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On the Roller Straightener

(Report 1, Straightening of Sections)*

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The mechanism of straightening of sections and rails by means of a roller straightener with axial adjustable rolls has not been clarified. The author studied this problem analytically and derived the relations between the largest radii of bending necessary for straightening, at the points of the second and third rolls of straightener and the results of straightening, when the modulus of elasticity and curvature of sections before operating are known.

The author also shows, as a special case, that the sections can be straightened not only in horizontal plane, but also vertical plane in accordance with severe bending in even horizontal plane by the straightener rolls.

The present paper deals with the repeated bending tests and the tension-compression tests for specimens of hard steel, mild steel and brass, as these characteristics of repeated cold works on these materials are the most important points for designing the straightening machine.

1. Introduction

Studies on the roller straightener have hitherto been made chiefly on the bending only about one axis of the section of some plates or sections^{(1)~(5)}. But the roller straightener for sections is different from that for plates; it is so made as to straighten the material by adjusting the rolls along their axes, too.

The aim of this study was to get the necessary data for designing of the straightener by clarifying the behaviour of the materials when they pass through it, taking the mechanism of the straightener into consideration.

2. Deformation of the material caused by alternating bending in the roller straightener

Fig. 1 represents the principle of the straightener with four rolls. The arrow shows the moving direction of the rolls and material. First, the material is bent to nearly the same curvature in one direction through the three rolls A, B and C, regardless of its initial curvature, and then it is bent again in the opposite direction by the roll D so that it may become straight when it springs back.

Generally, rolls with odd numbers are set on the upper or lower side, while the rolls of even numbers are set on the opposite side and adjusted vertically, so that the distance of off-set between the upper and lower rolls may be regulated. Besides, it is possible for each upper and lower roll to be adjusted along the axis of the roll. So the material may be straightened not only along one axis but also alternating bending may be performed along the axis at right angles to that axis. Thus it is possible to straighten along two axes at the same time.

Now in clarifying the deformation of the material inside the straightener, the following conditions are assumed to simplify mathematical treatment.

- (1) The shape of cross section is symmetric according to the two axes.
- (2) The cross sections remain on a plane during the process of bending, and always make right angles to the axis of the material.
- (3) The cross sections never change in their

Fig. 1 Principle of the roller straightener

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shape during the process of bending.

(4) Every component other than the stress in the longitudinal direction is zero.

(5) The relation between the longitudinal stress of the fiber and the longitudinal strain conforms to the plastic curve for simple tension and compression.

(6) The material is of perfect elastic-plastic material, and the stress-strain relations in uniaxial tension and compression are the same. It has the same modulus of elasticity and yield point in tension and compression.

(7) The material has no initial internal stress before straightening.

We consider that the assumptions (2) to (5) approximately hold.

Repeated tension and compression hysteresis curve of the material is deemed to be as Fig. 2 according to assumption (6).

In accordance with assumption (2), the strain at a certain point on the cross section is in proportion to the distance between the axis of bending and that point. According to assumption (1), (5) and (6), strain zero line inside the material passes the centre of cross section and is constant.

Hence the strain on the cross section follows the same process as indicated by Fig. 3 owing to the repeated bending.

In Fig. 3, the strain that is caused when the bent material is first straightened, is represented by $\overline{a0a'}$ line.

Now, when the strain caused by the first bending is represented by $\overline{101'}$ line, the plastic strain $\overline{1_e1_a01_a'1_e'}$ is left behind after the elastic strain ϵ_e is deducted from $\overline{101'}$ line.

Next, when the second bending, which is smaller than the first bending, is applied in the opposite direction and the strain caused in consequence is represented by $\overline{202'}$ line, the plastic

strain $\overline{2_e2_a1_a01_a'2_a'2_e'}$ is left behind after the elastic strain is deducted from it just as above.

When a still smaller bending $\overline{303'}$ is applied in the opposite direction to the second, the plastic strain $\overline{3_e3_a2_a1_a01_a'2_a'3_a'3_e'}$ is left behind.

If the initial strain is $\overline{b0b'}$ instead of $\overline{a0a'}$ and the three bendings are applied as in the former case, the final plastic strain distribution will be $\overline{3_e3_a2_a1_b01_b'2_a'3_a'3_e'}$, thus showing that the latter is different from the former in strain distribution only near the central.

When the material is made straight after straightening, the plastic strain distribution of the material is, regardless of its initial stress being positive or negative, to draw $\overline{3_e3_a2_a1_b}$ and $\overline{1_b'2_a'3_a'3_e'}$ in its outer layer, while in its centre parts it draws either $\overline{1_b1_a01_a'1_b'}$ or $\overline{1_b01_b'}$, thus showing different plastic and elastic strain distributions according to the positive or nature of the initial strain.

The straightened sections are at the equilibrium states of internal stress which produced proportional to the distance of the plastic or elastic strain line from axis $\overline{Y0Y'}$ and hold the straightness.

The moment of the outer layer about the bending centre may be nullified by regulating the off-set of the straightening rolls, but the initial strain on the centre parts cannot be eliminated.

Thus the straightening accuracy of the straightened material is decided by the balance on the internal stress between the centre parts and the whole section.

The left side of Fig. 4 denotes a section of materials to be straightened. When the material is bent about the axis $X-X'$ and then alternating bending about the axis $Y-Y'$ is given, the resultant bending axis will be the same as that

Fig. 2 Stress-strain curve due to alternating bending of the material

Fig. 3 Strain due to alternating bending

resulting from alternating bending about the axis $X_1-X'_1$ or the axis $X_2-X'_2$.

Let us consider the case in which the curvature of initial strain of the material is smaller than that of bending by means of the rolls. Now it is assumed that,

r_{1x}, r_{1y} : the bending radii of the neutral axis after the material has been bent about the axes $X-X'$ and $Y-Y'$ by the first and second rolls,

r_{0x}, r_{0y} : the initial bending radii of the neutral axis about the axes $X-X'$ and $Y-Y'$ before straightening of the material, i.e. straightness of the material,

ϵ_e : the strain at which the yielding of the material begins,

a_x, a_y : the distance from the point at which the yielding of the material begins when the straight material has been bent to the radii r_{1x} and r_{1y} to the centre of bending.

Then the following equation is obtained:

$$\epsilon_e = \frac{a_x}{r_{1x}} \dots \dots \dots (1)$$

When the curvature of the material before straightening is assumed to be equal both in the positive and the negative side and thus, in the right side of Fig. 4, it is assumed that $\overline{sb} = \overline{sa}$, the difference in the internal stress is roughly double the stress indicated by the strain distribution $\overline{c_1 0 c_1'}$ in the figure.

Because, in the figure,

$$a_x' = \frac{a_x}{r_{0x}} \left/ \left(\frac{1}{r_{1x}} - \frac{1}{r_{0x}} \right) \right. \dots \dots \dots (2)$$

and assuming that $r_{0x} \gg r_{1x}$, a_x' will tend to zero.

In case the material is made straight after straightening, when the locus of the point corresponding to the point c in the right side of Fig. 4 is plotted on the left side of Fig. 4 and it is indicated by $\overline{c_1 c_2 c_1' c_2'}$ line, the inside of this locus is subjected to the bending moment corresponding to the strain before straightening about the central

Fig. 4 Combined alternating bending for straightening of sections

axis of bending of the material because that area has not suffered any plastic deformation due to alternating bending.

Now assuming that the moment about the axis $X-X'$ is M_{1x} , we obtain

$$\begin{aligned} M_{1x} &= \int_{(c_1 c_2 c_1' c_2')} \epsilon E x d f \\ &= 4E \int_0^{a_y} \frac{2}{3} \epsilon_e \frac{r_{1x}}{r_{0x}} \cdot \frac{a_x^3}{a_y^3} (a_y - x)^3 dx \\ &= \frac{1}{3} \epsilon_e^4 \frac{r_{1x}^3 r_{1y}}{r_{0x}} E \dots \dots \dots (3) \end{aligned}$$

where, E denotes the modulus of elasticity and $d f$ denotes the element of area in the cross section.

In the same way about the axis $Y-Y'$, we obtain

$$M_{1y} = \frac{1}{3} \epsilon_e^4 \frac{r_{1x} r_{1y}^3}{r_{0y}} E \dots \dots \dots (4)$$

Now, the moments of inertia of the plastic region of the cross sectional area about the axes $X-X'$ and $Y-Y'$ are I_{ax} and I_{ay} respectively, and the residual radii of curvature of the material about the axes $X-X'$ and $Y-Y'$ after straightening are r_{sx} and r_{sy} , then the moments of these region about the axes $X-X'$ and $Y-Y'$ are as follow.

$$M_x = EI_{ax}/r_{sx}, \quad M_y = EI_{ay}/r_{sy} \dots \dots \dots (5)$$

When the material inside the elastic region $c_1 c_2 c_1' c_2'$ is bent back to the radii r_{sx} and r_{sy} , the moments with respect to the centre are to be M_{1x}' and M_{1y}' , then we obtain

$$M_{1x}' = \frac{1}{3} \epsilon_e^4 r_{1x}^3 r_{1y} \left(\frac{1}{r_{0x}} - \frac{1}{r_{sx}} \right) E \dots \dots \dots (6)$$

the M_x and M_y are equal to the M_{1x}' and M_{1y}' respectively,

$$\frac{1}{3} \epsilon_e^4 r_{1x}^3 r_{1y} \left(\frac{1}{r_{0x}} - \frac{1}{r_{sx}} \right) E = \frac{EI_{ax}}{r_{sx}} \dots \dots \dots (6)'$$

as shown in above

$$\frac{1}{3} \epsilon_e^4 r_{1x} r_{1y}^3 \left(\frac{1}{r_{0y}} - \frac{1}{r_{sy}} \right) E = \frac{EI_{ay}}{r_{sy}} \dots \dots \dots (6)''$$

and we can take that

$$\left. \begin{aligned} \frac{1}{3} \epsilon_e^4 r_{1x}^3 r_{1y} &= I_{ex}, \quad \frac{1}{3} \epsilon_e^4 r_{1x} r_{1y}^3 = I_{ey} \\ I_{ax} + I_{ex} &= I_x, \quad I_{ay} + I_{ey} = I_y \end{aligned} \right\} \dots \dots \dots (6)'''$$

where I_{ex}, I_{ey} ; moments of inertia of elastic region about the axes $X-X'$ and $Y-Y'$ respectively.

I_x, I_y : moments of inertia of the all sectional area of the material about the axes $X-X'$ and $Y-Y'$.

Hence

$$\frac{I_x}{r_{sx}} = \frac{1}{3} \epsilon_e^4 \frac{r_{1x}^3 r_{1y}}{r_{0x}}, \quad \frac{I_y}{r_{sy}} = \frac{1}{3} \epsilon_e^4 \frac{r_{1x} r_{1y}^3}{r_{0y}} \dots \dots (7)$$

Or, solving the above equations about r_{1x} and r_{1y} , we obtain

$$\left. \begin{aligned} r_{1x} &= \frac{1}{\epsilon_e} \sqrt[3]{9 \frac{I_x^3}{I_y} \cdot \frac{r_{sy}}{r_{sz}^3} \cdot \frac{r_{0x}^3}{r_{0y}}} \\ r_{1y} &= \frac{1}{\epsilon_e} \sqrt[3]{9 \frac{I_y^3}{I_x} \cdot \frac{r_{sz}}{r_{sy}^3} \cdot \frac{r_{0y}^3}{r_{0x}}} \end{aligned} \right\} \dots\dots\dots (7)'$$

Now, assuming that the accuracies of straightening of the material about the axes $X-X'$ and $Y-Y'$ are the same i.e. $r_{sz}=r_{sy}$, we obtain from Eq. (7)

$$\frac{r_{1x}^2}{I_x r_{0x}} = \frac{r_{1y}^2}{I_y r_{0y}} \dots\dots\dots (8)$$

Moreover, assuming that the initial strains on the axes $X-X'$ and $Y-Y'$ are the same, we obtain $r_{0x}=r_{0y}$, therefore Eq. (8) is

$$r_{1x}^2/I_x = r_{1y}^2/I_y \dots\dots\dots (9)$$

Hence, using the two conditions

$$r_{sz}=r_{sy}, \quad r_{0x}=r_{0y}$$

we obtain from Eq. (7)

$$r_{1x} = \frac{1}{\epsilon_e} \sqrt[4]{3 \frac{r_{0x}}{r_{sz}} \left(\frac{I_x^3}{I_y} \right)^{1/2}} = \frac{E}{\sigma_e} \sqrt[4]{3 \frac{r_{0x}}{r_{sz}} \left(\frac{I_x^3}{I_y} \right)^{1/2}} \dots\dots\dots (10)$$

$$r_{1y} = \frac{1}{\epsilon_e} \sqrt[4]{3 \frac{r_{0y}}{r_{sy}} \left(\frac{I_y^3}{I_x} \right)^{1/2}} = \frac{E}{\sigma_e} \sqrt[4]{3 \frac{r_{0y}}{r_{sy}} \left(\frac{I_y^3}{I_x} \right)^{1/2}} \dots\dots\dots (11)$$

Thus from Eqs. (10) and (11), maximum bending radius for straightening is to be decided, provided the allowable residual radii of bent r_{sz} and r_{sy} after straightening, i.e., the accuracy of straightening of the material, and the bent of the material before straightening, r_{0x} and r_{0y} are known.

Now the above mentioned is the case in which alternating bending has been applied about two slanted axes in the left side of Fig. 4. When straightening is performed with the bending radius small enough, it may be satisfactorily carried out by alternating bending about the axis $X-X'$ of section of the material with the rolls adjusted along their longitudinal axes in such a way that they are all aligned.

Of course, before straightening, the material is bent about the axis $Y-Y'$ so in the course of straightening, it becomes straight about the axis $Y-Y'$ and consequently the bending moment about this direction is also added.

Again in Fig. 4, $c_1 c_2 c_1' c_2'$ is to be the area in which the elastic region remains when the material passes through the straightener, in case initial bending radii of the material are r_{0x} and r_{0y} and the bending radius for straightening about the axis $X-X'$ is r_{1x} only. Then we obtain

$$a_x = \epsilon_e r_{1x}, \quad a_y = \epsilon_e r_{0y} \dots\dots\dots (12)$$

When the material is made straight, the moment added upon the elastic region with respect to the centre parts of the material is, assuming

that $a_y \leq h$

$$M_x = \int_{(c_1 c_2 c_1' c_2')} \epsilon E d f = \frac{1}{3} E \epsilon_e^4 \frac{r_{0y} r_{1x}^3}{r_{0x}} \dots\dots\dots (13)$$

where, $2h$ denotes the width of the material.

Assuming that $a_y \geq h$, we obtain

$$\begin{aligned} M_x &= \int_{(c_1 c_2 c_1' c_2')} \epsilon E d f \\ &= \frac{1}{3} E \epsilon_e^4 \frac{r_{0y}}{r_{0x}} r_{1x}^3 \left\{ 1 - \left(1 - \frac{h}{\epsilon_e r_{0y}} \right)^4 \right\} \dots\dots\dots (14) \end{aligned}$$

Hence, according to the same notion as Eqs. (6)' and (6)'', assuming that $a_y \leq h$, we obtain

$$r_{1x} = \sqrt[3]{3 \frac{I_x}{\epsilon_e^4 r_{sz}} \frac{r_{0x}}{r_{0y}}} = \sqrt[3]{3 \frac{E^4}{\sigma_e^4} \frac{I_x}{r_{sz}} \frac{r_{0x}}{r_{0y}}} \dots\dots\dots (15)$$

Assuming that $a_y \geq h$, we obtain

$$r_{1x} = \sqrt[3]{3 \frac{I_x}{\epsilon_e^4 r_{sz}} \frac{r_{0x}}{r_{0y}} \frac{1}{\{1 - (1 - h/\epsilon_e r_{0y})^4\}}} \dots\dots\dots (16)$$

From Eqs. (15) and (16), it is possible to decide the bending radius requisite for straightening when the bending radii of the material before straightening, i.e. r_{0x} and r_{0y} and the allowable residual warping radius about the axis $X-X'$ after straightening, i.e. r_{sz} are known.

In the same way in the direction of the axis $Y-Y'$ assuming that $a_y \geq h$, we obtain

$$M_y = \frac{4}{3} \frac{E}{r_{0y}} h^3 \left(a_x - \frac{3}{4} \frac{a_x}{a_y} h \right) \dots\dots\dots (17)$$

Assuming that $a_y \leq h$, we obtain

$$M_y = \frac{1}{3} E r_{1x} \epsilon_e^4 r_{0y}^2 \dots\dots\dots (18)$$

Hence, we obtain

$$r_{1x} = \frac{3}{4} I_y \frac{r_{0y}}{r_{sy}} \frac{1}{h^3 (\epsilon_e - 3h/4r_{0y})} \dots\dots\dots (19)$$

for $a_y \geq h$

$$r_{1x} = 3 \frac{I_y}{r_{sy}} \frac{1}{\epsilon_e^4 r_{0y}^2} = 3 \frac{E^4}{\sigma_e^4} \frac{I_y}{r_{sy} r_{0y}^2} \dots\dots\dots (20)$$

for $a_y \leq h$

Thus it is possible to decide the bending radius for straightening by calculating the value of r_{1x} from Eqs. (16), (19) or (17), (18) and by adopting the smallest value of these radii.

3. Axial thrust upon the rolls of the roller straightener

Roller straightener has generally the mechanism to straighten the material whose axis with large section modulus is at right angles to the rolls. For instance, the I-beam is passed through the straightener with its web horizontal.

Therefore it seems that the axial thrust upon the rolls of the straightener is very large, but it is known to be extremely small as compared with the load along the direction of the radius⁽⁴⁾.

In Fig. 5 when the material to be straightened is bent about the axis $X-X'$ with the radius of

r_x , and about the axis $Y-Y'$ with the radius of r_y , the central axis of bending becomes the axis $X-X'$.

Stress and strain distribution along the line $A-A$ denoted on the figure of cross section of the material is indicated on the right side of Fig. 5.

That is, when the beam is subjected to the bending about the axis $X-X'$, strain becomes $\overline{101'}$ on the side $A-A$ of the beam as shown in the right side of the figure, and the stress is represented by $\overline{34056}$.

If the beam is subjected to the bending about the axis at the same time, strain increases to $\overline{20'2'}$ and stress becomes $\overline{34'0'5'6}$.

If the section $\overline{00'}$ upon the cross section is indicated on the left side of the figure, it becomes the shaded area between the axes $X-X'$ and X_1-X_1' .

Therefore, the necessary moment for bending about the axis $Y-Y'$ is represented by the following equation,

$$M_y = \frac{4}{3} h^3 \frac{r_{1x}}{r_{1y}} \sigma_e \quad (21)$$

Fig. 5 Stress and strain due to the resultant bending

Fig. 6 Specimen

using Eq. (9), we obtain

$$M_y = \frac{4}{3} h^3 \left(\frac{I_x}{I_y} \right)^{1/2} \cdot \sigma_e \quad (22)$$

4. Required power of the straightener

Net power required for straightening can be found as the sum of area bounded by tension and compression hysteresis curves when each point on the material passes through the straightener. From Fig. 2, we obtain

$$dW = \sigma_e (\epsilon - \epsilon_e) dV \quad (23)$$

where dW denotes the work done for volume dV to draw tension and compression hysteresis curve. Integrating this about all the sectional area, we obtain

$$\begin{aligned} W &= \sigma_e \int_{(Y)} \left(\frac{\eta}{r} - \epsilon_e \right) df \cdot \Delta l \\ &= \sum \left\{ \sigma_e \int_{(Y)} \left(\frac{\eta}{r} - \epsilon_e \right) df + \int_{(E)} \sigma \frac{\eta}{r} df \right. \\ &\quad \left. - \int_{(E)} \sigma \frac{\eta}{r} df \right\} \Delta l \\ &= \sum \left(\frac{M}{r} - \sigma_e \epsilon_e A_{(Y)} - \frac{M_e}{r} \right) \Delta l \quad (24) \end{aligned}$$

where the suffix (E) and (Y) of the integral, denote respectively the integration about the elastic region and the plastic region of the section of the material, and the notations are

r : the bending radius about centre of bending on the section

df : the element area of the section

η : the distance from centre of bending to df

$A_{(Y)}$: the yielding part of section, i.e. area of plastic region

M_e : the bending moment of the elastic region alone on the section

M : the total bending moment of the section
Therefore, neglecting the elastic strain of the material, the theoretical required power is,

$$P = v \frac{M}{r} \quad (25)$$

where v denotes velocity of straightening, and Σ means summing up of each alternating cycle.

5. Experiment

To decide the yield bending moment and hysteresis curve of the material, a tensile test, repeated tensile and compressive test and alternating bending test were performed using the low carbon steel (SS41), high carbon steel (S45C) and forged brass (BSBF). The specimens were cut off from the same raw material and normalized.

Fig. 6 shows the shape of each specimen.

Matsumura-type 30 ton universal testing machine was used for tensile test, and tensile and

compressive test, 30 ton Riehle-type universal testing machine for compression test, and Matsu-mura-type universal testing machine for tension test.

And for bending test, Riehle-type universal testing machine was used.

For the purpose of measuring the elastic strain of the specimen, Okhuizen-type mechanical strain gauge was used.

In the case of tensile test, two dial gages were fixed at the ends of specimen with large diameter as shown in Fig. 7, and the mean value of their

Fig. 7 Repeated tensile and compressive testing machine

readings was taken as the strain of the specimen.

In the case of bending test, using the deflection measuring instrument as shown in Fig. 8, we could read the deflection of the centre up to 1/100 mm scale for 100 mm gauge length.

Table 1 shows the results of tensile test.

Fig. 9 shows the tension and compression hysteresis curve.

The numbers marked in the figure shows the order of the deformation. The materials had no work hardening by repeated tension and compression, and hard steel had larger Baushinger's effect than mild steel.

Fig. 10 shows the principle of the bending test and Fig. 11 shows its results.

When the steel beam was subjected to first bending, and even the stress on the outer layer increased over the yield point which was shown

Table 1 Results of tensile tests

Material	Tensile strength kg/mm ²	Yield point kg/mm ²	Elonga- tion %	Contraction percentage of area %	Modulus of elasti- city kg/cm ² ×10 ⁶
Forged brass BSBF	47.2		20.8	19.5	0.89
Mild steel SS41	43.9	upp. 33.1 low. 26.7	37.8	63.0	2.1
Hard steel S45C	63.2	upp. 41.9 low. 37.6	31.2	55.1	2.1

Fig. 8 Deflection measuring instrument

Fig. 9 Repeated tension and compression hysteresis curve

in the case of tensile test, deflection of the beam continued to increase linearly at the same rate as in the elastic region and then sharply increased when the bending moment become half as large again as the moment to which the outer layer of the beam was subjected when the stress on it reached the yield point. Subjected to bending in opposite direction, the beam showed conspicuous Baushinger's effect after spring back and the hysteresis curve took the form as shown in the figure.

In the alternating bending test, the material is not subjected to work hardening and the steel has even an inclination to be softened.

In the case of alternating bending test, the length of neutral axis of the beam tended to elongate slightly.

Table 2 shows the results of measurement of that elongation.

This did not coincide with the inclination that the length of material which passed through the straightener became shorter.

It seemed to be not because of alternating bending, but because the off-set on the inlet side of the straightener was larger than that of the outlet and the bending radius for straightening on the inlet side was smaller than that of the outlet side, so that in case the circumferential speeds of rolls were equal, the centre speed of gravity of

material on the inlet side was larger than that of the outlet side and therefore the material was subjected to compression.

6. Conclusion

The data requisite for the designing of the roller straightener have been obtained.

That is:

(1) Knowing the modulus of elasticity of the material, yield stress, shape of the section and the condition of initial warp, the bending radius for straightening to obtain the proper accuracy requisite for straightening can be decided.

(2) In case the rolls line up in the direction of their central axes, if the proper bending radius for straightening is given, the accuracy requisite for straightening may be obtained not only in the horizontal direction but also in the longitudinal.

(3) The axial thrust on the roll can be calculated and the required power can be obtained as the sum of area bounded by the hysteresis curve for each point inside the material.

(4) The nature of the repeated cold work of the material requisite for designing the straightener has been experimentally clarified.

(5) It is not because the material is subjected to alternating bending that it becomes shortened after passing through the straightener.

In the case of simple bending, the material

Fig. 10 Principle of the bending test

Table 2 Variation of neutral axis due to alternating bending test

Material	Before bending test mm	After bending test mm	Difference μ
Forged brass BSBF	131.110 8	131.121 8	11.0
Mild steel SS41	120.053 5	120.100	46.5
Hard steel S45C	122.200 0	122.217 3	17.3

Fig. 11 Hysteresis curve for alternating bending

has on the contrary an inclination to elongate.

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