A Measuring Theory of Impact Force Acting on a Bar with Two Different Diameters *

by Naoto YANAGIHARA** and Hiroshi SAITO ***

The true value of impact force, in high speed deformation test of metals, is commonly measured by the Hopkinson-bar method. But, a drawback to this method is the restriction of measurable time because of the length of the bar. One of the authors suggested constructing a measuring theory for impact force acting on a bar with uniform cross sectional area, based on the theory of the propagation of longitudinal elastic stress waves. From the stresses detected at two points on the bar, two stress waves travelling in opposite directions along the bar axis are separated, and the impact force is estimated, without the restriction of the measuring time. This paper presents a new theory for measuring the impact force which acts on a bar having two different diameters. The results of the experiments clearly confirm the validity and usefulness of this theory.

Key words: Shock, Measurement, One Dimensional Elastic Stress Wave, Different Cross Sectional Area, Use of Micro Computer

1. Introduction

For the accurate measurement of the impact force, such as the high speed deformation tests of metals, Split-Hopkinson-Pressure-Bar (SHPB) method is commonly used. But, this method has a drawback in that the measurable time is restricted by the length of the bar. Therefore, in the case where long measuring time is needed or the length of the bar is restricted, we cannot measure the impact force because of the influence of the stress wave reflections.

New theories were advanced by authors before, (1)-(3) i.e., theory for estimating the impact force on the bar end, and one for calculating the velocity and the displacement at that spot. In these theories the impact force and other quantities are estimated without the restriction of the measuring time, from the stresses detected at two points on the bar. Based on these theories, we completed a new measuring system, Yanagihara-bar method, by which the impact force, the velocity and the displacement at the bar end are able to be measured with a short bar. But, this method is only applicable to a bar with uniform cross-sectional area, not to a bar with varying one.

In this paper, a new measuring theory of the impact force which can be applied to a bar having two different cross-sectional areas, is suggested. Namely, this theory is proposed to

consider the influence of the reflecting and the transmitting stress waves at the discontinuous surface in Yanagihara-bar method. The theory is applicable to a bar having two diameters, where the impact force acts on the bar end and the strain gauges are attached on the points with another diameter. Usefulness of this theory was confirmed by the experiments. And in a high speed compression test of metals by SHPB method, the relations between the measuring errors and the ratios of the specimen diameters to pressure bar one, are also shown.

2. Theory of Separation of Stress Waves (1)

As shown in Fig.1, we take two points a and b with a distance l_1 on a bar having uniform cross-sectional area and made of homogeneous material. When the stress waves are travelling along the bar, two values of stress waves passing through the point a, from the left and right directions, at time T , are denoted by $\sigma_L(T)$ and $\sigma_R(T)$ respectively. The stresses at the point a and b, $\sigma_a(T)$ and $\sigma_b(T)$, are given by the following equations.

Fig.1 Stress waves in elastic bar.

* Received 7th March, 1985.

** Associate Professor, Department of Mechanical Engineering, Tamagawa Univ. Machida, Tokyo, 194 Japan.

*** Professor, Department of Mechanical Engineering, Tamagawa Univ.

$$\sigma_a(T) = \sigma_{La}(T) + \sigma_{Ra}(T) \dots\dots\dots(1)$$

$$\sigma_b(T) = \sigma_{La}(T - t_1) + \sigma_{Ra}(T + t_1) \dots\dots\dots(2)$$

where, t_1 is the travelling time of the stress wave between the points a and b. From Eq.(1) and Eq.(2) in which time is shifted, we can derive Eq.(3).

$$\sigma_{La}(T) = \sigma_a(T) - \sigma_b(T - t_1) + \sigma_{La}(T - 2t_1) \dots\dots\dots(3)$$

In Eq.(3), times are shifted by $2t_1$ successively, and thus we can get a derived equation. Substituting this derived equation into the equation which is obtained by shifting the time in Eq.(2) and rearranging, we can find the following equations which express the values of the stress waves from the left and right directions respectively.

$$\sigma_{La}(T) = \sum_{n=0}^K \sigma_a(T - 2nt_1) - \sum_{n=0}^K \sigma_b(T - t_1 - 2nt_1) + \sigma_{La}(T - 2(K+1)t_1) \dots\dots\dots(4)$$

$$\sigma_{Ra}(T) = \sum_{n=0}^K \sigma_b(T - t_1 - 2nt_1) - \sum_{n=1}^K \sigma_a(T - 2nt_1) - \sigma_{La}(T - 2(K+1)t_1) \dots\dots\dots(5)$$

3. Measuring Theory of Impact Force

In Fig.2, stress waves travelling on a bar having two different cross-sectional areas A_1 and A_2 , are shown. Where, $\sigma_{La}(T)$ and $\sigma_{Ra}(T)$ denote the stress waves passing through the point d from left and right directions, and $\sigma_{Lc}(T)$ and $\sigma_{Rc}(T)$ express the starting stress wave, in the right direction, and the arriving one from the same direction at the point c located at a discontinuous spot in diameter. And all these stress wave values are considered at time T .

When the travelling stress waves pass through the point c, some of the stress waves are reflected and the others are transmitted. In this case α_1 and α_2 , the reflection coefficients of the stress waves from right and left directions, and β_1 and β_2 , their transmission coefficients, are given by the following equations.(4)

Fig.2 Stress waves in a bar with different diameters.

$$\alpha_1 = \frac{A_2 - A_1}{A_1 + A_2} \dots\dots\dots(6)$$

$$\alpha_2 = \frac{A_1 - A_2}{A_1 + A_2} \dots\dots\dots(7)$$

$$\beta_1 = \frac{2A_1}{A_1 + A_2} \dots\dots\dots(8)$$

$$\beta_2 = \frac{2A_2}{A_1 + A_2} \dots\dots\dots(9)$$

where A_1 and A_2 denote the cross-sectional areas of the right and left side bars.

Assuming that the travelling time of the stress wave from the point d to c, with a distance l_3 is t_3 , then the value of stress wave $\sigma_{Lc}(T)$ which starts from the point c in the right direction at time T , equals the sum of the reflected stress wave at the point c, the one transmitted from the right direction and the transmitted stress wave passing through the point d, t_3 before. Therefore $\sigma_{Lc}(T)$ is shown by the next equation.

$$\sigma_{Lc}(T) = \sigma_{Rc}(T) \cdot \alpha_1 + \sigma_{Ld}(T - t_3) \cdot \beta_2 \dots\dots\dots(10)$$

The stress wave $\sigma_{Ld}(T)$ transmitted from the right and passing through the point d at the time T , equals the sum of the two stress waves, i.e., the stress wave at the point c transmitted from the right at time $T - t_3$, and the reflected stress wave, which was caused at the point c by the one transmitted from the left and passing through the point d at time $T - 2t_3$. Then, $\sigma_{Rd}(T)$ is given by the following equation.

$$\sigma_{Rd}(T) = \sigma_{Rc}(T - t_3) \cdot \beta_1 + \sigma_{Ld}(T - 2t_3) \cdot \alpha_2 \dots\dots\dots(11)$$

Substituting the time-shifted equation derived from Eq.(10) into Eq.(11) and rearranging, we have the following equation.

$$\sigma_{Rd}(T) = \frac{1}{\beta_2} \{ \sigma_{Rc}(T - t_3) \cdot (\beta_1 \cdot \beta_2 - \alpha_1 \cdot \alpha_2) + \sigma_{Lc}(T - t_3) \cdot \alpha_2 \} \dots\dots\dots(12)$$

Considering the relation $\beta_1 \cdot \beta_2 - \alpha_1 \cdot \alpha_2 = 1$, Eq.(12) is rewritten as

$$\sigma_{Rd}(T) = \frac{1}{\beta_2} \{ \sigma_{Rc}(T - t_3) + \sigma_{Lc}(T - t_3) \cdot \alpha_2 \} \dots\dots\dots(13)$$

Shifting the time by t_3 in Eq.(10), we have

$$\sigma_{Ld}(T) = \frac{1}{\beta_2} \{ \sigma_{Lc}(T + t_3) - \sigma_{Rc}(T + t_3) \cdot \alpha_1 \} \dots\dots\dots(14)$$

The stress at the point d at time T equals the sum of the stresses due to the waves of opposite directions, at that time, and substituting the relation $\alpha_2 = -\alpha_1$ into the above equation, we have

$$\begin{aligned}\sigma_d(T) &= \sigma_{Ld}(T) + \sigma_{Ra}(T) \\ &= \frac{1}{\beta_2} \{ \sigma_{Lc}(T+t_3) + \sigma_{Rc}(T-t_3) \\ &\quad + [\sigma_{Rc}(T+t_3) + \sigma_{Lc}(T-t_3)] \cdot \alpha_2 \} \dots\dots\dots (15)\end{aligned}$$

When l_2 and l_3 denote the distances from the point c to a and to d, respectively, we have $l_2 = l_3$ then $t_2 = t_3$. And these stress waves are expressed by the ones passing through the point a as follows.

$$\begin{aligned}\sigma_d(T) &= \frac{1}{\beta_2} \{ \sigma_{La}(T+2t_3) + \sigma_{Ra}(T-2t_3) \\ &\quad + [\sigma_{Ra}(T) + \sigma_{La}(T)] \cdot \alpha_2 \} \dots\dots\dots (16)\end{aligned}$$

When l_1 , the distance from the point a to b, equals to l_3 , then we have $t_3 = t_1$. And substituting the shifted equations derived from Eqs.(4) and (5) into Eq.(16), rearranging, and using $1 + \alpha_2 = \beta_1$, we have

$$\begin{aligned}\sigma_d(T) &= \frac{1}{\beta_2} \{ \sigma_a(T+2t_1) - \sigma_b(T+t_1) + \sigma_a(T) \cdot \beta_1 \\ &\quad - \sigma_b(T-t_1) + \sigma_a(T-2t_1) \} \dots\dots\dots (17)\end{aligned}$$

In the above equation, if the bar has uniform cross-sectional area, then $\beta_2 = \beta_1 = 1$.

When $l_3 = l_1$ and $l_2 = 2l_1$, then $t_3 = t_1$ and $t_2 = 2t_1$. Rearranging the equation in the same way, we get

$$\begin{aligned}\sigma_d(T) &= \frac{1}{\beta_2} \{ \sigma_a(T+3t_1) - \sigma_b(T+2t_1) \\ &\quad + [\sigma_a(T+t_1) - \sigma_b(T) + \sigma_a(T-t_1)] \cdot \beta_1 \\ &\quad - \sigma_b(T-2t_1) + \sigma_a(T-3t_1) \} \dots\dots\dots (18)\end{aligned}$$

In the case of $l_3 = l_1$ and $l_2 = kl_1$, in which k denotes an integral number larger than unity, the stress at the point d is shown as follows

$$\begin{aligned}\sigma_d(T) &= \frac{1}{\beta_2} \{ \sigma_a(T+[k+1]t_1) - \sigma_b(T+kt_1) \\ &\quad + \left[\sum_{n=0}^{k-1} \sigma_a(T+[k-1-2n]t_1) \right. \\ &\quad \left. - \sum_{n=1}^{k-1} \sigma_b(T+[k-2n]t_1) \right] \cdot \beta_1 \\ &\quad \left. - \sigma_b(T-kt_1) + \sigma_a(T-[k+1]t_1) \right\} \dots\dots\dots (19)\end{aligned}$$

If $l_3 = 2l_1$ and $l_2 = l_1$, then $t_3 = 2t_1$ and $t_2 = t_1$, we have the next equation by substituting $t_3 = 2t_1$, in Eq. (15).

$$\begin{aligned}\sigma_d(T) &= \frac{1}{\beta_2} \{ \sigma_{Lc}(T+2t_1) + \sigma_{Rc}(T-2t_1) \\ &\quad + [\sigma_{Rc}(T+2t_1) + \sigma_{Lc}(T-2t_1)] \cdot \alpha_2 \} \dots\dots\dots (20)\end{aligned}$$

Rewriting the right hand side of the above equation with the stress wave passing through the point a, then we have

$$\begin{aligned}\sigma_d(T) &= \frac{1}{\beta_2} \{ \sigma_{La}(T+3t_1) + \sigma_{Ra}(T-3t_1) \\ &\quad + [\sigma_{Ra}(T+t_1) + \sigma_{La}(T-t_1)] \cdot \alpha_2 \} \dots\dots\dots (21)\end{aligned}$$

Putting the shifted time equation derived from Eq.(4) and (5) into the above equation, considering the relation $1 + \alpha_1 = \beta_1$, and rearranging it, we get

$$\begin{aligned}\sigma_d(T) &= \frac{1}{\beta_2} \{ \sigma_a(T+3t_1) - \sigma_b(T+2t_1) \\ &\quad + \sigma_a(T+t_1) - \sigma_b(T) \cdot \beta_1 + \sigma_a(T-t_1) \\ &\quad - \sigma_b(T-2t_1) + \sigma_a(T-3t_1) \} \dots\dots\dots (22)\end{aligned}$$

When k is an integral number larger than 2, and $l_3 = kl_1$, $l_2 = l_1$, $t_3 = kt_1$ and $t_2 = t_1$. Rearranging the equation in the same way, Eq.(22) is rewritten as follows.

$$\begin{aligned}\sigma_d(T) &= \frac{1}{\beta_2} \{ \sigma_a(T+[k+1]t_1) - \sigma_b(T+kt_1) \\ &\quad + \sigma_a(T+[k-1]t_1) \\ &\quad + \left[\sum_{n=1}^{k-2} \sigma_a(T+[k-1-2n]t_1) \right. \\ &\quad \left. - \sum_{n=1}^{k-1} \sigma_b(T+[k-2n]t_1) \right] \cdot \beta_1 \\ &\quad + \sigma_a(T-[k-1]t_1) - \sigma_b(T-kt_1) \\ &\quad + \sigma_a(T-[k+1]t_1) \} \dots\dots\dots (23)\end{aligned}$$

Thus, using stresses detected at the two points on the bar, the stress at the point d is found even in the case of the bar having two different diameters, i.e., the impact force acts on the bar with one diameter and strain gauges are on another diameter. In this case, the frequency of calculation increases with the distance from the point d to the measuring point. Of course, these equations are able to be applied to a case of the point d locating at the end of the bar.

4. Confirmation of the Theory by Impact Test

To confirm the above mentioned theory, the impact tests were carried out with the test equipment shown in Fig.3. The measuring bar for the impact force and the impact bar are set on the same axis with the guide blocks and the accelerator - the air-gun -. The impact bar is a round steel bar of 25mm in diameter and 400mm in length. The measuring bar is made of tool steel having diameter $D=25$ mm and length 270mm. As shown in Fig.4, the stress measuring bar has the following dimensions, i.e., the diameter D in the measuring portion is 25mm, and in the end

Fig.3 Block diagram of the measuring system of the impact test.

portion with the length 30.6mm, the diameters d are varied from 40 to 10mm, then the ratios between d and D are $d/D = 0.4$ to 1.6. The strain gauges are mounted at the measuring points a and b on the bar with the distances 30.6mm and $30.6 + 30.6$ mm respectively from the stepped end. The semiconductor strain gauges, P and N type, of 2mm in gauge length, are used. Two pairs of gauges have N and P attached on the symmetric surface at each points a and b . In the bridge boxes 4-gauge circuits are housed and the output signals from the bridge boxes are fed to a digital oscilloscope, in which the signals are sampled with 1 us intervals, and memorized. The oscilloscope has a performance of 150KHz frequency bandwidth. The memorized signals in that oscilloscope are sent to the micro-computer by RS-232C method at baud rate 2400, and are calculated by the theoretical equation. The calculated results are recorded on the printer. In the experiments, output voltage from the bridge circuits is 14mV per 10 MPa of stress of measuring bar.

On the upper side of Fig.5, curves of stress measured at the points a and b , in which the tail of the measuring bar of the ratio $d/D=0.8$ is struck by the impact bar with the velocity of 4m/s, are shown. And lower side of this figure shows the stress curves at the end of the measuring bar calculated from the upper stress curves. In the experiments the end surface of the measuring bar is stress-free; this means

Fig.4 Gauge positions and shape of the measuring bar.

Fig.5 Measured stresses at the points a and b , and calculated stresses on the top of the bar.

the stresses at this point are to be zero. But, the calculated stresses fluctuate a little and show some error. When the stress fluctuations at the points a and b are comparatively large, the calculated stress curves at the end of the measuring bar are as shown in the lower side of Fig.5, in which the ratios d/D are taken 0.7 and 0.6 and the errors increase as the ratio d/D decreases.

From the stresses measured at the points a and b , the impact force $F_c(T)$ acting on the stepped end of the measuring bar is calculated by the following equation (1).

$$F_c(T) = \{\sigma_a(T+t_1) - \sigma_b(T) + \sigma_a(T-t_1)\} \cdot A_1 \quad \dots\dots\dots (24)$$

We consider that the impact condition at the stepped end is identical to the one, in which the end of the bar having the diameter D is subjected to the impact force by the area with the diameter d . To examine the accuracy of this measuring method, we take the ratio $\Delta F_{dmax} / F_{cmax}$, where ΔF_{dmax} is the maximum error of the calculated impact force, $-F_d(T) = \sigma_d(T) \cdot A_2$ at the end of the bar, and F_{cmax} is the maximum impact force at the stepped end. The results are shown in Fig.6. In Fig.6 the maximum values of errors are under 3% for the ratio $d/D=0.8-1.2$ and less than 5% for $d/D=0.7-1.3$. Therefore, if the difference of diameter between d and D is comparatively small, this method is applicable to measure the impact force satisfactorily. Besides, the rise time of the stress is about 12us. In Fig.4, when the two measuring points a and b are shifted to the right side by 30.6mm, the error of the impact force calculated by Eq.(18) increases by about 0.5% over that of the previous measurement. From the experiments, we see that the one-dimensional elastic wave theory is well applicable to a bar with two different diameters.

We consider that the curve in Fig.6 shows the range of the errors contained in the measured values when the specimens of

Fig.6 Relation between error of measured stress and ratio d/D of two diameters of bar.

dmm in diameter are compressed at high speed by a bar of Dmm in diameter. In this condition, the rise time of the stress equals $12\mu\text{s}$, and the distance between the measuring point and the bar end surface is taken 1.2 times the diameter of the bar. And Fig.6. shows that the error values increase about 1% for $d/D=0.8$ and about 3% for $d/D=0.7$ compared to the error for $d/D=1$.

Dr. Fukazu and et al., discussed about this problem, (5) i.e., in the case of d/D varying, even if the ratio is taken $d/D=0.4$, the stresses distribute uniformly on the cross section more than D mm apart from the end of bar. From the experiments based on this measuring theory, also in the measurement of the impact stress based on the one-dimensional elastic stress wave theory, the influences of the ratio d/D on the measured impact forces are confirmed to be small.

5. Conclusions.

A theoretical equation for the impact forces, acting on the top surface and on any cross-sections including the discontinuous one, is obtained. The validity of the theory, and the measuring accuracy of Yanagihara-bar method based on this theory, are confirmed with the experiments. The measuring errors in the high speed compression test, in which the ratios between the pressure bar diameters and the specimen ones are changed, are also shown.

The main results are as follows.

Under the condition that the rise time of the stress is longer than $12\mu\text{s}$,
 (1) when the bar has two different diameters, $D(=25\text{mm})$ and dmm, and the impact force acts on the end of the bar with dmm in diameter, the measuring errors are within the range of about 3% for $d/D=0.8-1.2$ and about 5% for $d/D=0.7-1.3$,
 (2) in the high speed compression tests of metals, the measuring errors caused by the decrease of the ratio d/D , where d and D denote the diameters of the specimen and the pressure bar, increase within the range of about 1% for $d/D=0.8$ and about 3% for $d/D=0.7$.

Acknowledgement: We would like to thank Mr. Toshiki Yamaguchi [The Hohchiki co.] for his cooperation in the experiments, and also Mr. Takeshi Sotozaki, Mr. Juichi Yasuike [Engineers, Tamagawa Univ.] for preparing the equipment.

References

- (1). Yanagihara, N., Bulletin of JSME Vol.21, No.157 (1978-7) p.1085.
- (2). Yanagihara, N., et al., ibid., Vol.27, No.175 (1980-1), p.140.
- (3). Yanagihara, N., et al., Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol.29, No.464, C (1985-4), p.790
- (4). Matumoto, H., et al., ibid., Vol.29, No.197 (1963-1), p.49.
- (5). Fukazu, K., et al., ibid., Vol.50, No.453, A (1984-5), p.869.