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Theory of Heat Transfer in the Working Gases of Internal Combustion Engines*

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A great many theoretical researches are reported on the periodic heat transmission in the internal combustion engines. Many of them discuss the problem only from the point of view of heat conduction in the solid wall, but there are few reports on the theory of heat transfer in the working gases. H. Pfriem and K. Elser have derived the energy equation of the gases especially in the boundary layer near the cylinder wall and solved it. But they have calculated only the heat transferred from the gas space to the wall under the condition that there were not any heat sources in the gases and the adiabatic compression and expansion were repeated.

It is the object of the present paper to introduce the term of heat quantity in the energy equation by applying the conception of the polytropic change. By this means the heat flow from the gases to the wall of combustion chamber can be calculated when the gas cycle is quite similar to the spark-ignition engine. In the latter part of the present report, the numerical calculation is made, and the calculated results are compared with the experimental results.

1. Introduction

The theory of heat conduction in the solid wall of combustion chamber was developed early as the theoretical investigations of the heat transfer in the internal combustion engines, and in Japan A. Kobayashi investigated about the subject in detail⁽¹⁾. However there are many pending questions on the theory of heat transfer in the cylinder gases. The investigation of the subject was attempted at first by A. Herzfeld⁽²⁾ but worthy results were not obtained. H. Pfriem investigated widely the problems of nonsteady heat transfer from the working gases and laid the foundation to this theme^{(4)~(8)}. Recently K. Elser studied it improving Pfriem's results, and was able to compare the results of calculation with his own experiments^{(9)~(10)}. Both of them, however, calculated only the heat flow to the metallic wall from the gas zone which repeated adiabatic compression and expansion.

It is the object of the present paper to extend the results mentioned above and to calculate theoretically the quantities of the heat transferred to the metallic wall from the gas zone which

repeats the change of pressure and temperature quite similar to the working gas of internal combustion engine. Namely, the changes in the state of gas are the adiabatic compression from the bottom dead center to the top dead center and the adiabatic expansion from T. D. C. to B. D. C. and in the vicinity of T. D. C. the heat is generated almost under constant volume like in actual engine.

2. Derivation of fundamental equation

The following nomenclature it used in the paper :

$v=1/\rho(\text{m}^3/\text{kg})$: specific volume of the gas particles observed

$p(\text{kg}/\text{m}^2)$: gas pressure

$T(^{\circ}\text{K})$: temperature

$c_p(\text{kcal}/^{\circ}\text{K kg})$: specific heat of the gas under constant pressure change

κ : adiabatic constant

n : polytropic index

$R(\text{kg-m}/^{\circ}\text{K kg})$: gas constant

$W(\text{kcal}/\text{m}^3 \text{ hr})$: quantity of heat generated in the unit-time and the unit-volume of gas particles observed

i : unit of imaginary number

$\omega(\text{rad}/\text{hr})$: angular frequency

$t(\text{hr})$: time

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In deriving the fundamental equation it is assumed that :

(a) The radiant-heat is completely negligible,
 (b) the working gases are the ideal gas,
 (c) the principal phenomena of heat transfer between gases and wall present themselves in the thin layer near the wall, and the thickness of the layer is very small compared with the radius of curvature of the wall surface,

(d) x -axis of the rectangular co-ordinates is outwardly perpendicular to the surface of wall and y and z -axes in parallel with the surface. Then the motion of gas particles in the direction of x -axis (w_x) is assumed to be predominant in accordance with the change of specific volume. On the contrary the velocity components w_y and w_z are negligible as compared with w_x . So the intake and exhaust strokes are excluded since in these periods w_y or w_z are considered to be considerably large.

(e) In the zone of the working gases the pressure wave does not happen, so the value of pressure is dependent upon the time t only.

(f) In the region far-away from the wall, the temperature of gas is not influenced by the wall temperature and the polytropic change with index n goes on uniformly. As the wall is approached, however, the temperature differs gradually from the temperature of polytropic change and at last equals the wall temperature.

(g) The polytropic index n is a function of time or crank angle only, and its value is very large in the short interval before and after the T.D.C., and in the other greater part it equals the adiabatic constant κ . On these assumption the following energy-equation may be obtained in respect to the unit mass of gas particles,

$$\left(\frac{\partial T}{\partial t}\right)_x + w_x \left(\frac{\partial T}{\partial x}\right)_t - \frac{\kappa-1}{\kappa} \frac{T}{p} \left(\frac{\partial p}{\partial t}\right)_x = \frac{v}{c_p} \left[\left\{ \frac{\partial \left(\lambda \frac{\partial T}{\partial x} \right)}{\partial x} \right\}_t + W(x, t) \right] \quad \dots\dots\dots (1)$$

This equation is complicated by the term of w_x , but if the co-ordinate of mass particle (co-ordinate of Lagrange's method)* is used in place of the ordinary co-ordinate of space (co-ordinate of Euler's method)**, the trouble can be eliminated. When the specified particle of gas is indicated by the co-ordinate of mass particle ξ , the Eq. (1) is expressed as follows

$$\left(\frac{\partial T}{\partial t}\right)_\xi - \frac{\kappa-1}{\kappa} \frac{T}{p} \left(\frac{\partial p}{\partial t}\right)_\xi = \frac{v}{c_p} \left[\left\{ \frac{\partial \left(\lambda \frac{\partial T}{\partial \xi} \frac{d\xi}{dx} \right)}{\partial \xi} \right\}_t \left(\frac{d\xi}{dx}\right) + W(x, t) \right] \quad \dots\dots\dots (2)$$

where it must be assumed that an observer is moving with the particle observed. If the gas particle of plane sheet parallel with the surface of wall is viewed, and its mass is constant and its depth dx is variable, then the latter is in proportion with the specific volume of gas. So if the depth is assumed to be $d\xi$ under the standard condition which can be selected properly, the next relation can be obtained

$$d\xi/dx = v_1/v \quad \dots\dots\dots (3)$$

where the suffix 1 shows the quantities at the standard state. Using the relation $W(\xi, t) = vW(x, t)$ (kcal/kg hr) which shows the quantities of the heat generated in the gas particles of unit mass, the following may be obtained ;

$$\left(\frac{\partial T}{\partial t}\right)_\xi - \frac{\kappa-1}{\lambda} \frac{T}{p} \left(\frac{\partial p}{\partial t}\right)_\xi = \frac{v_1^2}{c_p} \frac{\partial \left(\frac{\lambda}{v} \frac{\partial T}{\partial \xi} \right)}{\partial \xi} + \frac{1}{c_p} W(\xi, t) \quad \dots\dots\dots (4)$$

Introducing the next parameter

$$\sigma = \frac{T}{T_1} \left(\frac{p}{p_1} \right)^{-(n-1)/n} \quad \dots\dots\dots (5)$$

the two dependent variables can be reduced to one, where σ must be considered in general as the function of time and position of particles. Giving up the idea of discussing the general change of n , it is considered only that the change of state is quite similar to the Otto cycle. Namely, in the uniform zone far-away from the wall, the change in the state of gas is an adiabatic one ($n=\kappa$) on both compression and expansion strokes except in the vicinity of T.D.C., and it is similar to the constant volume change near the T.D.C. (Fig. 1). If it is assumed that the absolute value of n there becomes very large, but its rate of change with respect to time stays at least in the same order of n , and the term of heat generated $W(\xi, t)$ is expressed independently of ξ as follows

$$W(\xi, t) = \frac{n-\kappa}{n-1} c_v \frac{\partial T}{\partial t} \text{ (kcal/kg hr)} \quad \dots\dots\dots (6)$$

next equation may be obtained

$$\frac{p_1}{p} \frac{\partial \sigma}{\partial t} = \frac{\kappa(n-1)}{n(\kappa-1)} \frac{v_1 T_1}{c_p} \frac{\partial \left(\frac{\lambda}{T} \frac{\partial \sigma}{\partial \xi} \right)}{\partial \xi} \quad \dots\dots\dots (7)$$

This non-linear second order differential equation shows the plane field of temperature of the perfect gas in the co-ordinate of mass particles. And if

* Massenpunktkoordinate, Lagrangeschekoordinate.

** Raumpunktkoordinate, Eulerschekoordinate.

n is taken here as κ i.e. $W(\xi, t) = 0$, the equation agrees perfectly with the one by Pfriem and Elser.

3. Assumption for changing the differential equation into linear form

As it is imaginable that the quantity of the heat transmitted in the swirling state of gases is larger than the value of product of the pure thermal conductivity multiplied by the temperature inclination there, it may be assumed that the apparent thermal conductivity λ is the sum of the pure conductivity λ_m and the rate of transfer by convection λ_t . So the following may be obtained,

$$\lambda = \lambda_m + \lambda_t = \lambda_m (1 + \lambda_t / \lambda_m) \quad \dots\dots\dots (8)$$

λ_m is known as the function of temperature only and independent of the pressure of gas in a wide range and λ_t may be estimated as the function of distance from the wall.

After various trials⁽¹¹⁾ about λ_t the author decided to follow Elser's estimation. Namely

$$\frac{\lambda}{\lambda_1} = \frac{T}{T_1} \left\{ 1 + \left(\frac{p}{p_1} \right)^{-1/2} \frac{\xi}{\xi_1} \right\} \quad \dots\dots\dots (9)$$

where λ_1 is the pure conductivity of gas under the condition of standard state 1, and in the case of $p = p_1$, ξ_1 shows the actual distance from the wall in which λ_t is equal to λ_m . In place of ξ the dimensionless co-ordinate is defined as follows

$$\zeta = 1 + \left(\frac{p}{p_1} \right)^{-1/2} \frac{\xi}{\xi_1} \quad \dots\dots\dots (10)$$

If the specific heat under constant pressure c_p is assumed to be constant and is equal to c_{p1} which is the value at the standard state, the next equation may be obtained using $\partial \zeta / \partial \xi = (1/\xi_1) (p/p_1)^{-1/2}$

$$\frac{1}{a_1} \frac{n(\kappa-1)}{(n-1)\kappa} \frac{\partial \sigma}{\partial t} = \frac{1}{\xi_1^2} \left(\zeta \frac{\partial^2 \sigma}{\partial \zeta^2} + \frac{\partial \sigma}{\partial \zeta} \right) \quad \dots\dots (11)$$

where a_1 shows the thermal diffusivity of gas $\lambda_1 v_1 / c_{p1}$ (m²/hr) at the standard state.

4. Density of heat flow

If the direction of heat flow from the gas to the wall is chosen to be positive, then the instantaneous value of heat flow at an arbitrary particle of gas ξ is expressed by the next relation

$$q = \lambda \frac{\partial T}{\partial x} = \lambda \frac{\partial T}{\partial \zeta} \frac{\partial \zeta}{\partial \xi} \frac{\partial \xi}{\partial x}$$

So

$$q = \frac{\lambda_1 \zeta}{\xi_1} \left(\frac{p}{p_1} \right)^{1/2} \frac{\partial T}{\partial \zeta} = \frac{\lambda_1 T_1 \zeta}{\xi_1} \left(\frac{p}{p_1} \right)^{1/2 + (n-1)/n} \frac{\partial \sigma}{\partial \zeta} \quad \dots\dots\dots (12)$$

and the heat flow at the wall surface is expressed as follows

Fig. 1 Change of polytropic index n on both compression and expansion strokes

$$q_w = \frac{\lambda_1}{\xi_1} \left(\frac{p}{p_1} \right)^{1/2} \left(\frac{\partial T}{\partial \zeta} \right)_{\zeta=1} = \frac{\lambda_1 T_1}{\xi_1} \left(\frac{p}{p_1} \right)^{1/2 + (n-1)/n} \left(\frac{\partial \sigma}{\partial \zeta} \right)_{\zeta=1} \quad \dots\dots\dots (13)$$

5. Integration of differential equation for the field of state of periodic changing

The Eq. (11) is integrated under the boundary conditions as follows*

$$\zeta \rightarrow \infty : T_{\infty} p^{-(n-1)/n} = T_1 p_1^{-(n-1)/n},$$

$$\zeta \rightarrow 1 : T = T_w$$

At first the particular solution for the case of steady state $\partial \sigma / \partial t = 0$ may be obtained from

$$\zeta (\partial^2 \sigma / \partial \zeta^2) + (\partial \sigma / \partial \zeta) = 0$$

as follows

$$\sigma_I : \text{constant}$$

If this constant is assumed to be unity, σ_I may be reduce to the next relation

$$T p^{-(n-1)/n} = T_1 p_1^{-(n-1)/n}$$

This equation shows the pure polytropic change, and the temperature and the pressure of gas are connected with each other independently of its distance from the wall. The condition for $\zeta \rightarrow \infty$ is satisfied by this solution. As the next particular solution the following relation may be obtained

$$\sigma_{II} = K \ln \zeta$$

$$\therefore T/T_1 = K (p/p_1)^{(n-1)/n} \ln \{ 1 + (p/p_1)^{-1/2} (\xi/\xi_1) \}$$

From this solution the density of heat flow is calculated by using the relation (12),

$$q = (K \lambda_1 T_1 / \xi_1) (p/p_1)^{1/2 + (n-1)/n}$$

* Strict examination of the boundary conditions will be discussed in the next chance.

Its value is the same at all gas particles independently of the distance from the wall. But it is considered that the gas space is extending $\zeta \rightarrow \infty$, so the constant of integration K must be zero since the gas temperature is finite at every point of the gas space.

Putting $\sigma(\zeta, t) = \phi(\zeta)\chi(t)$, the Eq. (11) becomes

$$\frac{1}{a_1} \frac{n(\kappa-1)}{(n-1)\kappa} \frac{1}{\chi} \frac{d\chi}{dt} = -\frac{1}{\xi_1^2 \phi} \left(\zeta \frac{d^2 \phi}{d\zeta^2} + \frac{d\phi}{d\zeta} \right) \quad (14)$$

Since the solution for periodic change is sought here, $-i\beta^2$ is chosen as the constant of each side of the equation. Thus

$$\frac{1}{a_1} \frac{n(\kappa-1)}{(n-1)\kappa} \frac{1}{\chi} \frac{d\chi}{dt} = -i\beta^2 \quad (15)$$

As $n(\kappa-1)/\{(n-1)\kappa\}$ is the periodic function of t (hr) or crank angle ωt (rad), so the following relation can be expanded into Fourier series.

$$f(t) = \frac{(n-1)\kappa}{n(\kappa-1)} = F(\omega t) \quad (16)$$

If the timing of generating heat in the gases is assumed to be ε (rad) symmetrically to T.D.C. and the process of n is shown as in Fig. 1, $F(\omega t)$ is expressed as in Fig. 2. Strictly speaking the point of T.D.C. must be excluded, but in the present case it is considered that the function includes the point.

$$f(t) = F(\omega t) = 1 + \frac{\varepsilon}{\pi(\kappa-1)} + \frac{2}{\pi(\kappa-1)} \sum_{m=1}^{\infty} \frac{1}{m} \sin m\varepsilon \cos m\omega t \quad (17)$$

Hence, the solution of (15) can be obtained as follows

$$\chi = e^{-ia_1\beta^2[\{1+\varepsilon/\pi(\kappa-1)\}t+2/\{\omega\pi(\kappa-1)\} \sum_{m=1}^{\infty} 1/m^2 \sin m\varepsilon \sin m\omega t]} \quad (18)$$

In respect to the equation of ϕ the next equation may be obtained by putting $z = 2\beta\xi_1\sqrt{i\zeta}$,

$$\frac{d^2\phi}{dz^2} + \frac{1}{z} \frac{d\phi}{dz} + \phi = 0$$

This is the Bessel function of order zero and when its solution is shown by $\phi = Z_0(2\beta\xi_1\sqrt{i\zeta})$, then

$$\sigma_{III} = Z_0(2\beta\xi_1\sqrt{i\zeta}) e^{-ia_1\beta^2[\{1+\varepsilon/\pi(\kappa-1)\}t+2/\{\omega\pi(\kappa-1)\} \sum_{m=1}^{\infty} 1/m^2 \sin m\varepsilon \sin m\omega t]}$$

Putting all the solutions together, σ can be expressed in the next form

$$\sigma = 1 + \sum_{\nu=1}^{\infty} C_{\nu} Z_0(2\beta_{\nu}\xi_1\sqrt{i\zeta}) e^{-ia_1\beta_{\nu}^2[\{1+\varepsilon/\pi(\kappa-1)\}t+2/\{\omega\pi(\kappa-1)\} \sum_{m=1}^{\infty} 1/m^2 \sin m\varepsilon \sin m\omega t]} \quad (19)$$

The state of gas changes periodically with the thermodynamical cycle of the engine, so it may be expanded in Fourier series of $\nu\omega t$ in the interval $(-\pi, \pi)$. In order to make the solution (19) consistent with the periodicity of engine cycle, the following condition must be attached to β_{ν} . In the case of the four stroke engine, however, only the compression and expansion strokes are repeated excluding the intake and exhaust strokes.

$$a_1\beta_{\nu}^2 \left\{ 1 + \frac{\varepsilon}{\pi(\kappa-1)} \right\} = \nu\omega \quad (\nu = \pm 1, \pm 2, \pm 3, \dots) \quad (20)$$

The meaning of periodic part of the solution (19) is the deviation of gas temperature from the polytropic temperature of the uniform zone, and the deviation is of such character that it disappears in $\zeta \rightarrow \infty$. So it is suitable to use Hankel's function in expressing the solution,

$$\begin{aligned} \sigma &= \frac{T}{T_1} \left(\frac{p}{p_1} \right)^{-(n-1)/n} \\ &= 1 + \sum_{\nu=1}^{\infty} A_{\nu} H_0^{(1)} \left(2\xi_1 \sqrt{\frac{i\nu\omega\zeta}{a_1} \frac{\pi(\kappa-1)}{\pi(\kappa-1)+\varepsilon}} \right) e^{-i\nu[\omega t+2/\{\pi(\kappa-1)+\varepsilon\} \sum_{m=1}^{\infty} 1/m^2 \sin m\varepsilon \sin m\omega t]} \\ &\quad + \sum_{\nu=1}^{\infty} B_{\nu} H_0^{(2)} \left(2\xi_1 \sqrt{\frac{-i\nu\omega\zeta}{a_1} \frac{\pi(\kappa-1)}{\pi(\kappa-1)+\varepsilon}} \right) e^{i\nu[\omega t+2/\{\pi(\kappa-1)+\varepsilon\} \sum_{m=1}^{\infty} 1/m^2 \sin m\varepsilon \sin m\omega t]} \quad (21) \end{aligned}$$

Fig. 2 Change of $F(\omega t)$ in the interval $(\omega t = -\pi, \pi)$

Using the following simplified form

$$\Psi = 2\tilde{\zeta}_1 \sqrt{\frac{\omega}{a_1} \frac{\pi(\kappa-1)}{\pi(\kappa-1)+\varepsilon}} \dots\dots\dots (22)$$

$$\begin{aligned} \Theta(\theta) &= \omega t + \frac{2}{\pi(\kappa-1)+\varepsilon} \sum_{m=1}^{\infty} \frac{1}{m^2} \sin m\varepsilon \sin m\omega t \\ &= \theta + \frac{2}{\pi(\kappa-1)+\varepsilon} \sum_{m=1}^{\infty} \frac{1}{m^2} \sin m\varepsilon \sin m\theta \dots\dots\dots (23) \end{aligned}$$

and the boundary condition for $\zeta \rightarrow \infty$

$$\sigma_{\infty} = (T_{\infty}/T_1)(p/p_1)^{-(n-1)/n} = 1$$

and diving the Eq. (21) by σ_{∞} the following relation can be obtained

$$T/T_{\infty} = 1 + \sum_{\nu=1}^{\infty} A_{\nu} H_0^{(1)}(\Psi \sqrt{\nu i \zeta}) e^{-i\nu \Theta(\theta)} + \sum_{\nu=1}^{\infty} B_{\nu} H_0^{(2)}(\Psi \sqrt{-\nu i \zeta}) e^{i\nu \Theta(\theta)} \dots\dots\dots (24)$$

By means of this equation the gas temperature T at an arbitrary distance from the wall may be given by the ratio T_{∞} , and the temperature of the wall can be obtained by putting $\zeta=1$, i.e.

$$T_W/T_{\infty} = 1 + \sum_{\nu=1}^{\infty} A_{\nu} H_0^{(1)}(\Psi \sqrt{\nu i}) e^{-i\nu \Theta(\theta)} + \sum_{\nu=1}^{\infty} B_{\nu} H_0^{(2)}(\Psi \sqrt{-\nu i}) e^{i\nu \Theta(\theta)} \dots\dots\dots (25)$$

Next these equations must be rewritten in the real forms which have physical meaning. Namely, from the fundamental theorem of Hankel's function

$$\begin{aligned} H_0^{(1)}(\Psi \sqrt{\nu i \zeta}) &= R_{\varepsilon} H_0^{(1)}(\Psi \sqrt{\nu i \zeta}) + i I_m H_0^{(1)}(\Psi \sqrt{\nu i \zeta}) \\ H_0^{(2)}(\Psi \sqrt{-\nu i \zeta}) &= R_{\varepsilon} H_0^{(1)}(\Psi \sqrt{\nu i \zeta}) - i I_m H_0^{(1)}(\Psi \sqrt{\nu i \zeta}) \end{aligned}$$

and putting as follows

$$A_{\nu} = (U_{\nu} - i V_{\nu})/2, \quad B_{\nu} = (U_{\nu} + i V_{\nu})/2$$

where U_{ν} and V_{ν} are the real constants, the Eq. (24) may be changed to

$$\begin{aligned} T/T_{\infty} &= 1 + \sum_{\nu=1}^{\infty} [\{U_{\nu} R_{\varepsilon} H_0^{(1)}(\Psi \sqrt{\nu i \zeta}) + V_{\nu} I_m H_0^{(1)}(\Psi \sqrt{\nu i \zeta})\} \cos \nu \Theta(\theta) \\ &\quad + \{U_{\nu} I_m H_0^{(1)}(\Psi \sqrt{\nu i \zeta}) - V_{\nu} R_{\varepsilon} H_0^{(1)}(\Psi \sqrt{\nu i \zeta})\} \sin \nu \Theta(\theta)] \dots\dots\dots (26) \end{aligned}$$

and the Eq. (25) reduces to

$$\begin{aligned} T_W/T_{\infty} &= 1 + \sum_{\nu=1}^{\infty} [\{U_{\nu} R_{\varepsilon} H_0^{(1)}(\Psi \sqrt{\nu i}) + V_{\nu} I_m H_0^{(1)}(\Psi \sqrt{\nu i})\} \cos \nu \Theta(\theta) \\ &\quad + \{U_{\nu} I_m H_0^{(1)}(\Psi \sqrt{\nu i}) - V_{\nu} R_{\varepsilon} H_0^{(1)}(\Psi \sqrt{\nu i})\} \sin \nu \Theta(\theta)] \dots\dots\dots (27) \end{aligned}$$

Then the density of heat flow can be calculated as follows

$$\begin{aligned} q &= -\lambda_1 \left(\frac{p}{p_1} \right)^{1/2} T_{\infty} \sqrt{\frac{\omega}{a_1} \frac{\pi(\kappa-1)\zeta}{\pi(\kappa-1)+\varepsilon}} \sum_{\nu=1}^{\infty} \sqrt{\nu} [\{U_{\nu} R_{\varepsilon} \sqrt{i} H_1^{(1)}(\Psi \sqrt{\nu i \zeta}) \\ &\quad + V_{\nu} I_m \sqrt{i} H_1^{(1)}(\Psi \sqrt{\nu i \zeta})\} \cos \nu \Theta(\theta) \\ &\quad + \{U_{\nu} I_m \sqrt{i} H_1^{(1)}(\Psi \sqrt{\nu i \zeta}) - V_{\nu} R_{\varepsilon} \sqrt{i} H_1^{(1)}(\Psi \sqrt{\nu i \zeta})\} \sin \nu \Theta(\theta)] \dots\dots\dots (28) \end{aligned}$$

Putting $\zeta=1$ in this equation the heat flow at the surface of wall is obtained and if the coefficient of heat transfer is defined as $\alpha = q_W/(T_{\infty} - T_W)$, α is expressed by the next equation

$$\begin{aligned} \alpha &= -\lambda_1 \left(\frac{p}{p_1} \right)^{1/2} \frac{1}{1 - (T_W/T_{\infty})} \sqrt{\frac{\omega}{a_1} \frac{\pi(\kappa-1)}{\pi(\kappa-1)+\varepsilon}} \\ &\quad \times \sum_{\nu=1}^{\infty} \sqrt{\nu} [\{U_{\nu} R_{\varepsilon} \sqrt{i} H_1^{(1)}(\Psi \sqrt{\nu i}) + V_{\nu} I_m \sqrt{i} H_1^{(1)}(\Psi \sqrt{\nu i})\} \cos \nu \Theta(\theta) \\ &\quad + \{U_{\nu} I_m \sqrt{i} H_1^{(1)}(\Psi \sqrt{\nu i}) - V_{\nu} R_{\varepsilon} \sqrt{i} H_1^{(1)}(\Psi \sqrt{\nu i})\} \sin \nu \Theta(\theta)] \dots\dots\dots (29) \end{aligned}$$

6. Example of numerical calculation

The numerical calculation is made by using the experimental results of the four stroke spark-ignition engine with a single cylinder illustrated in the previous report⁽²⁾. The value of ω is taken as 89.01×3600 rad/hr since it is assumed that only the compression and expansion strokes are repeated in the engine speed of 850 r. p. m..

6.1 Calculation of $\Theta(\theta)$

The relation between $\Theta(\theta)$ and θ is estimated by the Eq. (23). The adiabatic constant is taken as 1.29, being the mean value of the compression and expansion strokes. The period of heat generation is assumed to be symmetric to the T.D.C., and to be $2\varepsilon=20^\circ=0.3490$ rad. in the crank angle. Fig.3 is obtained by calculating to the extent of $\nu=35$.

6.2 Determination of ξ_1

If the suffix "W" is added to the state variables of the gas which touches the wall surface, since the standard state 1 is the constant one selected properly, T_1 is taken as T_W and p_1 is also taken as the value when T_∞ becomes equal to T_W . So from the relations of

$$a_1 = \lambda_1 / (c_{p1} \gamma_1), \quad \gamma_1 = p_1 / (RT_W), \quad \gamma_W = \gamma_1 p / p_1, \quad \lambda_1 = \lambda_W, \quad c_{p1} = c_{pW}$$

a_1 can be expressed by

$$a_1 = \{\lambda_W / (c_{pW} \gamma_W)\} (p/p_1) = a_W p / p_1$$

As in the previous report⁽²⁾, N_u and P_e' are expressed respectively as $\alpha L / \lambda_W$ and $\omega L^2 / a_W$ using the piston stroke L . Considering the Eq. (29) quite approximately to the extent of $\nu=1$, and if it is assumed that U_1 is nearly equal to V_1 from the results of preliminary calculation which show $U_1=0.5090$ and $V_1=0.5846^{(12)}$, the Eq. (29) may be written in the following form

$$\frac{N_u}{\sqrt{P_e'}} = \sqrt{\frac{\pi(\kappa-1)}{\pi(\kappa-1)+\varepsilon}} \left[\frac{\{R_e \sqrt{i} H_1^{(1)}(\Psi \sqrt{i}) + I_m \sqrt{i} H_1^{(1)}(\Psi \sqrt{i})\} \cos \Theta(\theta) + \{I_m \sqrt{i} H_1^{(1)}(\Psi \sqrt{i}) - R_e \sqrt{i} H_1^{(1)}(\Psi \sqrt{i})\} \sin \Theta(\theta)}{\{R_e H_0^{(1)}(\Psi \sqrt{i}) + I_m H_0^{(1)}(\Psi \sqrt{i})\} \cos \Theta(\theta) + \{I_m H_0^{(1)}(\Psi \sqrt{i}) - R_e H_0^{(1)}(\Psi \sqrt{i})\} \sin \Theta(\theta)} \right] \dots (30)$$

From the experimental results the value of $N_u / \sqrt{P_e'}$ is estimated in regard to $\theta=0^\circ$ (T.D.C.), 30° , 90° and 120° , and on the other hand the value of right side of the Eq. (30) is calculated in the vast range of Ψ . If the adjoining values of Ψ which satisfy the equation are obtained in these crank angles, ξ_1 can be determined from Ψ . For the purpose of this procedure Fig.4 is drawn up, and in it the hatched areas show the meeting points. Inconveniently there are two or more such points in regard to the same crank angle, but Ψ of the smaller side is considered to be reasonable here. As these points do not always converge in the same Ψ , the mean value of them is selected as 0.10. Then the corresponding ξ_1 becomes 0.0264×10^{-3} m.

6.3 Density of heat flow at the wall surface

The amplitude of temperature vibration at the wall surface is very small and is negligible in comparison

Fig. 3 Relation between $\Theta(\theta)$ and θ

Fig. 4 Right side of Eq. (30) shown as function of Ψ

with that of uniform gas zone, so $T_W = \text{constant}$. Since in the Eq. (27) the terms inside Σ are all pure harmonic functions of $\Theta(\theta)$, the integral of those from $-\pi$ to π becomes zero, and the following may be written

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{T_{\infty}} d\Theta(\theta) = \frac{1}{T_W} \quad \dots\dots\dots (31)$$

As the result there is no room for discussing the variation of the wall temperature under the various conditions of cooling, and there is the drawback that T_W is determined only by the process of T_{∞} . But in the present report T_W is determined in this way. $1/T_{\infty}$ is calculated from T_{∞} obtained in the previous report, and is shown in Fig. 5 as the function of $\Theta(\theta)$. By means of this figure it is estimated that $T_W = 837^{\circ}\text{K}$.

Next the function T_W/T_{∞} is expanded in Fourier series of $\Theta(\theta)$ to the extent of $\nu=35$, and the coefficients (a_{ν} , b_{ν}) of following equation are obtained

$$T_W/T_{\infty} = 1 + \sum_{\nu=1}^{35} \{a_{\nu} \cos \nu\Theta(\theta) + b_{\nu} \sin \nu\Theta(\theta)\} \quad \dots\dots\dots (32)$$

Equating these coefficients to those of same order of the Eq. (27), the values of U_{ν} and V_{ν} can be calculated simultaneously as follows

Fig. 5 Relation between $1/T_{\infty}$ and $\Theta(\theta)$

$$\left. \begin{aligned} a_{\nu} &= U_{\nu} g_{\nu} + V_{\nu} h_{\nu} \\ b_{\nu} &= U_{\nu} h_{\nu} - V_{\nu} g_{\nu} \end{aligned} \right\} \quad \dots\dots\dots (33)$$

where

$$g_{\nu} = R_e H_0^{(1)}(\Psi \sqrt{\nu i}), \quad h_{\nu} = I_m H_0^{(1)}(\Psi \sqrt{\nu i})$$

$\nu = 1 \sim 35$

Therefore the density of heat flow q_w may be obtained at once, and is shown in Fig. 6. For comparison its value obtained experimentally in the previous report is illustrated in the same figure.

6.4 Applicable range of the fundamental equation

The fundamental Eq. (11) is applicable in the gas zone from the wall surface to the infinite region. It does not, however, contain the intake and exhaust strokes as mentioned frequently. By putting in the Eq. (26), $\zeta = 2, 4, 9, 16, 25, 36, 49, 64, 100$; $\Theta(\theta) = 0^{\circ}, \pm 30^{\circ}, \pm 60^{\circ}, \pm 90^{\circ}, \pm 120^{\circ}, \pm 150^{\circ}$; i.e. $\theta = 0^{\circ}, \pm 7.9^{\circ}, \pm 37.0^{\circ}, \pm 72.8^{\circ}, \pm 108.5^{\circ}, \pm 144.3^{\circ}$, T/T_{∞} can be calculated to the extent of $\nu=35$ with the results as shown in Fig. 7. Though it is inconvenient to know intuitively the temperature field of gas as the abscissa is ζ , it will give many suggestions to inquire into the temperature

Fig. 6 Density of heat flow on the surface of wall as estimated by calculation and by experiment

near the wall. For example, $\zeta=100$ shows the value of $\xi=2.61$ mm in the case of $\xi_1=0.0264$ mm and $p=p_1=6.16$ kg/cm² abs.. Calculations are abandoned for the values of ζ more than 100 owing to the limitations of table of Bessel function, but it is imaginable that T very slowly becomes T_∞ as $\zeta \rightarrow \infty$.

7. Conclusions

(1) The processes of adiabatic change of gases and of heat generation in combustion are expressed by changing the polytropic index n . By this means the calculation becomes quite similar to that of the thermodynamical cycle of gases in Otto engine.

(2) The results of numerical calculation come considerably close to the experimental results. But the calculated value of the peak of heat flow is lower than that of the experiment, and the difference is presumably due to the neglect of the radiant heat transmission.

(3) The fundamental equation is applicable in the gas zone from the wall surface to the infinite region, but the temperature of wall T_w is determined only by the process of T_∞ and there is no room for discussing the variation of wall temperature under various conditions of cooling.

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Fig. 7 Temperature of gases in the vicinity of wall surface