Seismic Response of Liquid Sloshing in the Annular Region Formed by Coaxial Circular Cylinders*

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Although much research work has been performed on the liquid sloshing inside simple cylindrical vessels, with free surface at the time of an earthquake, few reports are published about the sloshing in intricate structures with inner structures. This paper deals with an analysis of liquid sloshing in the annular region of more intricate coaxial circular cylinders. In this analysis, by utilizing the velocity potential theory, the transient response is formulated as a boundary value problem in axisymmetrical vessels when the sine wave in resonance with the sloshing of the primary mode is applied which is known to be a convenient method to evaluate the seismic response of sloshing. These analytical results show very good agreement with the experimental results. Further, by utilizing this analytical method and FEM, the sloshings in more complicated structures are investigated.

Key Words : Fluid Vibration, Vibration of Continuous System, Coaxial Circular Cylinder, Seismic Response, Sloshing in Annular Region

1. Introduction

With regard to the liquid sloshing inside the liquid storage tank with free surface at the time of an earthquake, experimental data and reports by numerous researchers are available, and as a result analytical solutions and/or various analytical methods such as FEM, have been developed to an extent that the nonlinear response and/or coupled behaviour with the flexible shell surface have also been analysed^{(1),(2),(3),(4)}. It is shown that the actual phenomena can be estimated very accurately by these methods. However, all these analytical methods are, without exception, restricted to fairly simple structures such as simple cylindrical and/or rectangular tank, and few reports about intricate structures with inner structures are available. As practical examples of these complicated structures, various reaction towers of chemical plants, BWR's pressure suppression pool and/or reactor vessel of the fast breeder reactor can be cited, which have annular regions inside them, but no report is made on the sloshing phenomena in this

annular region, except for a paper by Aslam.⁽⁸⁾

This paper deals with an analysis of the seismic response of the liquid sloshing in the annular region of a coaxial circular cylinder, i.e., one of the complicated structures which can be seen in many places. In this analysis, the axisymmetric shell wall is assumed to be rigid, the liquid is assumed to be incompressible, inviscid, and irrotational. Thus it is considered an ideal fluid, and the linear theory is applied. Under these conditions, we obtained the velocity potential satisfying the boundary condition, and analyzed the transient response for the N sine wave in resonance with the sloshing of the first mode, which is called "N resonance sine wave". In addition, a coaxial circular cylinder with two kinds of inner/outer diameter ratios was installed on a large scale shaking table (6m x 6m, 3-dimensional simultaneously) for the vibration tests in which the vibration characteristics of sloshing in the annular region as well as response distribution of liquid pressures were experimentally obtained when a resonance sine wave was given.

Furthermore, the vibration characteristics of the sloshing inside an axisymmetric tank with more intricate inner construction were obtained experimentally. When applied to these liquid-tank systems, the analytical method thus developed was found to give a remarkably good agreement with measured values as well as the analytical results by means of FEM.

These experimental and analytical results are presented here.

2. Analytical Study

In the same manner as the sloshing in simple circular cylinders, the velocity

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potential theory was applied and the analytical equations for sloshing in the annular region were derived.⁽⁸⁾

2.1 Basic equations

As shown in Figure 1, it is assumed that (i) the tank wall is rigid; (ii) the liquid in the annular region is incompressible, inviscid and irrotational; and (iii) thus it is an ideal fluid to be treated by linear theory. In this case, the liquid has the velocity potential ϕ satisfying

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (1)$$

where (r, θ, z) for this equation are denoted as cylindrical coordinates in Figure 1.

2.2 Natural frequency of sloshing in the annular region

In eigen value analysis, free vibration will be considered. In this case the velocity potential ϕ_0 will also satisfy Eq.(1), thus, we have

$$\frac{\partial^2 \phi_0}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_0}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi_0}{\partial \theta^2} + \frac{\partial^2 \phi_0}{\partial z^2} = 0 \quad (2)$$

Also, the liquid velocity is considered the same as the shell wall velocity at the tank wall. But in the free vibration problem,

the shell wall velocity is equal to zero, therefore the boundary condition will be as follows;

$$\left. \frac{\partial \phi_0}{\partial r} \right|_{r=a} = 0, \quad \left. \frac{\partial \phi_0}{\partial r} \right|_{r=b} = 0 \quad (3)$$

$$\left. \frac{\partial \phi_0}{\partial z} \right|_{z=-H} = 0 \quad (4)$$

$$\frac{\partial^2 \phi_0}{\partial t^2} + g \frac{\partial \phi_0}{\partial z} = 0 \quad (\text{at } z=0) \quad (5)$$

where a is outer diameter, b inner diameter, H liquid depth, g acceleration of gravity, and $\dot{}$ denotes a derivative of time.

The velocity potential ϕ_0 satisfying Eq.(2) ~ Eq.(5) is obtained as follows.

$$\phi_0 = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} e^{i\omega_{mn}t} \{ C_{mn} \sin(m\theta) + D_{mn} \cos(m\theta) \} \\ \times \frac{\cosh \left\{ \xi_{mn} \left(\frac{z}{a} + \frac{H}{a} \right) \right\} G_{mn}(r)}{\cosh \left(\xi_{mn} \frac{H}{a} \right) Y'_m(\xi_{mn})} \quad (6)$$

$$G_{mn}(r) = J_m \left(\xi_{mn} \frac{r}{a} \right) Y'_m(\xi_{mn}) - J'_m(\xi_{mn}) Y_m \left(\xi_{mn} \frac{r}{a} \right) \quad (7)$$

where ξ_{mn} is a constant depending on the mode and the radius ratio b/a . Figure 2 shows the value of ξ_{mn} in case of $m=1$ where ξ_{1n} is expressed as ξ_n for convenience. Substituting Eq.(6) into Eq.(5), the boundary condition at the free surface, and putting the coefficients of sine and cosine terms as zero, we get the natural frequency as follows.

$$\omega_{mn}^2 = \left(\frac{g}{a} \right) \xi_{mn} \tanh \left(\xi_{mn} \frac{H}{a} \right), \quad m=0, 1, 2, \dots, \infty, \\ n=1, 2, 3, \dots, \infty \quad (8)$$

2.3 Seismic response of sloshing in the annular region

In this analysis, the seismic response will be derived by taking into account the damping of sloshing which Aslam et al. did not. When the ground motion $X(t)$ is applied, the boundary conditions at the tank wall are,

$$\left. \frac{\partial \phi}{\partial r} \right|_{r=a} = \dot{X}(t) \cos \theta, \quad \left. \frac{\partial \phi}{\partial r} \right|_{r=b} = \dot{X}(t) \cos \theta \quad (9)$$

$$\left. \frac{\partial \phi}{\partial z} \right|_{z=-H} = 0 \quad (10)$$

and the free surface boundary conditions considering the damping of sloshing are,⁽⁴⁾

$$\frac{\partial^2 \phi}{\partial t^2} + \mu \frac{\partial \phi}{\partial t} + g \frac{\partial \phi}{\partial z} = 0 \quad (\text{at } z=0) \quad (11)$$

where μ is the damping coefficient of sloshing. Here, it is assumed that the velocity potential ϕ is equal to the sum of the velocity potentials ϕ_1 and ϕ_F which satisfy the Laplace equation, thus, we have

Fig.1 Model of Annular Region

Fig.2 Relationship between ξ_1, ξ_2, ξ_3 and $K (=b/a)$

$$\phi(r, \theta, z, t) = \phi_1(r, \theta, z, t) + \phi_F(r, \theta, t) \quad (12)$$

$$\left. \frac{\partial \phi_F}{\partial r} \right|_{r=a} = \dot{X}(t) \cos \theta, \quad \left. \frac{\partial \phi_F}{\partial r} \right|_{r=b} = \dot{X}(t) \cos \theta \quad (13)$$

And ϕ_F also satisfies Eq.(1), then the following equation is obtained.

$$\frac{\partial^2 \phi_F}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_F}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi_F}{\partial \theta^2} = 0 \quad (14)$$

Thus, from Eq.(13), (14), ϕ_F is obtained as follows.

$$\phi_F = \dot{X} r \cos \theta \quad (15)$$

Next, ϕ will be derived. In general, the seismically induced mode is only an asymmetric mode. Therefore, in this paper, we will only consider the case of $m=1$. Then the next equations referred to ϕ_1 are obtained from Eqs.(1), (9), (10), (13).

$$\frac{\partial^2 \phi_1}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_1}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi_1}{\partial \theta^2} + \frac{\partial^2 \phi_1}{\partial z^2} = 0 \quad (16)$$

$$\left. \frac{\partial \phi_1}{\partial r} \right|_{r=a} = 0, \quad \left. \frac{\partial \phi_1}{\partial r} \right|_{r=b} = 0, \quad \left. \frac{\partial \phi_1}{\partial z} \right|_{z=-H} = 0 \quad (17)$$

The velocity potential ϕ_1 satisfying Eqs. (16), (17) is expressed by the following equation.

$$\phi_1 = \sum_{n=1}^{\infty} (C_n \sin \theta + D_n \cos \theta) \times \frac{\cosh\left\{\xi_n \left(\frac{z}{a} + \frac{H}{a}\right)\right\} G_n(r)}{\cosh\left(\xi_n \frac{H}{a}\right) Y_1'(\xi_n)} T_n(t) \quad (18)$$

where

$$G_n(r) = J_1\left(\xi_n \frac{r}{a}\right) Y_1'(\xi_n) - J_1'(\xi_n) Y_1\left(\xi_n \frac{r}{a}\right) \quad (19)$$

and C_n , D_n , ξ_n , ω_n are C_{mn} , D_{mn} , ξ_{mn} , ω_{mn} respectively in the case of $m=1$, with subscript 1 omitted for simplification. $T_n(t)$ is a function depending on the mode. From Eqs.(11), (12), (15), the boundary conditions at the free surface referring to ϕ_1 are given as follows.

$$\frac{\partial^2 \phi_1}{\partial t^2} + \mu \frac{\partial \phi_1}{\partial t} + g \frac{\partial \phi_1}{\partial z} = -r \cos \theta \{\ddot{X}(t) + \mu \dot{X}(t)\}, \quad (\text{at } z=0) \quad (20)$$

Substituting Eq.(18) into Eq.(20), and using the relations $\mu=2\xi_n\omega_n$ where ξ_n is the damping ratio of n th mode and $\omega_n^2=g/a \xi_n \tanh(\xi_n H/a)$, we obtain the following equation.

$$\sum_{n=1}^{\infty} (C_n \sin \theta + D_n \cos \theta) \frac{G_n(r)}{Y_1'(\xi_n)} \{\ddot{T}_n(t) + 2\xi_n\omega_n \dot{T}_n(t) + \omega_n^2 T_n(t)\} = -r \cos \theta \{\ddot{X}(t) + 2\xi_n\omega_n \dot{X}(t)\} \quad (21)$$

Further, r is expressed as Bessel's Fourier series.

$$r = a \sum_{n=1}^{\infty} Q_n G_n(r) \quad (22)$$

The coefficient Q_n is decided from the following equation by multiplying both sides of Eq.(22) by $rG_n(r)$, integrating them from b to a and utilizing the orthogonality of $G_n(r)$.

$$Q_n = 2 \left\{ \frac{2}{\pi \xi_n} - k G_n(b) \right\} / \left\{ \frac{4}{\pi^2} \xi_n^2 (\xi_n^2 - 1) + G_n^2(b) (1 - k^2 \xi_n^2) \right\}, \quad k = b/a \quad (23)$$

Substituting Eq.(22) into Eq.(21) and comparing the coefficients of sine and cosine terms, we find the coefficient C_n of sine term to be zero. The following equations are obtained.

$$\ddot{T}_n(t) + 2\xi_n\omega_n \dot{T}_n(t) + \omega_n^2 T_n(t) = -\beta_n \{\ddot{X}(t) + 2\xi_n\omega_n \dot{X}(t)\} \quad (24)$$

$$\beta_n = a Q_n Y_1'(\xi_n) / D_n \quad (25)$$

β_n corresponds to the participation factor in modal analysis. In general, as the damping of sloshing is very small, Eq.(24) becomes as follows by putting $\xi_n=0$.

$$\ddot{T}_n(t) + \omega_n^2 T_n(t) = -\beta_n \ddot{X}(t) \quad (26)$$

The solution of Eq.(26) which is a linear differential equation is obtained as follows by taking into account the initial condition at $t=0$.

$$T_n(t) = -\frac{\beta_n}{\omega_n} \sin \omega_n t \int_0^t \ddot{X}(\tau) \cos \omega_n \tau d\tau + \frac{\beta_n}{\omega_n} \cos \omega_n t \int_0^t \ddot{X}(\tau) \sin \omega_n \tau d\tau \quad (27)$$

Equation (26) can also be solved by the Duhamel formula, and the solution agrees with Eq.(27). From Eqs.(12), (15), (18), (27) and $C_n=0$, the velocity potential is derived as follows.

$$\phi = r \dot{X}(t) \cosh - \sum_{n=1}^{\infty} D_n \cos \theta \frac{\cosh\left\{\xi_n \left(\frac{z}{a} + \frac{H}{a}\right)\right\} G_n(r)}{\cosh\left(\xi_n \frac{H}{a}\right) Y_1'(\xi_n)} \times \frac{\beta_n}{\omega_n} \left\{ \sin \omega_n t \int_0^t \ddot{X}(\tau) \cos \omega_n \tau d\tau - \cos \omega_n t \int_0^t \ddot{X}(\tau) \sin \omega_n \tau d\tau \right\} \quad (28)$$

Once the expression for the velocity potential is given, we can easily derive the seismic response wave height and the pressure distribution of the sloshing in the annular region by using Eqs.(25), (28) and the relations, $\eta=1/\rho (\partial \phi / \partial t)$, $P=-\rho (\partial \phi / \partial t)$ and by using the partial integration. Thus Eqs.(29), (30) are obtained.

$$\eta(r, \theta, z, t) = -\frac{1}{g} \left\{ r \ddot{X}(t) - a \sum_{n=1}^{\infty} Q_n G_n(r) \left[\ddot{X}(t) + \omega_n \cos \omega_n t \int_0^t \ddot{X}(\tau) \sin \omega_n \tau d\tau \right] \right\}$$

$$-\omega_n \sin \omega_n t \int_0^t \ddot{X}(\tau) \cos \omega_n \tau d\tau \Bigg\} \quad \dots\dots\dots(29)$$

$$\begin{aligned} p(r, \theta, z, t) \Big|_{r=0} = & -\rho \left\{ a \ddot{X}(t) \right. \\ & - a \sum_{n=1}^{\infty} \frac{Q_n G_n(a) \cosh \left\{ \xi_n \left(\frac{z}{a} + \frac{H}{a} \right) \right\}}{\cosh \left[\xi_n \frac{H}{a} \right]} \left[\ddot{X}(t) \right. \\ & + \omega_n \cos \omega_n t \int_0^t \ddot{X}(\tau) \sin \omega_n \tau d\tau \\ & \left. \left. - \omega_n \sin \omega_n t \int_0^t \ddot{X}(\tau) \cos \omega_n \tau d\tau \right] \right\} \quad \dots\dots\dots(30) \end{aligned}$$

Substituting Eq.(22) into Eqs.(29), (30) and simplifying them, we get the response wave height and pressure, as follows.

$$\eta(r, \theta, z, t) \Big|_{r=0} = -\frac{a}{g} \sum_{n=1}^{\infty} Q_n G_n(r) \omega_n \int_0^t \ddot{X}(\tau) \sin \omega_n (t-\tau) d\tau \quad \dots\dots\dots(31)$$

$$\begin{aligned} p(r, \theta, z, t) \Big|_{r=0} = & -\rho a \left\{ \ddot{X}(t) \right. \\ & - \sum_{n=1}^{\infty} \frac{Q_n G_n(a) \cosh \left\{ \xi_n \left(\frac{z}{a} + \frac{H}{a} \right) \right\}}{\cosh \left[\xi_n \frac{H}{a} \right]} \left[\ddot{X}(t) \right. \\ & \left. \left. - \omega_n \int_0^t \ddot{X}(\tau) \sin \omega_n (t-\tau) d\tau \right] \right\} \quad \dots\dots\dots(32) \end{aligned}$$

If excited by sine wave with the same frequency as the natural frequency of the fundamental mode, the response will be obtained by substituting $X(t) = Dg \sin \omega_1 t$ into Eqs.(31), (32). In this case, the contribution of higher mode such as $n \geq 2$ is

Fig.3 Measured Points

Fig.4 Outline of Test Apparatus

considered to be extremely small in Eqs. (31), (32), thus the response value for the "N resonance sine wave" is considered to be mostly influenced by the mode $n=1$. Therefore, we have

$$\eta(r, \theta, z, t) \Big|_{r=0} = -\frac{a}{g} Q_1 G_1(r) \omega_1 \int_0^t \ddot{X}(\tau) \sin \omega_1 (t-\tau) d\tau \quad \dots\dots\dots(33)$$

$$\begin{aligned} p(r, \theta, z, t) \Big|_{r=0} = & -\rho a \left\{ \ddot{X}(t) - \frac{Q_1 G_1(a) \cosh \left\{ \xi_1 \left(\frac{z}{a} + \frac{H}{a} \right) \right\}}{\cosh \left[\xi_1 \frac{H}{a} \right]} \right. \\ & \left. \times \left[\ddot{X}(t) - \omega_1 \int_0^t \ddot{X}(\tau) \sin \omega_1 (t-\tau) d\tau \right] \right\} \quad \dots\dots\dots(34) \end{aligned}$$

Table 1 Natural Frequency of Sloshing (Hz)

Case	Case 1 (b=82.5cm, H=74.5cm)		Case 2 (b=23.0cm, H=112.5cm)	
	Cal.	Exp.	Cal.	Exp.
1	0.452	0.43	0.67	0.66
2	3.867	—	1.182	1.17
3	5.468	—	1.558	1.59

$$\begin{aligned} \text{Case 1 : } \xi_1 &= 1.03, \xi_2 = 53, \xi_3 = 106 \\ \text{Case 2 : } \xi_1 &= 1.64, \xi_2 = 4.95, \xi_3 = 8.6 \end{aligned}$$

Fig.5 Frequency Response Curve

Fig.6 Radius Ratio (b/a) and Frequencies

Fig.7 Natural Frequencies and Liquid Depth

Substituting $X=Dg \sin \omega_1 t$, Eqs.(33), (34) become,

$$\eta(r, \theta, z, t) \Big|_{\substack{\theta=0 \\ z=0}} = \frac{a Q_1 G_1(r) D_2 \omega_1^2}{2g} (\sin \omega_1 t - \omega_1 t \cos \omega_1 t) \quad (35)$$

$$p(r, \theta, z, t) \Big|_{\substack{r=a \\ \theta=0}} = \rho a D_2 \omega_1^2 \left\{ \sin \omega_1 t - \frac{Q_1 G_1(a) \cosh \left\{ \xi_1 \left(\frac{z}{a} + \frac{H}{a} \right) \right\}}{\cosh \left[\xi_1 \frac{H}{a} \right]} \left[\sin \omega_1 t - \frac{1}{2} (\sin \omega_1 t - \omega_1 t \cos \omega_1 t) \right] \right\} \quad (36)$$

The integral term in the right part of Eq. (35) reaches maxima at the time of $t=i\pi/\omega$, and the maximum value between $0 \leq t \leq lT$ ($T=2\pi/\omega_1$) is obtained at the time of $t=2l\pi/\omega_1$, where both i and l are positive integers and l is the number of input sine wave. When three sine waves are given, l equals 3, and the maximum value is obtained at the time of $t=6\pi/\omega_1$.

Fig.8 Relationship between 1st Mode Natural Frequency and Liquid Depth

Fig.9 Sloshing Mode (CASE 1, $b/a=0.9375$)

Fig.10 Sloshing Mode (CASE 2, $b/a=0.2875$)

Therefore, the maximum response wave height will be given by the following equation:

$$\eta_{\max}(r) = \frac{a}{g} Q_1 G_1(r) 3\pi D_2 \omega_1^2 \quad (37)$$

Specifically, the maximum wave height at the shell wall and the maximum pressure at the free surface will be obtained by substituting $r=a$ and $z=0$ into Eqs.(36) and (37).

$$\eta_{\max} \Big|_{r=a} = \frac{a}{g} Q_1 G_1(a) 3\pi D_2 \omega_1^2 \quad (38)$$

$$p_{\max} \Big|_{z=0} = \gamma \cdot \eta_{\max}, \quad \gamma = \rho g \quad (39)$$

3. Characteristics of Sloshing in the Annular Region

We investigated the influences of the radius ratio and the liquid depth, on the vibration characteristics of sloshing and also the validity of analysis by the experiments using the cylindrical tank model.

3.1 Vibration characteristics of sloshing in the annular region

3.1.1 Test procedure

A cylindrical vessel model of hard vinyl chloride was installed on a large scale earthquake simulator and the tests

Fig.11 Sloshing Mode ($b/a=0.05$)

Fig.12 Time History of Wave Height for CASE 1

Fig.13 Distribution of Response Wave Height for Three Resonance Sine Wave (Values at the Wall Surface of Outer Cylinder)

were performed by shaking it.

In this test, the radius of the outer cylinder was kept constant and the radius of the inner cylinder was varied into two dimensions, with the liquid depth simultaneously changed.

Figure 4 shows the dimensions of these vessels as well as an outline of the test apparatus. Both the inner and outer cylinder walls have enough thickness to be considered rigid against sloshing. Water was used as liquid inside the cylinders. As shown in Figure 3, we mounted liquid level indicators inside the outer cylinder at 4 points in the circumferential direction and measured the response wave height. In addition, the response pressure was measured by the pressure transducers which were installed on the inside wall of the outer cylinder in the same direction as the exciting direction.

Natural frequencies of sloshing were obtained from the frequency response curve for the wave height at the free surface, and the displacement of the shaking table was maintained constant in the tests.

3.1.2 Natural frequencies of sloshing

By utilizing the test apparatus as shown in Figure 4, we obtained natural frequencies of sloshing in the annular region experimentally and compared them with the theoretical values.

Figure 5 shows the frequency response curve obtained experimentally. Table 1 indicates comparison between the measured and the theoretical values. Both are found in good agreement, and the validity of the theoretical values is confirmed. Figures 6 and 7 show the analytical results about the relationship between the natural frequencies and both the radius ratio b/a and the liquid depth. It will be known that the most important 1st mode when discussing the seismic safety has a slightly different tendency from those of higher modes and that with an increasing radius

ratio, the natural frequencies decrease. It is also confirmed that the analytical values of the natural frequencies for $b=0$, which is the case of a simple circular cylinder, agree with those for a simple circular cylinder. Figure 8 shows the effect of the liquid depth on the 1st mode with various values of b/a . Generally, it is recognized that in the case of simple circular cylinders, the influence will be reduced to zero in the range of $H/a \geq 1.5$, and it is understood that the same criteria can be applied in the annular region of the coaxial circular cylinder. Further, the effect of H/a seems to increase with an increasing b/a .

3.1.3 Vibration mode of sloshing

It is known that sloshing in the annular region have both non-axisymmetric and axisymmetric mode; however, it is only the non-axisymmetric mode that is induced by a horizontal earthquake. Therefore, only non-axisymmetric modes will be studied here. Figures 9~11 show the analytical results about the vibration modes for three values of radius ratio. As is known from these Figures, the vibration modes in the coaxial circular cylinder become similar to those of the simple circular cylinder with a decreasing radius ratio and the mode shape of the 1st mode in the case of $b/a \neq 1$ is similar to the part of that for $b/a = 0$, while the mode shapes of higher modes are not so simple as that of the 1st mode.

3.2 Response characteristics of sloshing due to three resonance sine waves in the annular region

3.2.1 Test procedure

The transient response characteristics of sloshing for sine wave input were studied by using the same test apparatus and the same measuring systems as shown in Figure 4.

3.2.2 Response wave height

Figure 12 shows the time history of response wave height of sloshing for three resonance sine waves with an amplitude of 1.98cm and a frequency of 0.43Hz. Figure 13 shows the response wave height distribution in this case. In Figure 12, FEM solution by NASTRAN using the axisymmetric elements is also shown. This figure reveals that the present solution, being in good agreement both with the measured values and with FEM solution, is valid. These numerical values are given in Table 2.

Next, the effect of radius ratio and liquid depth on response wave height is

Table 2 Response Wave Height and Pressure

Case (Item)	Case 1			Case 2		
	Cal.*		Exp.	Cal.*		Exp.
	No. 1	No. 2		No. 1	No. 2	
Max. wave height (cm)	12.9	11.6	11.0	13.0	22.3	29.0
Max. pressure (MPa)	1.27 (12.9 kgf/cm ²)	1.14 (11.6 kgf/cm ²)	1.08 (11.0 kgf/cm ²)			

* In case of Cal. No.1, analytical natural frequency is used, in case of Cal. No.2 experimental value is used.

* * Damping ratio (Exp.): Case 1 = 1.95%, Case 2 = 0.68%

Fig.14 Radius Ratio b/a and Response Wave Height

Fig.15 Relationship between Liquid Depth and Response Wave Height

shown in Figures 14 and 15, in which it is known that as b becomes close to zero, then the response wave height tends to increase.

In the case of $b/a \geq 0.8$, a slightly different tendency is shown, but it is anticipated that the damping ratio also will practically increase with an increasing b/a ; therefore, an abrupt jump in the response wave height will not occur. Further, in consequence of increased liquid depth, the response wave height will also increase but the increasing ratio itself has a tendency of decreasing.

3.2.3 Response pressure at shell wall

The distribution of response pressures at the shell wall is shown in Figure 16, where the analytical solution by FEM is also given, which indicates that the present solution regarding the response pressure is also in good agreement both with the analytical results by FEM and with the measured values. In addition, the relationship between response pressure and radius ratio or liquid depth as quoted in Eq.(39) seems to have a similar tendency to the effect on the response wave height, as shown in Figures 14 and 15.

Further, in the case 1 of Table 2, analytical values are found to be slightly higher than the measured values. This is considered to come from the fact that the structure-liquid system was assumed to have 0% damping ratio in the analysis while the

measured value was about 2%. Also, in the case 2 of the same Table, the phenomena that the measured values are higher than the theoretical values come from a strong non-linear effect of sloshing.

4. Sloshing in a more complicated structure

Here, the sloshing in a more complicated axisymmetric vessel is investigated.

4.1 Test apparatus

Figure 17 shows an outline of the axisymmetric vessel composed of complicated structure in which both the inner and outer cylinders have some stages, and studies are performed on sloshing inside such axisymmetric vessels.

The wave height mode of sloshing, the pressure distribution at the shell wall and natural frequencies are given in Figure 20, where the analytical result by FEM is indicated by a solid line. The analysis was performed by NASTRAN by using axisymmetric finite elements, i.e. CFLUID elements as shown in Figure 18. It was difficult to apply the analytical solution to these complicated systems, therefore we treated the liquid system as an annular region shown in Figure 19. The analytical result is also given in Figure 20 for reference. As shown in Figure 20, the theoretical value

Fig.16 Response pressure distribution

Fig.17 Complicated Axisymmetric Vessel

Fig.18 Analytical model for FEM

Fig.19 Analytical model for approximation

Fig.20 Distribution of Wave Height and Pressure, Natural Frequency

Fig.21 Time History for Response Wave Height

Fig.22 Distribution of Response Pressure

by FEM shows good agreement with the measured value, and it is confirmed that the vibration characteristics of sloshing in these complicated structures can be estimated with a fairly good accuracy. Besides, the reference analytical solution was also found to be in a remarkably good agreement with the measured values. This means that an irregular surface of the shell wall located at a considerable depth of liquid inside the vessel can be practically ignored with no problem.

4.3 Transient response for sine wave

The characteristics of the transient response when the axisymmetric vessel shown in Figure 17 is excited by sine wave are also studied. Figure 21 shows the record of the wave height, and Figure 22 shows the distribution of response pressures at the shell wall. All these transient responses were obtained when three resonance sine waves were given. With regard to the response in wave height, the theoretical values are in good agreement with the measured values, as a whole, while the measured values are found slightly higher than the analytical values in the positive side because of the non-linear effect.

Also, the response pressure obtained analytically by FEM is evidently in good agreement with the experimental values. From these results, the analytical methods presented in this paper are found to give accurate values even for the transient response due to sine wave input, although the structure is very complicated.

5. Conclusions

The vibration characteristics and seismic response for the sloshing in the annular region of the coaxial circular cylinder were examined by three methods, i.e., experiments, analytical solution and FEM analysis. Further, sloshing phenomena inside more intricate axisymmetric structures were studied. As a result, the following facts were revealed:

- (1) By applying the analytical method for sloshing at the annular region developed by Aslam and others, an analytical solution for the response to three resonance sine waves, which is a convenient evaluation method of seismic response, has been obtained; hence response wave height and liquid pressure distribution have been formulated.
- (2) These evaluation equations are themselves simple, facilitating the seismic response of sloshing inside various cylindrical tanks with the annular region to be evaluated.
- (3) The response wave height and the liquid pressure distribution which are obtained from these equations show good correspondence with the measured values, and confirm the validity of this analytical solution, besides indicating favourable coincidence with FEM solution. This also verifies the reliability of these analytical methods.
- (4) Taking into account the damping and the nonlinearity of sloshing we can obtain more accurate solutions.

- (5) The influence of the liquid depth and the radius ratio on sloshing in the annular region has been made clear.
- (6) It is verified that the analytical methods we have reported will give proper appraisalment on the seismic response characteristics of sloshing in more complicated structures.

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References

- (1) Shiraki, K. and Fujita, K., Mitsubishi Juko Giho, (in Japanese), Vol.12, No.3 (1975), P.282.
- (2) Fujita, K., Trans. Japan Soc. Mech. Eng. (in Japanese), Vol.47, No.413, C(1981), P.20.
- (3) Sogabe, K. and Shibata, H., Seisan Kenkyu (in Japanese), Vol.26, No.3 (1974), P.119.
- (4) Senda, K. and Nakagawa, K., Proc. 4th Jpn. Natl. Congr. Appl. Mech., (1954), P.247.
- (5) Shaaban, S.H. and Nash, W.A., U. Massachusetts Rep. Nat. Sci. Found. (1976)
- (6) Ghassan R. Khabbaz, AIAA J., Vol.9, No.10 (1971), P.1985.
- (7) Nakayama, T., J. Japan Soc. Areronaut. Space Sci., (in Japanese), Vol.27, No.310 (1979), P.22.
- (8) Aslam, M. et al., Proc., Am. Soc. Civ. Eng., 105-EM3 (1979), P.371.