Dynamic and Static Characteristics of Rolling Rigid Bodies*

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Motion of a prismatic part, which rolls across a surface, is analyzed theoretically in consideration of stiffness and viscous damping about elastic deformation of edges of the part and the surface, and the motion is also investigated experimentally. Probability that the part will come to rest in a particular natural resting aspect is derived in consideration of dissipation of energy when the part bounces up and down.

Key Words : Dynamics of Machinery, Elastic Deformation, Viscous Damping, Dissipation of Energy, Probability

1. Introduction

A vibratory-bowl feeder is the commonest feeding and orienting device used in automatic assembly. The feed rate of oriented parts is influenced by probability that the parts will come to rest in a particular natural resting aspect. The probability of the resting aspect of the parts was obtained in consideration of shape and center of gravity of the parts⁽¹⁾⁻⁽⁵⁾, but a difference in the probability when the parts dropped onto different surfaces was not considered sufficiently.

In this paper, the motion of a prismatic part is analyzed theoretically in consideration of stiffness and viscous damping about elastic deformation of the edges of the part and its surface and the motion of the part is investigated experimentally. The probability that the part will come to rest in a particular natural resting aspect is derived in consideration of the dissipation of energy when the part bounces up and down.

2. Equations of motion of the rolling part

Figure 1 shows a prismatic part whose edges are replaced approximately by spring and damper. The parameters of the prismatic part are as follows.

$G(x, y)$: center of gravity of the prismatic part

- I : moment of inertia about the center of gravity
- m : mass of the prismatic part
- θ_j : angular displacement of the prismatic part
- r_j : length between an edge of the prismatic part and its center of gravity
- $A_j(x_j, y_j)$: coordinates of the edge of the prismatic part
- K : stiffness about elastic deformation of the edge
- C : viscous damping coefficient about elastic deformation of the edge
- l_j : reaction normal to the contact surface

Fig.1 Prismatic part whose edges are replaced by spring and damper

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- F_j : frictional force of the contact surface
 μ : coefficient of friction
 ϕ : slant angle of the plate
 a, b : length of the side of the prismatic part
 E_c : dissipation of energy when the prismatic part comes into collision with the plate
 E_μ : dissipation of energy when the prismatic part slips on the plate
 h : height of the center of gravity of the prismatic part
 P : probability of the natural resting aspect of the part

Then, the equations of motion of the rolling part are

$$m\ddot{x} = F_j 1(-y_j) 1(R_j) + F_{j+1} 1(-y_{j+1}) 1(R_{j+1}) + mg \sin \phi \quad (1)$$

$$m\ddot{y} = R_j 1(-y_j) 1(R_j) + R_{j+1} 1(-y_{j+1}) 1(R_{j+1}) - mg \cos \phi \quad (2)$$

$$I\ddot{\theta}_j = (R_j r_j \sin \theta_j - F_j r_j \cos \theta_j) 1(-y_j) 1(R_j) + (R_{j+1} r_{j+1} \sin \theta_{j+1} - F_{j+1} r_{j+1} \cos \theta_{j+1}) 1(-y_{j+1}) 1(R_{j+1}) \quad (3)$$

($j=1, 2, 3, \dots$)

The reaction normal to the contact surface is

$$R_k = -Ky_k - C\dot{y}_k \quad (4)$$

($k=j$ or $k=j+1$)

When $\dot{x}_k \neq 0$, the frictional force of the contact surface is

$$F_k = F_{k\max} \text{sign}(\dot{x}_k) \quad (5)$$

When $\dot{x}_{k+1} \neq 0$, $\dot{x}_k = 0$ and $F_k \leq F_{k\max}$,

$$F_k = m r_k (\ddot{\theta}_k \cos \theta_k - \dot{\theta}_k^2 \sin \theta_k) - mg \sin \phi - F_{k+1} 1(-y_{k+1}) 1(R_{k+1}) \quad (6)$$

When $\dot{x}_{k+1} = 0$, $\dot{x}_k = 0$ and $F_k \leq F_{k\max}$,

$$F_k = -\frac{mg \sin \phi}{\sigma} [\sigma - r_k \sin(\theta_k + \phi)] \times 1(-y_{k+1}) 1(R_{k+1}) \quad (7)$$

($k=j$ or $k=j+1$)

Where

$$1(x) = \begin{cases} 1 & (x \geq 0) \\ 0 & (x < 0) \end{cases}$$

$$\sigma = r_j \sin(\theta_j + \phi) + r_{j+1} \sin(\theta_{j+1} + \phi)$$

$$F_{k\max} = -\mu \frac{mg \cos \phi}{\sigma} [\sigma - r_k \sin(\theta_k + \phi)] \times 1(-y_{k+1}) 1(R_{k+1}) \quad (8)$$

3. Analysis of the motion of the rolling part

Figure 2 shows a prismatic part whose sectional shape is a symmetric hexagon. The parameters of the part are shown in Table 1, and the parameters of the plate are shown in Table 2. Theoretical results are obtained by Eqs. (1) ~ (8) which are calculated by the Runge-Kutta method. The

angular acceleration is measured by two accelerometers, and the angular displacement is measured by a stroboscope. When the initial conditions are $\theta = \theta_i$ and $\dot{\theta} = 0$, the theoretical and experimental results are as follows.

Figure 3 shows the angular acceleration of the prismatic part which rolls across a rubber plate. When the slant angle of the rubber plate is small as shown in Fig. 3 (a) and (b), the rotational direction of the part changes after edge A_2 has come into collision with the plate. When the slant angle of the rubber plate is large as shown in Fig. 3 (c), the rotational direction of the part changes after edge A_3 has come into collision with the plate. The experimental results (solid line) are the same as the theoretical results (dashed line) before the edge of the part comes into collision two or three times with the rubber plate.

Figure 4 shows the angular displacement of the prismatic part which rolls across the rubber plate. C/C_c is the damped ratio when the part bounces up and down under condition that two edges of the part contact with the plate. The circles (o) express the experimental results before the time when the rotational direction changes, and the experimental results are the same as the theoretical ones represented by the solid line.

Fig. 2 Prismatic part whose sectional shape is symmetric hexagon

Table 1 Parameters of prismatic part

Parameter	Value	Parameter	Value
a	m	m	kg
b	m	I	kgm ²
			1.2
			1.2×10^{-3}

Table 2 Parameters of plate

Parameter		Value	
		Acryl	Rubber
K	N/m	2.0×10^5	6.0×10^3
C	Ns/m	400	25
μ		0.21	0.50

Fig.3 Angular acceleration of part which rolls across rubber plate

Fig.4 Angular displacement of part which rolls across rubber plate

Fig.5 Angular displacement of part which rolls across acrylic plate

Figure 5 shows the angular displacement of the prismatic part which rolls across the acrylic plate. We shall express only the results when the slant angle of the plate is zero, because the acrylic part is easy to slip on the acrylic plate. The experimental results are the same as the theoretical ones.

From these results, it is clear that the theoretical results of the motion of the prismatic part, whose edges are replaced approximately by the spring and the damper, are the same as the experimental results before the edge of the part comes into collision two or three times with the plate.

4. Probability of the natural resting aspect of the parts

4.1 Relation between the motion of the rolling part and the dissipation of energy

Here, we shall define the dissipation of energy which is nondimensional as follows.

Table 3 Parameters of part and plate

Parameter	Value	Parameter	Value
a m	3.0×10^{-2}	K N/m	6.0×10^3
I kgm ²	2.9×10^{-4}	C Ns/m	23.0
m kg	0.56	μ	0.50

Fig.6 Motion of rolling part when ratio of length to side changes

$$E_c = \frac{E'_c}{mg(h_i - h_f)} \times 100 [\%] \dots \dots \dots (9)$$

$$E_\mu = \frac{E'_\mu}{mg(h_i - h_f)} \times 100 [\%] \dots \dots \dots (10)$$

where

E'_c : dissipation of energy when the prismatic part comes into collision with the plate

E'_μ : dissipation of energy when the prismatic part slips on the plate

h_i : initial height of the center of gravity of the prismatic part

h_f : final height of the center of gravity of the prismatic part

We shall take the parameters of the prismatic part, whose sectional shape is a symmetric hexagon, as shown in Table 3. When the initial conditions are $\theta = \theta_i$ and $\dot{\theta} = 0$, the theoretical results are as follows.

Figure 6 shows the motion of the rolling part when the ratio of the length of the side b/a changes. When edge A_3 contacts with the plate, the dissipation of energy E_c increases as b/a increases. And, the dissipation of energy E_μ is small.

Figure 7 shows the motion of the rolling part when the slant angle of the plate changes. The dissipation of energy E_μ increases as the slant angle of the plate increases.

From these results, it is clear that most of potential energy of the rolling

Fig.7 Motion of rolling part when slant angle of plate changes

part dissipates before the rotational direction of the part changes.

4.2 Parameters which influence the dissipation of energy

When the initial conditions are $\theta = \theta_i$ and $\dot{\theta} = 0$, the theoretical results of the rolling part, whose sectional shape is a regular hexagon ($a = 0.03\text{m}$, $m = 0.42\text{kg}$), are as follows.

Figure 8 shows the relation between the coefficient of friction and the dissipation of energy before the rotational direction of the part changes. When the coefficient of friction is smaller than 0.2, the dissipation of energy E_μ is large. When the coefficient of friction is larger than 0.4, most of potential energy of the part dissipates when the part comes into collision with the plate.

Figure 9 shows the relation between the damped ratio C/C_c and the dissipation

Fig.8 Relation between coefficient of friction and dissipation of energy

Fig.9 Relation between damped ratio and dissipation of energy

of energy. The ratio of the dissipation of energy E_μ/E_c decreases as the damped ratio increases.

Figure 10 shows the relation between the initial height of the center of gravity of the part and the dissipation of energy. When the coefficient of friction is large and the damped ratio is small, the dissipation of energy E_c increases as the height of the center of gravity increases, and it is expressed approximately as follows.

$$E_c \approx \alpha_1 h + \beta_1 \quad (\alpha_1, \beta_1 \text{ are constant})$$

When the coefficient of friction is small and the damped ratio is large, the dissipation of energy decreases as the height of the center of gravity increases, and it is expressed approximately as follows.

$$E_c \approx \frac{\alpha_2}{h} + \beta_2, \quad E_\mu \approx \frac{\alpha_3}{h} + \beta_3$$

($\alpha_2, \alpha_3, \beta_2, \beta_3$ are constant)

Figure 11 shows the relation between the ratio of the length to the side a/a_i ($a_i = 0.03 \text{ m}$) and the dissipation of energy. The dissipation of energy E_c increases as a/a_i increases, and it is expressed as follows.

$$E_c \approx \alpha_4 \frac{a}{a_i} + \beta_4 \quad (\alpha_4, \beta_4 \text{ are constant})$$

From these results, we can get an approximated expression for the dissipation of energy $E_c = E_{cm}gh$ as follows.

Fig.10 Relation between height of center of gravity and dissipation of energy

When μ is large and C/C_c is small,

$$E_c' = \alpha_5 h^2 \frac{a}{a_i} \dots\dots\dots (11)$$

When μ is small and C/C_c is large,

$$E_c' = \alpha_6 h \frac{a}{a_i} \dots\dots\dots (12)$$

(α_5, α_6 are constant)

4.3 Probability of the natural resting aspect of the rolling parts

Figure 12 shows a trajectory of the center of gravity of the rolling part whose sectional shape is a symmetric hexagon. The variations of the height of the center of gravity are expressed as follows.

$$h_a = d - a' = \frac{\sqrt{3a^2 + b^2}}{2} - \frac{\sqrt{3}}{4}(a+b) \dots\dots (13)$$

$$h_b = d - b' = \frac{\sqrt{3a^2 + b^2}}{2} - \frac{\sqrt{3}}{2}a \dots\dots\dots (14)$$

In the previous work⁽³⁾⁽⁴⁾, two methods were proposed to calculate the probability that the part will come to lie on side b as follows.

$$P_b = \frac{\left(\frac{b}{a}\right)^2 (\theta_{b1} + \theta_{b2})}{2(\theta_{a1} + \theta_{a2}) + \left(\frac{b}{a}\right)^2 (\theta_{b1} + \theta_{b2})} \dots\dots\dots (15)$$

$$P_b = \frac{h_b}{2h_a + h_b} \dots\dots\dots (16)$$

where

$$\frac{b}{a} < 1 \quad \text{then} \quad h'_a = \frac{a+b}{2} \left(1 - \frac{\sqrt{3}}{2}\right)$$

$$\frac{b}{a} \geq 1 \quad \text{then} \quad h'_a = h_a$$

Fig.11 Relation between ratio of length of side and dissipation of energy

Fig.12 Trajectory of center of gravity of rolling part

In these equations, the shape and the center of gravity of the parts are considered, but the difference in the probability when the parts drop onto different surfaces is not considered.

From equations (11) and (12), we shall propose the method to calculate the probability that the parts will come to lie on side b as follows.

When μ is small and C/C_c is large,

$$P_b = \frac{\left(\frac{b}{a}\right)h_b}{2h_a + \left(\frac{b}{a}\right)h_b} \quad (17)$$

When μ is large and C/C_c is small,

$$P_b = \frac{\left(\frac{b}{a}\right)h_b^2}{2h_a^2 + \left(\frac{b}{a}\right)h_b^2} \quad (18)$$

We shall investigate the probability of the natural resting aspect of the prismatic part whose sectional shape is a symmetric hexagon. The diameter of the circumscribed circle of the part is 0.03 m, and the length of the part is 0.16 m.

Fig.13 Probability of natural resting aspect

Figure 13 shows the results when the prismatic parts roll 600 times across the rubber or the acrylic plate. The experimental results ($\odot$ and $\circ$) agree with the theoretical results which are calculated from Eqs.(17) and (18).

From these results, it is clear that the method to calculate the probability of the natural resting aspect of the parts, which is derived in consideration of the dissipation of energy when the parts bounce up and down, is effective.

5. Conclusions

The results obtained in this paper are summarized as follows.

(1) The theoretical results about the motion of the prismatic part, whose edges are replaced approximately by the spring and the damper, are the same as the experimental results before the edge of the part comes into collision two or three times with the plate.

(2) It has been clarified that most of potential energy of the prismatic part dissipates before the rotational direction of the part changes.

(3) It has been clarified that the method to calculate the probability of the natural resting aspect of the parts, which is derived in consideration of the dissipation of energy when the parts bounce up and down, is effective.

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