effect is more severe. But the stress dependence of the value of $\sum(n/N)$ is not linear, and the increment of its value saturates with the increase of the peak stress.

(3) In the case in which two stress levels are very close to each other, if the primary stress is absent, the value of $\sum(n/N)$ is also less than unity as described in (1), but it increases rapidly with the cycle ratio of the primary stress, reaching a maximum value at 20% in cycle ratio. Further increase of the cycle ratio makes the value of $\sum(n/N)$ smaller.

(4) The cycle ratio of the peak stress does not appear to influence on the value of $\sum(n/N)$.

(5) All the general trends of test results can be well explained by the fatigue damage-cycle ratio curves (*D-R* curves) for both cases of increasing and decreasing stress.

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Analysis of Waterhammer in Case the Water Level of Reservoir Fluctuates*

By Takeshi KOBORI**

For waterhammer analysis, the water level of reservoir has been hitherto generally assumed to be constant. But, in the case of surge tank, the water level as a rule fluctuates so widely that it is indispensable to investigate its effects.

If the reservoir water level is expressed as $H=H_L(t)$, that is, a function of time, the waterhammer equation can be derived as follows:

$$H_i - H_0 = \frac{a}{g} (V_0 - V_i) - 2 \sum_{i=1}^{t-1} (H_i - H_0) + 2 \sum_{i=1}^t \{(H_L)_{i-(1/2)} - H_0\}$$

Applying this equation instead of the usual one, and using equations concerning surge tank together, waterhammer in pumping plant with surge tank is computed in consideration of level fluctuation.

In practice, however, the conventional method, by which the flow and pressure transition is first calculated under the assumption that surge tank level remains constant, then level fluctuation is computed under the flow condition that is obtained above, and the pressure is modified, will give satisfactory results.

1. Preface

Concerning the waterhammer problems, there have been many literatures hitherto published, not only for hydraulic power plants but also for pumping plants especially of late. In any of these cases, however, the assumption that the water level of reservoir remains constant during the transitional period is provided. If the water

surface of reservoir is comparatively large, the above assumption will be applicable with good accuracy. On the other hand, in such a case as where the plant is equipped with a surge tank, it is questionable whether the above assumption may be applicable or not. Especially, if there is possibility of separation of water column in pipe line, due to pressure drop caused by power failure when the pump is running, it is dangerous to neglect the down surge of the water level in the surge tank.

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The simple method to calculate the fluctuation of the water level in the surge tank is as follows:

First the transitions of flow and pressure will be computed by ordinary method, assuming the water level of surge tank is constant, and then the fluctuation of water level is calculated under the same flow condition that was set first. But it can not be assured easily that the results of such computation will have good accuracy in every case.

The writer derived a general waterhammer equation for the case where the water level of reservoir fluctuates. Applying this equation, he attained a somewhat accurate method to compute the transitions of flow, pressure, pump speed and the water level of surge tank. And with an actual pump installation, the computed results by this method were compared with those by the simple one.

2. Waterhammer equations in case the water level of reservoir fluctuates

In Fig. 1, if the valve at the end of pipe line is partially opened or closed instantaneously, a sudden change in the water velocity causes an F type pressure wave to form adjacent to the valve. This pressure wave of the F type will travel in x -direction with a velocity a . Upon reaching the reservoir a reflected wave of the f type is produced, and comes back in negative x -direction.

Neglecting the hydraulic losses, the fundamental waterhammer equations will be as follows:

$$H - H_0 = F\left(t - \frac{x}{a}\right) + f\left(t + \frac{x}{a}\right) \dots\dots\dots (1)$$

$$V - V_0 = -\frac{g}{a} \left\{ F\left(t - \frac{x}{a}\right) - f\left(t - \frac{x}{a}\right) \right\} \dots\dots\dots (2)$$

where

H_0, V_0 : pressure head and water velocity in pipe line at initial steady condition

H, V : pressure head and water velocity at transient condition

a : velocity of pressure wave

x : distance measured positive from valve along pipe line

$$H - H_0 = F(t) + H_L\left(t - \frac{L}{a}\right) - H_0 - F\left(t - \frac{2L}{a}\right) \dots\dots\dots (4)$$

$$V - V_0 = -\frac{g}{a} \left\{ F(t) - H_L\left(t - \frac{L}{a}\right) + H_0 + F\left(t - \frac{2L}{a}\right) \right\} \dots\dots\dots (5)$$

At the point of $x=x$

$$H - H_0 = F\left(t - \frac{x}{a}\right) + H_L\left(t - \frac{L}{a} + \frac{x}{a}\right) - H_0 - F\left(t - \frac{2L}{a} + \frac{x}{a}\right) \dots\dots\dots (6)$$

$$V - V_0 = -\frac{g}{a} \left\{ F\left(t - \frac{x}{a}\right) - H_L\left(t - \frac{L}{a} + \frac{x}{a}\right) + H_0 + F\left(t - \frac{2L}{a} + \frac{x}{a}\right) \right\} \dots\dots\dots (7)$$

The value of function F is 0 when its variable is 0 or negative, so at the point of $x=0$

Fig. 1

t : time measured from beginning of valve movement

F : pressure wave travelling in direction of $+x$, a function of $\left(t - \frac{x}{a}\right)$

f : pressure wave travelling in direction of $-x$, a function of $\left(t + \frac{x}{a}\right)$

Eqs. (1) and (2) give the pressure head and water velocity at a point on the pipe line distant x from the valve. In deriving these equations, there is not provided the condition that the water level of reservoir remains constant, so that Eqs. (1) and (2) can be used either when the water level fluctuates or when not.

If it is assumed, as usual, that the water level of reservoir remains constant, it becomes $H=H_0$, and from Eq. (1) the relation

$$f\left(t + \frac{L}{a}\right) = -F\left(t - \frac{L}{a}\right)$$

will be obtained. Under this condition the waterhammer problems have been hitherto computed.

On the contrary, if the water level of reservoir fluctuates, and is expressed as $H=H_L(t)$ at $x=L$, where H_L is a function of t , then from Eq. (1) we get

$$H_L(t) - H_0 = F\left(t - \frac{L}{a}\right) + f\left(t + \frac{L}{a}\right)$$

In place of t , put $(t - L/a)$ then we have

$$f(t) = H_L\left(t - \frac{L}{a}\right) - H_0 - F\left(t - \frac{2L}{a}\right) \dots\dots\dots (3)$$

At the point of $x=0$, substituting for $f(t)$ in Eq. (1) and (2) the right hand side of Eq. (3)

when

$$t \leq \frac{2L}{a}$$

$$H - H_0 = F(t) + H_L \left(t - \frac{L}{a} \right) - H_0 \dots \dots \dots (8)$$

$$V - V_0 = -\frac{g}{a} \left\{ F(t) - H_L \left(t - \frac{L}{a} \right) + H_0 \right\} \dots \dots (9)$$

and at the point of $x=x$

when

$$t \leq \frac{x}{a}$$

$$H - H_0 = H_L \left(t - \frac{L}{a} + \frac{x}{a} \right) - H_0 \dots \dots \dots (10)$$

$$V - V_0 = -\frac{g}{a} \left\{ -H_L \left(t - \frac{L}{a} + \frac{x}{a} \right) + H_0 \right\} \dots \dots (11)$$

when

$$\frac{x}{a} < t \leq \frac{2L-x}{a}$$

$$H - H_0 = F \left(t - \frac{x}{a} \right) + H_L \left(t - \frac{L}{a} + \frac{x}{a} \right) - H_0 \dots (12)$$

$$V - V_0 = -\frac{g}{a} \left\{ F \left(t - \frac{x}{a} \right) - H_L \left(t - \frac{L}{a} + \frac{x}{a} \right) + H_0 \right\} \dots \dots \dots (13)$$

3. Waterhammer equations for sudden valve closure

When the closing time of valve is shorter than $2L/a$, it is called sudden valve closure, and the pressure and velocity of water at $x=0$ are given by Eqs. (8) and (9). Eliminating the term $F(t)$

from these two equations, we have

$$H - H_0 = \frac{a}{g} (V_0 - V) + 2 \left\{ H_L \left(t - \frac{L}{a} \right) - H_0 \right\} \dots (14)$$

That is to say, when the water level of reservoir fluctuates, the pressure change at $x=0$ and at the time t should be larger by twice the change of level at the time L/a before t , i.e. at $(t-L/a)$, than the pressure change when the level does not fluctuate.

For complete closure of valve $V=0$, so Eq. (14) becomes

$$H - H_0 = \frac{aV_0}{g} + 2 \left\{ H_L \left(t - \frac{L}{a} \right) - H_0 \right\} \dots \dots \dots (15)$$

The pressure changes during the operating time of valve $0 < t < T_s$ (T_s is the total closing time) can be derived in the following way:

Let S be the valve opening at any instant, and A be the sectional area of pipe, and let

$$\frac{S}{A} = \phi$$

then ϕ is a function of time t . If c is a coefficient of discharge for optional valve opening, the velocity V in the pipe will be

$$V = c\phi\sqrt{2gH} \dots \dots \dots (16)$$

From Eqs. (14) and (16), we have

$$H - H_0 = \frac{a}{g} \{ V_0 - c\phi\sqrt{2gH} \} + 2 \left\{ H_L \left(t - \frac{L}{a} \right) - H_0 \right\}$$

or

$$H^2 - 2H \left[H_0 + \frac{aV_0}{g} + \frac{a^2c^2\phi^2}{g} + 2 \left\{ H_L \left(t - \frac{L}{a} \right) - H_0 \right\} \right] + \left[H_0 + \frac{aV_0}{g} + 2 \left\{ H_L \left(t - \frac{L}{a} \right) - H_0 \right\} \right]^2 = 0 \dots \dots \dots (17)$$

Solving above equation for H

$$H = \left[H_0 + \frac{aV_0}{g} + \frac{a^2c^2\phi^2}{g} + 2 \left\{ H_L \left(t - \frac{L}{a} \right) - H_0 \right\} \right] \pm \sqrt{\left[H_0 + \frac{aV_0}{g} + \frac{a^2c^2\phi^2}{g} + 2 \left\{ H_L \left(t - \frac{L}{a} \right) - H_0 \right\} \right]^2 - \left[H_0 + \frac{aV_0}{g} + 2 \left\{ H_L \left(t - \frac{L}{a} \right) - H_0 \right\} \right]^2}$$

But as will be known from Eq. (15), H does not become larger than

$$H_0 + \frac{aV_0}{g} + 2 \left\{ H_L \left(t - \frac{L}{a} \right) - H_0 \right\}$$

so in the above equation + sign should not be taken, and the equation of H should be

$$H = \left[H_0 + \frac{aV_0}{g} + \frac{a^2c^2\phi^2}{g} + 2 \left\{ H_L \left(t - \frac{L}{a} \right) - H_0 \right\} \right] - \sqrt{\left[H_0 + \frac{aV_0}{g} + \frac{a^2c^2\phi^2}{g} + 2 \left\{ H_L \left(t - \frac{L}{a} \right) - H_0 \right\} \right]^2 - \left[H_0 + \frac{aV_0}{g} + 2 \left\{ H_L \left(t - \frac{L}{a} \right) - H_0 \right\} \right]^2} \dots \dots \dots (18)$$

4. Waterhammer equations for gradual valve closure

When the valve is gradually closed, or the valve closing time T_s is longer than $2L/a$, it will be treated as follows:

$$(1) \quad 0 < t < \frac{2L}{a}$$

In this period, the equation of pressure at just before the valve ($x=0$) is the same as in the foregoing

section. Now let the symbols for the quantities in this period be suffixed with 1, then from Eq. (18)

$$H_1 = \left[H_0 + \frac{aV_0}{g} + \frac{a^2 c_1^2 \psi_1^2}{g} + 2 \left\{ H_L \left(t_1 - \frac{L}{a} \right) - H_0 \right\} \right] - \sqrt{\left[H_0 + \frac{aV_0}{g} + \frac{a^2 c_1^2 \psi_1^2}{g} + 2 \left\{ H_L \left(t_1 - \frac{L}{a} \right) - H_0 \right\} \right]^2 - \left[H_0 + \frac{aV_0}{g} + 2 \left\{ H_L \left(t_1 - \frac{L}{a} \right) - H_0 \right\} \right]^2} \dots (19)$$

$$(2) \quad (i-1) \frac{2L}{a} < t < i \frac{2L}{a}$$

Let t be expressed as

$$t_i = t_{i-1} + \frac{2L}{a}$$

and H be suffixed with i , then Eq. (4) will be

$$H_i - H_0 = F(t_i) - F\left(t_i - \frac{2L}{a}\right) + H_L\left(t_i - \frac{L}{a}\right) - H_0 = F(t_i) - F(t_{i-1}) + H_L(t_{i-1/2}) - H_0$$

Put

$$F(t_i) = F_i, \quad F(t_{i-1}) = F_{i-1}, \quad H_L(t_{i-1/2}) = (H_L)_{i-1/2}$$

then the above equation will be written as

$$H_i - H_0 = F_i - F_{i-1} + (H_L)_{i-1/2} - H_0 \dots (20)$$

As $F_0 = 0$, the following relations will be obtained from Eq. (20)

$$\begin{aligned} H_1 - H_0 &= F_1 + (H_L)_{1-1/2} - H_0 \\ H_2 - H_0 &= F_2 - F_1 + (H_L)_{2-1/2} - H_0 \\ H_3 - H_0 &= F_3 - F_2 + (H_L)_{3-1/2} - H_0 \\ &\dots \dots \dots \end{aligned}$$

$$\begin{aligned} H_{i-1} - H_0 &= F_{i-1} - F_{i-2} + (H_L)_{i-1-1/2} - H_0 \\ H_i - H_0 &= F_i - F_{i-1} + (H_L)_{i-1/2} - H_0 \end{aligned}$$

Addition of these equations gives

$$H_1 + H_2 + H_3 + \dots + H_i = F_i + \sum_{i=1}^i (H_L)_{i-1/2} \dots (21)$$

From Eq. (5)

$$F_i = \frac{a}{g} (V_0 - V_i) - F_{i-1} + (H_L)_{i-1/2} - H_0$$

Substituting this relation in Eq. (20)

$$H_i - H_0 = \frac{a}{g} (V_0 - V_i) - 2F_{i-1} + 2\{(H_L)_{i-1/2} - H_0\}$$

On the other hand, from Eq. (21)

$$F_{i-1} = H_1 + H_2 + H_3 + \dots + H_{i-1} - \sum_{i=1}^{i-1} (H_L)_{i-1/2}$$

and substituting this relation in the above equation

$$H_i - H_0 = \frac{a}{g} (V_0 - V_i) - 2\left\{ H_1 + H_2 + \dots + H_{i-1} - \sum_{i=1}^{i-1} (H_L)_{i-1/2} \right\} + 2\{(H_L)_{i-1/2} - H_0\}$$

or

$$H_i - H_0 = \frac{a}{g} (V_0 - V_i) - 2 \sum_{i=1}^{i-1} (H_i - H_0) + 2 \sum_{i=1}^i \{(H_L)_{i-1/2} - H_0\} \dots (22)$$

In order to express the above equation in dimensionless form, let Eq. (22) be divided by the normal head H_n , and let Q_n be the normal discharge, also put $V = Q/A$ (Q is an optional discharge and A is the sectional area of the pipe), then Eq. (22) becomes

$$\frac{H_i - H_0}{H_n} = \frac{aQ_n}{gAH_n} \left(\frac{Q_0 - Q_i}{Q_n} \right) - 2 \sum_{i=1}^{i-1} \left(\frac{H_i - H_0}{H_n} \right) + 2 \sum_{i=1}^i \left\{ \frac{(H_L)_{i-1/2} - H_0}{H_n} \right\}$$

Putting

$$\frac{H_i - H_0}{H_n} = h_i - h_0 = \xi_i, \quad \frac{Q_0 - Q_i}{Q_n} = q_0 - q_i,$$

$$\frac{(HL)_{i-1/2} - H_0}{H_n} = \frac{Y_{i-1/2}}{H_n} = y_{i-1/2}$$

the above equation will be written

$$\xi_i = \frac{aQ_n}{gAH_n} (q_0 - q_i) - 2 \sum_{i=1}^{i-1} \xi_i + 2 \sum_{i=1}^i y_{i-1/2} \quad \dots (23)$$

5. Fluctuation of the water level of surge tank

Fig. 2 shows a part of the pipe line with a surge tank. It is assumed that the water is flowing out of the reservoir, and through the surge tank down. If the hydraulic losses in the tunnel between the reservoir and surge tank are neglected, the water levels in both are the same in steady condition.

The velocity V in the pipe line decreases due to the closure of valve at the lower end of it. But the water in the tunnel between the reservoir and surge tank is to continue its velocity V' due to the inertia, so the water level in surge tank will rise, and after a certain time there will be a level difference Y between the surge tank and reservoir.

If the velocity V' increases by $\Delta V'$ in a short time Δt , the acceleration is approximately equal to $\Delta V' / \Delta t$. Let Y_a and Y_b be the values of Y at the beginning and end of Δt , then approximately

$$\Delta V' = -\frac{g}{L'} \cdot \frac{Y_a + Y_b}{2} \Delta t \quad \dots (24)$$

Putting

$$v' = \frac{V'}{V_n}, \quad \left(V_n = \frac{Q_n}{A} \right), \quad y_a = \frac{Y_a}{H_n}$$

and $y_b = Y_b / H_n$, then Eq. (24) will be expressed

$$\text{as } \Delta v' = -\frac{g}{L'} \cdot \frac{H_n}{V_n} \cdot \frac{y_a + y_b}{2} \Delta t$$

Let Q' be the flow in the tunnel, and put $q' = Q' / Q_n$, then

$$v' = \frac{V'}{V_n} = \frac{Q'}{A' V_n} = \frac{Q' A}{A' A V_n} = \frac{A Q'}{A' Q_n} = \frac{A}{A_1} q'$$

Substituting this relation in the above equation, we have

$$\Delta q' = -\frac{g A' H_n}{L' Q_n} \cdot \frac{y_a + y_b}{2} \Delta t \quad \dots (25)$$

Fig. 2

Next let Q_a and Q_b be the flow in the pipe line at the beginning and end of Δt ; Q_a' and Q_b' be the same in the tunnel, then the change in the water level of surge tank ΔY during the time Δt is nearly

$$\Delta Y = \frac{1}{A'} \left(\frac{Q_a' + Q_b'}{2} - \frac{Q_a - Q_b}{2} \right) \Delta t$$

Put $\Delta y = \Delta Y / H_n$, $q_a = Q_a / Q_n$, $q_b = Q_b / Q_n$,

$$q_a' = Q_a' / Q_n \quad \text{and} \quad q_b' = Q_b' / Q_n,$$

then we get

$$\Delta y = \frac{Q_n}{2 A' H_n} \{ (q_a' + q_b') - (q_a + q_b) \} \quad \dots (26)$$

6. Analysis of waterhammer in pumping plant in case the water level of surge tank fluctuates

In the pumping plant, it is the custom to take the flow and velocity positive in direction of pumping up. In compliance with the custom, Eqs. (23), (25) and (26) will be written as

$$\xi_i = \frac{aQ_n}{gAH_n} (q_i - q_0) - 2 \sum_{i=1}^{i-1} \xi_i + 2 \sum_{i=1}^i y_{i-1/2} \quad \dots (23)'$$

$$\Delta q' = \frac{g A' H_n}{L' Q_n} \cdot \frac{y_a + y_b}{2} \Delta t \quad \dots (25)'$$

$$\Delta y = \frac{Q_n}{2 A' H_n} \{ (q_a + q_b) - (q_a' + q_b') \} \quad \dots (26)'$$

Analysis of waterhammer in the pumping plant in case the water level of surge tank fluctuates does not differ fundamentally from the usual one⁽¹⁾ in which the water level of reservoir is assumed to be constant. But in the former, as the waterhammer equation, Eq. (23)' is used instead of the usual one, and also Eqs. (25)' and (26)' are supplemented as the equations concerning the surge tank. These equations are combined with those concerning the pump characteristics and valve resistance, and solved numerically as the simultaneous equations.

By the above method comparatively accurate results will be obtained, but it is extremely troublesome to calculate. As a simpler method, the one described in the preface will be considered instead of the complicated one. When the pumping head is large enough compared with the water level fluctuation in the surge tank, the effect of the fluctuation upon the flow is so small that it is supposed the simple method can be used without serious errors.

Now on the actual pump plant at Numazawa-numa power station, the difference between the numerical results calculated by the accurate method and those by the simple one, will be shown next. The numerical values essential for the waterhammer calculation of the Numazawa-numa

pump are as follows⁽²⁾⁽³⁾:

N_n : normal speed of pump=500 r.p.m.

H_n : normal head of pump=211 m

Q_n : normal discharge of pump=7.9 m³/sec

L : length of pipe line=605 m

A : average sectional area of pipe=3.8 m²

a : velocity of pressure wave=1 045 m/sec

M_n : normal torque of pump=40 000 m·kg

WR^2 : flywheel effect of rotating parts=48 980 kg·m²

L' : length of tunnel=1 020 m

A' : sectional area of tunnel=8.04 m²

Table 1 gives the calculated results of the time history of the pressure, flow, speed and the water level of surge tank at power failure, when operating at $H_0=190.5$ m and $Q_0=8.88$ m³/sec. The column A is the result calculated by the accurate method, B is the result calculated for the constant water level and C is the result by the

simple method.

The pressures in the column B are slightly higher than those in the column A, but such differences are practically negligible. However, at the time when the pressure becomes minimum ($h=59.5\%$), the fluctuation of the water level in the surge tank is $y=-0.27\%$, or the actual drop of the level is $211 \times 0.0027 = 0.57$ m. When we are considering the water column separation in pipe line, it can not absolutely be said that the drop of level is negligible. However, the pressures and level fluctuations in the column C are almost equal to those in the column A.

7. Conclusion

The waterhammer equation in case the water level of reservoir fluctuates is obtained as Eq. (22) or (23). Using this equation it is possible to calculate the waterhammer in the pumping plant

Table 1 Calculated results

Time $2L/a$	A				B			C		
	Accurate method				Constant level			Simple method		
	$h\%$	$q\%$	$n\%$	$y\%$	$h\%$	$q\%$	$n\%$	$q\%$	$y\%$	$n\%$
0	90.3	112.5	100	0	90.3	112.5	100	112.5	0	90.3
$1/4$	84.2	106.7	95.6	-0.002	84.2	106.7	95.6	106.7	-0.002	84.2
$1/2$	78.5	101.3	91.6	-0.007	78.5	101.3	91.6	101.3	-0.007	78.5
$3/4$	72.5	95.6	87.9	-0.015	72.5	95.6	87.9	95.6	-0.015	72.5
1	67.5	90.8	84.6	-0.026	69.5	90.8	84.6	90.8	-0.026	67.5
$1 1/4$	67.7	79.4	81.6	-0.041	67.7	79.4	81.6	79.4	-0.041	67.7
$1 1/2$	67.5	68.4	78.9	-0.062	67.5	68.4	78.9	68.4	-0.062	67.4
$1 3/4$	66.5	56.1	76.5	-0.090	66.5	56.0	76.5	56.0	-0.090	66.4
2	64.0	44.2	74.4	-0.124	64.0	44.1	74.4	44.1	-0.125	63.9
$2 1/4$	62.3	31.4	72.7	-0.165	62.3	31.3	72.7	31.3	-0.166	62.1
$2 1/2$	60.4	18.5	71.3	-0.213	61.0	18.8	71.3	18.8	-0.214	60.8
$2 3/4$	59.5	4.5	70.1	-0.269	60.0	4.5	70.1	4.5	-0.270	59.7
3	60.2	-9.0	69.1	-0.332	60.5	-9.3	69.1	-9.3	-0.333	60.2
$3 1/4$	63.0	-20.7	68.0	-0.402	63.5	-20.9	68.0	-20.9	-0.403	63.1
$3 1/2$	67.5	-31.0	66.8	-0.478	67.0	-31.3	66.8	-31.3	-0.479	66.5
$3 3/4$	72.5	-41.1	65.3	-0.560	72.5	-41.2	65.3	-41.2	-0.561	71.9
4	77.0	-49.4	63.5	-0.646	77.8	-49.5	63.5	-49.5	-0.648	77.2
$4 1/4$	80.5	-54.9	61.5	-0.736	81.0	-55.1	61.4	-55.1	-0.738	80.3
$4 1/2$	82.5	-58.8	59.3	-0.778	83.3	-60.1	59.1	-60.1	-0.780	82.5
$4 3/4$	83.5	-63.0	56.9	-0.814	84.0	-64.2	56.6	-64.2	-0.816	83.2
5	84.3	-66.3	54.4	-0.847	84.8	-66.6	54.0	-66.6	-0.849	84.0
$5 1/2$	84.5	-70.2	49.0	-0.908	85.5	-71.3	48.6	-71.3	-0.910	84.6
6	86.8	-73.5	43.3	-0.964	87.7	-74.3	42.9	-74.3	-0.966	86.7
$6 1/2$	89.7	-74.4	37.6	-1.016	90.7	-75.5	37.1	-75.5	-1.018	89.7
7	90.5	-74.8	32.1	-1.065	91.7	-75.5	31.3	-75.5	-1.068	90.6
$7 1/2$	88.8	-74.4	26.7	-1.112	90.0	-75.4	25.7	-75.4	-1.115	88.9
8	88.5	-74.3	21.4	-1.157	89.5	-75.0	20.3	-75.0	-1.160	88.3
$8 1/2$	89.9	-74.1	16.2	-1.200	91.2	-74.8	15.0	-74.8	-1.203	90.0
9	91.0	-73.0	11.2	-1.241	92.2	-73.8	9.9	-73.8	-1.245	91.0
$9 1/2$	91.5	-70.9	6.4	-1.281	92.8	-71.6	5.0	-71.6	-1.285	91.5
10	91.8	-68.5	2.0	-1.319	93.4	-69.1	0.5	-69.1	-1.323	92.1
$10 1/2$	93.0	-64.8	-2.0	-1.355						
11	93.7	-61.2	-5.5	-1.389	95.1	-61.6	-7.0	-61.6	-1.394	93.7
$11 1/2$	94.1	-55.9	-8.5	-1.422						
12	96.1	-49.7	-10.9	-1.453	97.7	-50.0	-12.4	-50.0	-1.458	96.2
$12 1/2$	97.3	-42.9	-12.8	-1.482						
13	98.6	-33.6	-14.0	-1.508	100.2	-33.5	-15.4	-33.5	-1.513	98.7
$13 1/2$	98.2	-25.7	-14.8	-1.532						
14	98.3	-15.0	-15.2	-1.553	100.4	-14.5	-16.5	-14.5	-1.558	98.8
$14 1/2$	99.9	-6.1	-15.3	-1.572						
15	94.5	-0.5	-15.2	-1.589	94.4	-1.0	-16.5	-1.0	-1.594	92.8

equipped with a surge tank, in which the water level fluctuates. This calculation is, however, very much troublesome.

According to a numerical example, the change in flow due to the level fluctuation in the surge tank is so small that the results by the simple method will practically give good accuracy, when the pump head is comparatively high.

For the low head pump installations, however, it can not be judged from only the above example whether the simple method can be used or not.

In these cases also, if a few examples have been checked, the judgement will be easily done.

The writer wishes to express his thanks to Mr. A. Kojima, who made the numerical calculations in the example.

References

- (1) For example, R.M. Peabody: *Trans. ASME*, Vol. 61, No. 2 (1939-2), p. 117.
- (2) Tateuchi: *Hitachi Hyoron*, Vol. 35, No. 4 (1953-4), p. 627.
- (3) Kobori: *Hitachi Hyoron*, Vol. 35, No. 6 (1953-6), p. 919.

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Pressure Recovery Efficiency of Short Conical Diffusers and of Roughened Diffusers*

By Yoshimasa FURUYA** and Takao SATO***

Two methods to delay the separation of flow in short conical diffusers are described.

The 10° conical diffusers, having a suddenly enlarged outlet, have great advantages over the simple conical diffusers of same length, when the equivalent angle of divergence is greater than 20°. When the interior wall near the separation point is roughened, the loss of flow becomes lower than in the case of smooth wall because of delay of separation point.

A method of analysing the effects of the separation positions and boundary layer thicknesses, on the loss of flow, is described.

1. Introduction

When the diffuser length is small or the diverging angle is larger than 10°, the loss of flow greatly increases. In order to minimize the loss in the short diffusers, many methods⁽¹⁾ have been considered. Gibson⁽²⁾ proposed a short conical diffuser whose cross section increases gradually and then is suddenly enlarged to its final section. However, these effects are not clear.

This paper is concerned with experiments and theory on the loss of flow in the short conical diffusers and in the roughened diffusers, especially on the effects of the flow separation and of the boundary layer thicknesses.

2. Nomenclature

x : distance along the diffuser axis from the entrance
 x_k : distance between the entrance and the

rough wall boundary
 y : distance from the wall
 r_0 : inner radius of pipe
 D : diameter of pipe
 A : cross sectional area of pipe
 l : length of diffuser
 u : velocity in boundary layer
 $\bar{U}$: cross-sectional mean velocity
 U : velocity outside the boundary layer
 p : pressure
 k : mean radius of sand grains in rough wall
 ζ : coefficient of loss of flow in the diffuser
 η : pressure recovery efficiency of the diffuser

Boundary layer thicknesses :

δ^* : displacement thickness

$$\int_0^{r_0} \left(1 - \frac{u}{U}\right) \left(1 - \frac{y}{r_0}\right) dy$$

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