this value, this effect will be subdued.

(b) The spring constant can be made smaller by using a bellows with small values of α . But, it should be noted that β will increase when α decreases as shown in Figs. 4 and 5, because θ_0 approaches to 180° . This effect is very distinct when the value of V_c is not large. Hence, the spring constant becomes large under the above mentioned condition in spite of adoption of small α . When α is positive and the mounting number n increases, k becomes small.

(c) The limit to the effect of lowering the spring constant by the elongation of the bellows δ is as follows

$$-\frac{m\gamma\delta(p_s+p_a)}{V_s+V_c} + \zeta\delta p_s < 0$$

That is,

$$m\frac{\gamma}{\zeta}\left(1+\frac{p_a}{p_s}\right) > V_s+V_c \dots\dots\dots(27)$$

7. Conclusion

General equations of spring constants of flat plate type and tire type air spring bellows are obtained by using the coefficients of bellows form. Applying these equations for the calculation of spring constants of commercial air spring bellows and bellows model, we get good coincidence with experimental data. This means the appropriateness of the equation and possibility of its utilization for the design of air spring bellows.

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Photoelastic Study on Flat Bars with V-notches on Both Edges Subjected to Pure Bending*

By Yasunori MURAKAMI** and Toshio KAWABE***

Flat bar specimens of Phenolite with V-notches on both edges were subjected to pure bending and the stress concentration factors under various conditions were measured by the photoelastic method. To determine the fringe orders at the base of notch, several bending moments were applied. Similar to the experimental results in tension, the stress concentration factor is the larger, the smaller the root radius, its variation being extremely steep for the values of ρ/b less than 0.3. The included angle has a little influence in the region between 0° and 90° , and as it exceeds 90° , the stress concentration factor diminishes gradually. The depth of notch has such an effect as to give the stress concentration factor its maximum value when the value of t/b is nearly 0.15 to 0.25.

1. Introduction

The stress concentration due to notches in many machine parts is a subject of paramount importance in engineering, and numerous studies have been carried out in connection with fatigue and creep phenomena. For the estimation of stress concentration factors, Neuber's theoretical formulas⁽¹⁾ are widely used. In the experimental field, photoelastic studies⁽²⁾ and other methods by strain

gauges or electroplating⁽³⁾ have been employed. Neuber's theoretical formulas seem to give a little higher values than those obtained by experiments, probably due to the difference in notch configuration.

Photoelastic study on flat bars with a single V-notch subjected to pure bending was carried out by M. Nishida, the effect of the root radius, the included angle and the depth of notch on the stress concentration factor being examined⁽⁴⁾. In this experiment, flat bar specimens with various root radii, included angles and depths of notch were subjected to uniform bending, and the effects of these variables on the stress concentration factors were investigated.

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** Lecturer, Faculty of Science and Engineering, Ritumeikan University, Kita-ku, Kyoto.

*** Assistance, Faculty of Science and Engineering, Ritumeikan University.

2. Experimental procedures

2.1 Specimens

Specimens tested were of Phenolite plate with a thickness of 6 mm, and the shape of specimen is as illustrated in Fig. 1. The width of specimen $2b$ was kept constant (20 mm), and the root radius ρ , included angle θ , and the depth t were varied in several steps. Specimens were finished by hand, with special caution to avoid dimensional errors and prevent edge stresses. Dimensions of specimens after hand finishing were measured precisely by a universal projector with twenty magnifications. The data obtained are indicated in Table 1. The maximum difference in dimensions between the notches on both edges is confined to about 5%. Photoelastic investigation was performed immediately after finishing in order to prevent

Fig. 1 Shape of specimen

the time-edge effect.

2.2 Loading apparatus

Loading apparatus used is reproduced in Fig. 2. To give a uniform bending moment to specimens, these were supported on two knife-edges equally distant from the center of specimens. Both ends of specimens were connected to arms through chucks, and weights were applied to these arms. In this experiment, the length of specimen was 168 mm, and the distance between two supports was 80 mm.

2.3 Calculation of the stress concentration factors

Fig. 2 Loading apparatus

Table 1 Dimensions of specimens

Specimen number			1	2	3	4	5	6	7	8
ρ	$\theta = 60^\circ$ $t/b = 0.25$	ρ	∞	1.00	2.05	2.85	4.33	6.80	7.53	—
		$2a$	15.00	15.00	14.90	14.90	14.49	14.61	14.50	—
		ρ/b	—	0.100	0.204	0.283	0.433	0.682	0.772	—
	$\theta = 60^\circ$ $t/b = 0.50$	ρ	∞	0.99	1.35	2.00	2.78	3.60	4.94	7.40
		$2a$	10.05	10.36	10.00	10.04	9.62	9.97	10.24	9.97
		ρ/b	—	0.099	0.135	0.200	0.280	0.360	0.496	0.740
	$\theta = 60^\circ$ $t/b = 0.75$	ρ	∞	1.09	1.25	1.95	2.93	3.98	4.88	7.50
		$2a$	5.06	5.03	5.04	5.23	5.05	5.00	5.30	5.06
		ρ/b	—	0.109	0.125	0.195	0.293	0.398	0.488	0.750
θ	$\rho = 1$ $t/b = 0.25$	θ	—	29°33'	62°35'	96°19'	120°28'	149°15'	—	—
		$2a$	15.00	14.68	15.22	14.92	14.98	14.55	—	—
	$\rho = 1$ $t/b = 0.50$	θ	—	30°15'	42°50'	59°00'	90°30'	120°00'	150°00'	—
		$2a$	10.05	10.13	10.25	10.36	10.00	10.00	10.00	—
	$\rho = 1$ $t/b = 0.75$	θ	—	30°46'	60°00'	85°01'	121°15'	147°35'	—	—
		$2a$	5.00	5.00	4.63	4.74	5.00	4.19	—	—
t	$\theta = 60^\circ$ $\rho/b = 0.10$	t	0.55	1.52	2.50	3.59	4.86	7.46	—	—
		$2a$	18.80	16.91	15.00	12.84	10.36	5.03	—	—
		t/b	0.055	0.152	0.250	0.359	0.486	0.746	—	—
	$\theta = 60^\circ$ $\rho/b = 0.20$	t	0.63	1.58	2.59	3.55	4.83	7.44	—	—
		$2a$	18.57	16.85	14.90	12.71	10.04	5.23	—	—
		t/b	0.063	0.158	0.259	0.356	0.483	0.744	—	—
	$\theta = 60^\circ$ $\rho/b = 0.30$	t	0.61	1.59	2.60	3.47	4.54	7.45	—	—
		$2a$	18.58	16.92	14.90	12.94	9.62	5.05	—	—
		t/b	0.061	0.159	0.260	0.348	0.454	0.745	—	—
	Comparative specimen	$2a$	18.85	17.05	15.00	13.02	10.05	5.06	—	—

Fringe patterns were photographed on the notched specimens, and also on the comparative specimens of the same material with the minimum width of $2(b-t)$. Examples of fringe patterns are shown in Fig. 3, (a) to (d) being for the notched specimens under different conditions and (e) for a comparative specimen. Basing on these fringe patterns, the distribution curves of fringe orders were drawn as shown in Fig. 4 and Fig. 5. Fig. 4 gives the distribution curve for a comparative specimen and all points lie on a straight line. Fig. 5 indicates that for a notched specimen. In order to obtain a higher experimental accuracy, several fringe patterns were photographed by applying the bending moments in four to six steps. From these fringe patterns, r/a versus fringe

order curves were drawn as shown in Fig. 6 and Fig. 7, where r is the distance between zero and any fringe order and a that from zero fringe order to the base of notch. In both Figures, solid and dotted lines correspond to the tension and compression sides of specimens, respectively. In Fig. 6, these two lines show little deviation, but in Fig. 7 there exists a little deviation in the case of larger bending moments. From these diagrams, we determined the fringe orders at the base of notch as the intersecting points of these curves and the vertical lines at $r/a=1$. The curves representing the relation between the fringe orders at the base of notch thus determined and the bending moments applied, were obtained as shown in Fig. 8 and Fig. 9. In these Figures, each point

Fig. 3 Photographs of isochromatic lines

Fig. 4 Distribution curve of fringe orders (Comparative specimen)

Fig. 5 Distribution curve of fringe orders (Notched specimen)

at the tension and compression sides lies nearly on a straight line, and drawing a mean line between these two lines, the final fringe orders were determined.

In Fig. 8, this mean line passes through the origin, but in Fig. 9 it deviates a little from the origin. This deviation may be due to the initial stress or edge stress, and to eliminate these influences, correction was made so that this mean line might pass through the origin. The calculation of stress concentration factor was carried out by

Fig. 6 r/a versus fringe order curves (Comparative specimen)

Fig. 7 r/a versus fringe order curves (Notched specimen)

Fig. 8 Correction curve (Comparative specimen)

the following formula⁽⁵⁾

$$\alpha = (n/\sigma) / (n_0/\sigma_0)$$

where n and σ represent the fringe order and nominal bending stress at the notch root and n_0 , σ_0 are the maximum fringe order and the maximum bending stress for the comparative specimens, respectively.

3. Results of experiment

3.1 The effect of the root radius ρ

Experiments were carried out for the three cases of $t/b=0.25$, 0.5 and 0.75, the included angle θ being kept at 60° and ρ/b being varied in six stages. The results are shown in Table 2 and

Table 2 Effect of the root radius, ρ

$\theta=60^\circ$ $t/b=0.25$	ρ/b	0.100	0.204	0.283	0.433	0.682	0.772	—
	α	2.41	1.81	1.65	1.41	1.32	1.31	—
$\theta=60^\circ$ $t/b=0.50$	ρ/b	0.099	0.135	0.200	0.280	0.360	0.496	0.740
	α	1.87	1.66	1.53	1.37	1.32	1.28	1.25
$\theta=60^\circ$ $t/b=0.75$	ρ/b	0.109	0.125	0.195	0.293	0.398	0.488	0.750
	α	1.39	1.31	1.23	1.11	1.06	1.07	1.02

Fig. 9 Correction curve (Notched specimen)

Fig. 10 Effect of the root radius, ρ

Fig. 10. For the sake of comparison, Neuber's theoretical values are indicated in Fig. 10. From this diagram, it is understood that the change in stress concentration factors is noticeable for the sharp notches where ρ/b is less than about 0.3, i.e. a little variation in the root radius gives a considerable effect on the magnitude of stress concentration factor. For the values of ρ/b larger than 0.3, the gradient of the curve becomes mild, and the stress concentration factor has the tendency of a monotonous diminution. All the experimental values are smaller than Neuber's theoretical ones, and this difference is conspicuous for the larger values of t/b . As is previously mentioned, this trend may be due to the difference in notch configuration. In the case of shallow notches, for example $t/b=0.25$, since the difference in notch shape gives little effect, experimental and theoretical values show a little divergence, but in the case of deep notches, for instance $t/b=0.75$, it seems that a considerable effect of the deviation in notch configuration comes to appear.

3.2 The effect of the included angle θ

As to the effect of the included angle, experiments were made for the three cases of $t/b=0.25, 0.50$ and 0.75 , where $\rho/b=0.1$ was kept constant and the included angle was varied in several steps. The results obtained are shown in Table 3 and Fig. 11. The stress concentration factor diminishes very slowly between the included angle of zero to 90° , thus the effect of included angle may be considered small. This trend is in good agreement with that of theoretical values.

Table 3 Effect of the included angle, θ

$t/b=0.25$ $\rho/b=0.10$	θ	29°33'	62°35'	96°19'	120°28'	149°15'	—
	α	2.36	2.26	2.16	1.92	1.53	—
$t/b=0.50$ $\rho/b=0.10$	θ	30°15'	42°50'	59°00'	90°30'	120°00'	150°00'
	α	2.00	1.99	1.87	1.79	1.54	1.29
$t/b=0.75$ $\rho/b=0.10$	θ	30°46'	60°00'	85°01'	121°15'	147°35'	—
	α	1.50	1.43	1.40	1.31	1.05	—

Fig. 11 Effect of the included angle, θ

For the values of included angle larger than 90° , stress concentration factor decreases gradually as the included angle increases, and above 120° , its diminution becomes rather steep forming nearly a straight line. In this case also, all the experimental values are below the theoretical ones and this trend is more evident for the larger values of t/b .

3.3 The effect of the depth of notch t

The effect of the depth of notch was examined for the three cases of $\rho/b=0.1, 0.2$ and 0.3 , the included angle being kept at 60° , and the values of t/b being varied in six steps. The results obtained are indicated in Table 4 and Fig. 12. In Fig. 12 the dotted lines show Neuber's theoretical values, and a chain line indicates the experimental results by M. Nishida for the case of $\rho/b=0.2$. As is apparent in this Figure, stress concentration factors increase steeply as the depth t grows large, and reach their maximum values, thereafter diminishing gradually to unity. These maximum values appear when the values of t/b are nearly 0.15 to 0.25, the peak being shifted to the smaller values of t/b for the larger values of ρ/b . The discrepancy between the experimental and theoretical values is narrow while the values of t/b are small, but grows wider for the larger values of t/b , presumably due to the influence of the difference in notch configuration. This discrepancy is wider for the smaller values of ρ/b , probably owing to the effect of the root radius.

Table 4 Effect of the depth of notch, t

$\theta=60^\circ$ $\rho/b=0.1$	t/b	0.055	0.152	0.250	0.359	0.486	0.746
	α	2.11	2.30	2.41	2.23	1.87	1.39
$\theta=60^\circ$ $\rho/b=0.2$	t/b	0.063	0.158	0.259	0.356	0.483	0.744
	α	1.72	1.80	1.81	1.71	1.53	1.23
$\theta=60^\circ$ $\rho/b=0.3$	t/b	0.061	0.159	0.260	0.348	0.454	0.745
	α	1.65	1.68	1.65	1.58	1.37	1.11

Fig. 12 Effect of the depth of notch, t

4. Conclusions

From the above experimental results, the following conclusions may be drawn:

(1) As to the effect of the root radius, the stress concentration factor decreases steeply until ρ/b is nearly 0.3, and thereafter its diminution becomes rather slow.

(2) The effect of variation in the included angle is little in the range between 0° and 90° . For the values of angle larger than 90° , the stress concentration factor decreases gradually till it reaches unity.

(3) With respect to the effect of the depth of notch, the stress concentration factor has its maximum value when t/b is nearly 0.15 to 0.25.

(4) In all cases investigated, the experimen-

tal values obtained were smaller than Neuber's theoretical ones, and this difference is considerable, especially, in the deeper notches.

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On the Detection Range of Strain Figures and Their Characteristics*

By Motoichi KODAMA** and Masanosuke SAKAMOTO***

We have studied on the following items:

1. The detection range of strain figures.
2. The relations between Lüders' line and strain figures.
3. The equivalency of strain figures between a strain caused by mechanical stress and that caused by thermal stress.

As a result of our researches, we have reached the following conclusions:

1. The yielding of steel or Fe-C alloy is due to the failure by slip of one phase of the two phases, such as ferrite and pearlite or pearlite and cementite. In an appropriate structure, we can detect the strain figures of steels from 0.05% to 3% in its carbon-content and some special steels. The strain energy of yielding disappears at the temperature of A_1 point.
2. Lüders' line corresponds in position to a strain figure at a low temperature; in other words, a strain figure appears at the same place that Lüders' line runs; but not at a high temperature, when it becomes a false figure.
3. The strain energy of mechanical stress is equivalent to that of thermal stress.

Note: The details of their characteristics shall be illustrated by photographs.

The strain figures of mild steel were detected by Fry⁽¹⁾, in 1921, and were used by the investigators for several years after that. But the etching

reagent in his method not being improved, the application of the study on strain figures to that of yielding was unsatisfactory. We have studied on the etching reagents for the strain figures of both this steel and some special steels, and the detection-range of the strain figures of steel is enlarged thereby from mild steel to malleable cast

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** Professor, Junior College of Engineering, University of Osaka Prefecture, Neyagawa, Osaka.

*** Assistant, Junior College of Engineering, University of Osaka Prefecture.