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On the Thicknesses of Smooth Jets Discharged from a Fixed and a Rotating Nozzle*

By Tosio KURABAYASI**

The sizes of droplets produced from a continuous liquid jet depend mainly on its thickness. This paper, deals with the contractions of smooth liquid jets discharged from a fixed and a rotating nozzle through acceleration and surface tension. Formulae for the thicknesses of jets are derived theoretically, and the condition is found under which the influences of physical properties of liquid on the thickness become negligibly small. Relationship between the cases of a fixed nozzle and a rotating one is also discussed. The values computed from the formulae show a good agreement with those experimentally obtained.

The results may also be applicable to filamentation of viscous liquid by means of a rotating nozzle.

1. Introduction

As shown in the references(1), the thickness of a liquid jet affects not only the sizes of droplets produced therefrom, but also the transition point of disintegration mechanism.

This paper deals with the contracting phenomena through acceleration and surface tension force, of smooth liquid jets discharged from a fixed and a rotating nozzle.

The results obtained may also be useful in filamentation of viscous liquid by means of a rotating nozzle.

2. Theoretical analyses

2.1 Case of fixed nozzle

A liquid jet discharged from a vertically fixed nozzle is contracted through surface tension and acceleration of gravity.

A unit volume of a jet with the length Δx_0 , the diameter D_0 and the velocity v_0 in Fig. 1 has the following energies at the position I at a distance x_0 from the nozzle :

- (i) velocity energy $\rho v_0^2/2$
- (ii) surface tension energy $\pi D_0 \sigma \Delta x_0$
- (iii) potential energy measured from the nozzle $-\rho g x_0$

where ρ is a density and σ is a surface tension of the liquid.

The sum of these energies should not be

changed at any other position, for instance, II at a distance x . Hence equalizing both, we have

$$\rho g(x-x_0) = \pi \sigma (D \Delta x - D_0 \Delta x_0) + \rho(v^2 - v_0^2)/2 \quad \dots\dots\dots (1)$$

Substituting the relations $\pi D^2 \Delta x/4 = \pi D_0^2 \Delta x_0/4 = 1$ and $v D^2 = v_0 D_0^2$ into the Eq. (1) and bringing the position I to the nozzle end, and then putting $x_0=0$, $D_0=D_n$ and $v_0=v_n$, the following result may be obtained

$$\frac{2gx}{v_n^2} = \left(\frac{D_n}{D}\right)^4 - 1 + 8 \left\{ \left(\frac{D_n}{D} - 1\right) / W_e^2 \right\} \quad \dots\dots\dots (2)$$

$$W_e = \frac{v_n \sqrt{D_n}}{\sqrt{g}} \left(\frac{\gamma}{\sigma}\right)^{1/2}$$

where $\gamma = \rho g$; and W_e is a Weber number, within which, as seen, all the physical properties of liquid are included.

From this Eq. (2) the thickness of a jet D at any point x is easily obtained for given values of

Fig. 1 Contraction of the jet from a fixed nozzle

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D_n , v_n , γ and σ .

The relations between the dimensionless values $Y=2gx/v_n^2$ and $X=D/D_n$ for various values of We are represented in Fig. 2. It is easily seen from this figure that as the value of We is increased, its influence on X , a representative value of thickness, becomes negligibly small and the curves finally tend to converge to $We=\infty$.

Hence when the value of We is large, the formula obtained above may be reduced by neglecting the term including We to a simpler form

$$\frac{2gx}{v_n^2} = \left(\frac{D_n}{D}\right)^4 - 1 \dots\dots\dots (3)$$

In order to check the approximation of this equation, the values of We giving an error of 1 per cent are calculated by using a relation

$$(X-X')/X=0.01$$

where X is the value of D/D_n obtained by the Eq. (2) and X' is the approximate one obtained by the Eq. (3). The results are shown in Fig. 3. They show that when We exceeds nearly the value of 4, the influences of any physical properties of liquid become negligibly small and the thickness of a jet can be given by the simple form (3).

As for the value of a continuous jet length l_j Tanasawa and Toyoda have given the following formula

$$\frac{l_j}{D_n} = 22.4 \left\{ (v_n - v_k) \sqrt{\frac{\gamma D_n}{\sigma g}} \right\}^{0.8} \left(1 + \frac{1}{250} \frac{1}{\mu} \sqrt{\frac{\sigma \gamma D_n}{g}} \right) \dots\dots\dots (4)$$

where μ is the viscosity and v_k is the critical discharge velocity defined by

$$v_k = \frac{0.99}{D_n} \left(\frac{\sigma}{\gamma} \right)^{3/4} \sqrt{g} + \frac{0.45}{\sqrt{D_n}} \left(\frac{\sigma}{\gamma} \right)^{2/4} \sqrt{g} - \frac{0.57}{\sqrt[4]{D_n}} \left(\frac{\sigma}{\gamma} \right)^{3/8} \sqrt{g} \dots\dots\dots (5)$$

Hence substituting the value of l_j from this formula into the Eq. (2) and assuming that the liquid column is simply cut off at a breaking point without being accompanied by the vibrational deformation, we can obtain the terminal diameter of the liquid column D_t . In Fig. 4 its dimensionless representative value D_t/D_n is plotted against the velocity ratio v_n/v_k for the cases of water and kerosine ($\sigma=24$ dyne/cm, $\mu=1.24$ c. P. and $\gamma=0.89$ g/cm³). An inspection of these curves shows that when the discharge velocity v_n becomes nearly equal to twice the value of the critical velocity v_k , D_t becomes minimum, hence at this point minimum droplets may be produced, assuming that a jet is broken up with the wave length proportional to D_t .

2.2 Case of rotating nozzle

Taking account of the fact that in the case of a rotating nozzle the value of We , in general,

Fig. 2 Variation of $2gx/v_n^2$ as a function of D/D_n for various values of We

Fig. 3 Value of We which produces error of 1 per cent

does not become so small when a continuous jet is formed, we shall in the beginning derive a formula which neglects the effect of surface tension force.

In Fig. 5, N is the present position of a rotating nozzle, D and E are its former positions with angle θ and $\theta+d\theta$ respectively; R a rotating radius and ω an angular velocity. A part of liquid being discharged at a position, for instance, E , flies in the ambient air with the resultant velocity

v of a discharge velocity v_n and a peripheral velocity v_p , and at present it has arrived at the point A.

Hence a small elongation of the jet $dl=AB$ during the interval dt is given by

$$dl = \sqrt{AC^2 + BC^2} \dots\dots\dots (6)$$

Putting $BD = \rho = vt \doteq CF$ and $GD \parallel AE$,

$$AC = d\rho - EF = (v - R\omega \cos \alpha)dt$$

In the same way,

$$BC = (\rho\omega + R\omega \sin \alpha)dt$$

Substituting these relations into the Eq. (6) and using the relations $R\omega = v_p$, $\cos \alpha = v_p/v$ and $\sin \alpha = v_n/v$, we have

$$\frac{v_j}{v_n} = \left[1 + 2\omega t \left(\frac{v_p}{v_n} \right) + (\omega t)^2 \left\{ 1 + \left(\frac{v_p}{v_n} \right)^2 \right\} \right]^{1/2} \dots\dots\dots (7)$$

Combining the condition of continuity $D_j^2 v_j = D_n^2 v_n$ with this equation we obtain

$$\frac{D_j}{D_n} = \left[1 + 2\omega t \left(\frac{v_p}{v_n} \right) + (\omega t)^2 \left\{ 1 + \left(\frac{v_p}{v_n} \right)^2 \right\} \right]^{-1/4} \dots\dots\dots (8)$$

or when $\omega t < 1$, approximately it becomes like this,

$$\frac{D_j}{D_n} \doteq 1 - \frac{1}{2} \left(\frac{v_p}{v_n} \right) \omega t \dots\dots\dots (9)$$

where D_j and v_j are the values of a jet.

Integrating the Eq. (7) we can obtain the length of a jet also approximately as,

$$l \doteq \frac{R}{(v_p/v_n)} \left\{ 1 + \frac{1}{2} \left(\frac{v_p}{v_n} \right) \omega t \right\} \dots\dots\dots (10)$$

In order to check the influence of neglect of the term with higher order of ωt , the comparison of the approximate values which are obtained by $v_j/v_n = 1 + (v_p/v_n)\omega t \dots\dots\dots (11)$ with those by the Eq. (7) is made in Fig. 6. From this figure we can see that, in the case of $v_p/v_n > 2$, errors become smaller than 5 per cent even in $\omega t = 1$.

With these equations the thickness of a jet D_j is obtainable for various positions on a jet.

Differentiating the Eq. (11) we have an acceleration along a jet as

$$\alpha = dv_j/dt \doteq v_p \omega \dots\dots\dots (12)$$

And when ωt is sufficiently small, the Eq. (10) gives

$$l \doteq vt$$

Combining these relations with an approximately reduced form of the Eq. (8), we have

$$(D_n/D_j)^4 \doteq 1 + (2\alpha l/v_n^2) \dots\dots\dots (13)$$

We can easily see that the form of this equation coincides with that of the Eq. (3), and that both cases of a fixed and a rotating nozzle do not differ in principle. Therefore the formula considering the effects of liquid properties may be derived from the Eq. (2) only by replacing g by variable acceleration α which is given from the Eq. (12) or more strictly from

$$\alpha = v_n \omega \left(\frac{v_p}{v_n} + \omega t \right) \dots\dots\dots (14)$$

Fig. 4 Relationship between terminal diameter and discharge velocity

Fig. 5 Liquid jet from a rotating nozzle

Fig. 6 Comparison of the approximate Eq. (11) and Eq. (7)

(a) $v_n = 44.4$ cm/sec, water

(b) $v_n = 3.38$ cm/sec, $\sigma = 42$ dyne/cm

Fig. 8 Comparison of theoretical and experimental results in the case of a rotating nozzle

Fig. 9 Water jet from a fixed nozzle with a thick wall

as

$$\frac{2\alpha l}{v_n^2} = \left(\frac{D_n}{D_j}\right)^4 - 1 + 8 \left\{ \left(\frac{D_n}{D_j} - 1\right) / W_e^2 \right\} \dots (15)$$

3. Comparisons of theoretical and experimental results

As shown in Fig. 7 (case of a fixed nozzle) or in Fig. 8 (case of a rotating nozzle), the plotted points fairly lie on the curves computed from the Eq. (2) or (15). Comparisons are also made for the case of a fixed nozzle in Fig. 9 when a pipe wall is thick. The difference between the curves associated with outer and inner diameter is not so large especially at some distance from the nozzle, and it has not been confirmed in our experiment what diameter should be taken to define the form

of a jet. Configurations of the jets corresponding to this case are shown in Fig. 10.

4. Conclusions

As a result of an investigation of the contracting phenomena of a liquid jet through acceleration and surface tension force, the following conclusions were drawn :

1. Formulae for a fixed and a rotating nozzle have almost the same form as the Eqs. (2) and (15), and if we replace the variable acceleration α in the Eq. (15) with the constant acceleration g , the Eq. (2) can be derived directly.

2. Physical properties of a liquid are all included in the form of Weber number W_e . With

Fig. 10 Configurations of water jets

the increase in this value, these effects on the thickness gradually decrease, and when it exceeds the value of 4 or so, they become negligibly small and the Eq. (2) or (15) can be reduced to the Eq. (3) or (13) respectively.

3. The values thus obtained theoretically

show a good agreement with the values experimentally obtained.

References

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On the Coefficient of Heat Transfer between Gases and Cylinder Walls of the Spark-Ignition Engine*

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This investigation is concerned with the rates of heat transfer from the working gases to the combustion chamber walls of the internal combustion engines. The test engine used is a four stroke spark-ignition engine with a water cooled single cylinder, bore×stroke being 114.3×140 mm.

On the basis of author's experimental results, it was ascertained that the coefficients of heat transfer in this engine agreed well with those which were calculated by Eichelberg's formula — $\alpha = 2.1 \sqrt[3]{C_m} \sqrt{pT}$ — in the expansion stroke, but did not so well in the compression stroke.

Law of similarity has been rarely applied to the problems of periodic heat transfer in the internal combustion engines, but recently Karl Elser developed this research. In the present report, improving Elser's method, the relation between the dimensionless groups was found as follows,

$$Nu/\sqrt{P_e'} = 1.75(1 + \Delta s/c_p) \{2 + \cos(\theta - 20^\circ)\}$$

where

Δs : entropy change

c_p : specific heat under constant pressure change

θ : crank angle

1. Introduction

W. Nusselt⁽¹⁾ and G. Eichelberg⁽²⁾ each advocated their own famous empirical formulae of the coefficient of heat transfer between the gases and the cylinder wall. Nusselt's formula is based on the experiments of combustion in the pressure vessel, and Eichelberg's is based on the experiments of heat transfer in Diesel engines. The latter can be used conveniently, but its basic experiments

do not always cover the wide range of engine dimensions and of running condition of engines. The accuracies of these formulae were only examined by the investigators who were under Eichelberg's tuition. The author has determined the coefficients of heat transfer by means of the measurement of the periodic change of temperature of piston as mentioned in the previous report⁽³⁾, and has compared them with those calculated by Eichelberg's formula.

In the latter half of the present report he has tried to find the relation between the dimensionless groups applying the law of similarity to the non-

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