H_5 = (m_1^* m_2^* - m_{A1}^* m_{A2}^*) \omega^4 - [m_2^* (k_1 + k_2) + m_1^* (k_2 + k_1) - 2m_{A1}^* k_1 - c_1 c_2 + c_{so} (c_1^* + c_2^*)] \omega^2 + k_1 k_2 + k_0 (k_1 + k_2)$$

$$H_6 = -[m_2^* (c_1^* + c_2^*) + m_1^* (c_2^* + c_{so}) - 2m_{A1}^* c_1^*] \omega^3 + [c_1 (k_2 + k_0) + c_2^* (k_1 + k_0) + c_{so} (k_1 + k_2)] \omega$$

Dimensionless quantities

$$A_i = A_{pi}/A_{oi}, \quad \lambda_i = h_i \sqrt{\omega / (2\nu)}$$

$$C_{ui} = \cosh \lambda_i + \cos \lambda_i, \quad S_{ui} = \sinh \lambda_i + \sin \lambda_i$$

$$C_{di} = \cosh \lambda_i - \cos \lambda_i, \quad S_{di} = \sinh \lambda_i - \sin \lambda_i$$

$$P_{2i} = \frac{1}{2} \lambda_i (2A_i + 1) C_{ui} S_{di}, \quad P_{2i} = \frac{1}{2} \lambda_i (C_{ui} - S_{ui}) \{ 2A_i A_{ci} C_{ui} + S_{ui} \}$$

$$S_{2i} = \frac{1}{2} \lambda_i (2A_i + 1) C_{ui} S_{di}, \quad P_{2i} = \frac{1}{2} \lambda_i (C_{ui} - S_{ui}) \{ 2A_i A_{ci} C_{ui} + S_{ui} \}$$

Fig.16 Equations of motion and solution of the equipment shown in Fig.15

Fig.17 Experimental results of searching for the optimum condition to prevent the mass m_2 from vibration; dashed lines ①, ② are calculated by Eqs.(2) and (3), and solid lines ③, ④ by the equations shown in Fig.16

theoretical ones by different equations; curve ① and curve ② are calculated by Eqs. (2) and (3), respectively. The numerical values to be necessary for the calculation are obtained from the following equations:

$$\left. \begin{aligned} \mu &= m_2 / m_1 \\ \beta &= \sqrt{m_1 k_2} / (m_2 k_1) \\ \zeta_1 &= (c_1 + c_{s1}) / (2\sqrt{m_1 k_1}) \\ \zeta_2 &= (c_2 + c_{s2}) / (2\sqrt{m_2 k_2}) \end{aligned} \right\} \dots\dots\dots (15)$$

and they are shown in each graph. Namely, the mass effects of the springs and the oil are neglected in the broken lines. The design of vibration isolators is carried out using the broken lines, but the actual characteristics of the isolators are represented by the solid lines. Therefore, the difference between the two kinds of the theoretical curves represents a gap between the theoret-

ical and the practical design of isolators.

In the experiment shown in Fig.17, we searched for the optimum suppression condition of vibration of the mass m_2 . The experiment was performed changing the piston diameter dp_2 of the oil damper attached between two disks (thereby the damping ζ_2 varies), and keeping the values of μ , ζ_1 and β constant; the tuning parameter β is set at the optimum value for given values of μ and ζ_1 . Comparing the four graphs in Fig.17, it is obvious that the optimum condition is accomplished in graph (c). Thereby, the optimum tuning and damping conditions are equal to 0.474 and 1.050, respectively. On the other hand, the theoretical values of them are calculated as $\beta_{opt}=0.477$ and $\zeta_{2opt}=1.048$ at $\mu=1.324$ and $\zeta_1=1.38 \times 10^{-3}$. Hence, there is little difference between the two optimum conditions obtained by the theoretical and the experimental methods.

Fig.18 Experimental results of searching for the optimum condition to prevent the mass m_1 from vibrating; upper three graphs (a)~(c) are different from lower three graphs (d)~(f) in the values of β and ζ_1

In the experiment shown in Fig.18, we searched for the optimum suppression condition of vibration of the mass m_1 . The experiment was performed in the same manner as Fig.17. However, β has no optimum value in the case of suppression of vibration of mass m_1 , so we have obtained only the optimum damping $\zeta_{2\text{opt}}$ for two different values of β . On the other hand, the mass ratio μ is set at a constant value throughout the experiment. In the first three graphs (a)~(c) of Fig.18, we have set $\beta \approx 1.05$ and $\zeta_1 \approx 0.08$, and achieved the optimum suppression condition of vibration in graph (b); whereas, in the last three graphs (d)~(f), we have set $\beta \approx 0.53$ and $\zeta_1 \approx 0$, and achieved the optimum condition in graph (e). The experimental values of the optimum damping ratio $\zeta_{2\text{opt}}$ are equal to 0.795 in (b) and 0.464 in (e), whereas the theoretical values of them are calculated as 0.892 and 0.483, respectively. They agree well with each other in spite of the mass effect of springs and oil being neglected in the theoretical values.

6. Conclusions

We have discussed the theory of suppression of vibration of a linear system with two degrees of freedom shown in Fig.1(a), in which the mass m_2 is subject to a harmonic exciting force. Two kinds of theories are necessary according as the main mass to be prevented from vibration is m_1 or m_2 . If m_2 is the main mass, there are two optimum design parameters (optimum tuning frequency ratio β_{opt} and damping ratio $\zeta_{2\text{opt}}$). These parameters were determined analytically ($\zeta_1 = 0$) or numerically ($\zeta_1 \neq 0$), and summarized in Figs.4(b), 7 and 8. However, if m_1 is the main mass, there is one optimum parameter $\zeta_{2\text{opt}}$. It was also determined analytically ($\zeta_1 = 0$) or numerically ($\zeta_1 \neq 0$), and summarized in Figs.5, 11, 13 and 14.

Next, we examined the optimum conditions

obtained theoretically for their practicality using a mechanical vibratory model. From the examination, it was found that there is little difference between the optimum conditions obtained by the theoretical and the experimental methods; hence that the theory proposed here is very useful for the design of vibration isolators.

The authors would like to thank T.Ito, H. Kojima, Y.Fukui and T.Watanabe for their assistance in carrying out the experiment.

References

- (1) Frahm, H., *U.S. Patent*, No.989, 958 (1911), p.3576.
- (2) Ormondroyd, J. and Den Hartog, J. P., *Trans. ASME, J. Appl. Mech.*, 50-7 (1927), p.9.
- (3) Den Hartog, J.P., *Mechanical Vibrations* 4th ed. (1956), p.93, McGraw-Hill.
- (4) Hahnkamm, H., *Ing. Arch.*, 4 (1933), p.192.
- (5) Brock, J. E., *Trans. ASME, J. Appl. Mech.*, 13-4 (1946), A-284.
- (6) Randall, S.E. et al., *Trans. ASME, J. Mech. Des.*, 103-4 (1981), p.908.
- (7) Thompson, A.G., *J. Sound Vib.*, 77-3 (1981), p.403.
- (8) Soom, A. and Lee, Ming-san, *Trans. ASME, J. Vib. Acoust. Stress Reliab. Des.*, 105-1 (1983), p.112.
- (9) Ikeda, K. and Ioi, T., *Trans. Jpn. Soc. Mech. Eng.* (in Japanese), Vol.43, No.369 (1977), p.1707.
- (10) Ioi, T., *Reports of Chiba Ins. of Tech.* (in Japanese), 27 (1981), p.49.
- (11) Sekiguchi, H. and Asami, T., *Bull. JSME*, Vol.27, No.234 (1984), p.2839.
- (12) Yamahara, H., *Handbook of Mechanical Vibrations* (in Japanese), (1976), p.1196, Yohkendo.
- (13) Asami, T. and Sekiguchi, H., *Bull. JSME*, Vol.28, No.238 (1985), p.663.