diffuser is increased.

References

- (1) T. Ueda: *Trans. Japan Soc. Mech. Engrs.*, Vol. 18, No. 67, (1952), p. 103.
 (2) T. Ueda: *Chem. Engng. Japan*, Vol. 24, No. 5, (1960), p. 264 or T. Ueda: *Trans. Japan Soc. Mech.*

- Engrs.*, Vol. 19, No. 80, (1953), p. 31 and Vol. 18, No. 67, (1952), p. 103.
 (3) K. Hayami: *Trans. Japan Soc. Mech. Engrs.*, Vol. 8, No. 31, (1942), p. II-25.
 (4) T. Ueda: *Trans. Japan Soc. Mech. Engrs.*, Vol. 19, No. 80, (1953), p. 31 and Vol. 18, No. 72, (1952), p. 56.

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On the Emissivity of a Gas which Contains Particles*

By Susumu YOKOBORI**

The author has found a new theoretical way of calculating the emissivity of a gas which contains particles. The emissivity of a gas, which contains a group of particles, is calculated by the following equations. ϵ_{Ga} is the emissivity by the same gas when it does not contain particles.

$$\begin{aligned}\epsilon_{Ga} &= 1 - e^{-(K_{GL}L)} \\ \epsilon_{Ga}' &= 1 - e^{-(K_{GL}L)'} \\ (K_{GL}L)' &= (K_{GL}L)(1 + \gamma) \\ \gamma &= \frac{1}{K_{G\Delta L}} \cdot \frac{\epsilon_P}{\Delta L} \cdot \frac{A_P}{A_B}\end{aligned}$$

where L is the thickness of gas layer. The particles are assumed to distribute uniformly and constitute spheres of equal diameter. A_P is the area of a sphere surface. Side planes of an imaginary cube, which corresponds to a sphere, are perfectly black and A_B is the area of a cube surface. As ΔL is the length of edge of a cube, A_B is $6(\Delta L)^2$. $(K_{G\Delta L}\Delta L)$ and $(K_{GL}L)$ are factors corresponding to the thickness of gas layers ΔL and L for ϵ_{Ga} of a gas as in above equations. These are given by Hottel and others for non-luminous gases.

The experiments by the author show that the results agree well with his theoretical analysis. The experimental results by Sherman, being analysed by author's method, show no contradiction. The experimental formula by Lindmark and Wohlenberg are expressed as special cases of author's theory. For a gas, which contains two or more groups of particles calculation can be made by repeating the above method. From the calculation it is revealed that an emissivity of a gas, which contains a group of particles, is greater than that of a gas alone. In this case the temperature of particles is assumed to be not lower than that of a gas. The increment of emissivity is expressed by the above equations.

1. The aim of the study

In order to calculate the amount of heat transferred in a combustion chamber of boiler or similar cases, one should consider the heat transferred by thermal radiation. In these cases, in general, gas in the chamber contains coal particles, oil droplets, segregated carbon particles, coke dust, ash particles and so on. It is the aim of the author's study to

formulate theoretically the equations to calculate the emissivity of the gas, using the dimensions, distribution and amount of these particles. Some experiments are carried out by the author, with the results showing good agreement with the theoretical analysis.

2. Theory

2.1 The aim

The aim of this paper is to get the theoretical formulae for the equivalent emissivity of gas which contains particles.

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2.2 The assumptions

In order to treat the problems theoretically, the author assumes the following conditions:-

(1) The conditions (i.e. temperature, composition, emissivity and so on) of the gases within the scope which is under consideration are uniform.

(2) The particles are distributed uniformly in the gas.

(3) The particles are spherical, of uniform quality and of equal diameter.

(4) The amounts of particles in the gas are so little that the states of the gas are not affected by their existence.

(5) The size of particle under consideration lies between 0.02 and 6 mm in diameter. Particles, whose diameters are smaller than 10μ or 0.01 mm, are ignored for the present. The reason why the author so assumes is that $(1\sim 10)\mu$ is the wave length for radiant heat and the diameter of a particle, 10μ , is nearly equal to $(1\sim 10)\mu$. And for this reason there are too many ambiguous points to apply the theory for radiant heat transfer to this wave length range. And if one can assume that in this domain the theory of radiation also holds, the following conclusions are applicable to this range.

2.3 Theoretical declarations

2.3.1 Theory for one group of particles in gas

(1) Theory As the particles are distributed uniformly in gas, the author considers an arbitrary particle and the corresponding gas volume. Let us call the volume of the gas as "unit volume" of the gas. In the "unit volume" B_0 , a particle P is surrounded by gas G . At the boundary surface of the "unit volume", he assumes black surfaces B . The space within the black surfaces B is filled with the gas G . The space under consideration is occupied with these "unit volumes" leaving no room. And he now considers the "unit volume" B_0 (Fig. 1). The author calculates the amount of heat Q , transferred to this imaginary black surface from the particle P in the center of the unit volume and from the gas G which fills the unit volume. Hereby it is assumed that the amount of convection heat transfer is negligible as compared with that of radiant heat transfer.

Then another volume B_0' is imagined whose dimensions and form are perfectly the same as the above unit volume. But, in this case, it is assumed that within the volume B_0' is uniformly occupied by another imaginary gas G' and that there is no particle in it (Fig. 2). Then the amount of heat Q' transferred to the surrounding imaginary black surface from the gas G' is calculated. Assuming that the temperatures of the two surfaces

B and B' are the same, and those of the two gases G & G' are also the same, the author can obtain the emissivity $\Delta\epsilon_G'$ of an imaginary gas containing no particle, by equating the two quantities Q_G' and Q_G . In this way we can replace an imaginary volume of gas G' in B_0 instead of a real gas G and a particle P in the same volume. Then, it is possible to write in terms of $\Delta\epsilon_G$, ϵ_P and the form and dimensions of gas volume G_0 , the particle P and the surfaces B (Nomenclature in Table 1).

(2) Calculation for a group of particles

In the previous section it was assumed that an imaginary gas G' of a small "unit volume" can be substituted for gas G and a particle P . Then, in the same way, it is possible to replace gas G and particle P in the whole space under consideration with the gas G' .

(3) Calculation for different kinds of particle groups Supposing, as the first step, that there is only one group of particles, the method of calculation is the same as expressed in the previous section. Gas G' is a substitute for gas G and a particle group P . Then the coexistence of particle group No. 2 in the gas G' is assumed. In the same way the imaginary gas G'' is obtained. Repeating the procedure, one can calculate the emissivity of the gas, which contains several groups of particles, which can be mathematically proved. But in this paper it is omitted for brevity.

2.4 Theoretical formulae

2.4.1 Theoretical calculation for a group of particles

(1) Assumptions and notations Particle P is a sphere of diameter d . The "unit volume" of gas B_0 that envelopes a particle P is a "unit cube" (Fig. 3). The space under consideration is filled up with these "unit cubes" and there is no room left unfilled.

Notations for the case are tabulated in Table 1.

For the temperature in the "unit cube", it is assumed as shown in Fig. 4. The notations are also described in the same figure.

Fig. 1 Unit volume B_0

Fig. 2 Imaginary unit volume B_0'

Fig. 3 Unit cube \$B_0\$

Fig. 4 Temperature distribution in volume \$B_0\$ (at section \$abcb'a'\$)

Fig. 5 Relation between \$K\$, \$KL\$ and \$L\$

$$\epsilon_c = 1 - e^{-KL}$$

\$\epsilon_c\$: emissivity of \$CO_2\$ gas

\$L\$: thickness of gas layer

\$Pc\$: partial pressure of gas (at)

(McAdams: *Heat Transmission*, 3rd edit, (1955); Hottel: *Radiant Heat Transmission*, P. 83, Fig. 4-13)

Fig. 6 Relation between \$KL\$ and \$PcL\$

(2) Formulae The formula is deduced as follows: Let emissivity of gas alone in a small "unit volume or cube" be \$\Delta\epsilon_{Ga}\$ and that of an

Table 1 Notations

	Suffix	Area	Temperature	Emissivity
Particle	\$P\$	\$A_P\$	\$T_P\$	\$\epsilon_P\$
Imaginary black surface	\$B\$	\$A_B\$	\$T_B\$	\$\epsilon_B=1\$
Gas (real)	\$G\$	\$A_G\$	\$T_G\$	\$\Delta\epsilon_{Ga}, \epsilon_G\$
Gas (imaginary)	\$G'\$	\$A_{G'}\$	\$T_{G'}\$	\$\Delta\epsilon_{Ga'}, \epsilon_{G'}\$
Total area		\$A_T = A_G = A_P + A_B\$		

imaginary gas \$\Delta\epsilon_{Ga'}\$ (Table 1)

$$\Delta\epsilon_{Ga'} = \epsilon_P (1 - \Delta\epsilon_{Ga}) \cdot \frac{A_P}{A_B} \cdot \left(\frac{T_P}{100} \right)^4 - \left(\frac{T_B}{100} \right)^4$$

$$+ \Delta\epsilon_{Ga} \left\{ 1 + (1 - \Delta\epsilon_{Ga}) (1 - \epsilon_P) \frac{A_P}{A_B} \right\}$$

$$\times \left(\frac{T_G}{100} \right)^4 - \left(\frac{T_B}{100} \right)^4 \dots (1)$$

where \$T_{G'}\$ is the temperature of the imaginary gas and equals to \$T_G\$. From the equation the relation between \$K_{GAL}\$ and \$K_{GAL}'\$ is obtained corresponding to \$\Delta\epsilon_{Ga}\$ and \$\Delta\epsilon_{Ga}'\$. Then the relation \$\epsilon_{Ga}\$ is obtained by calculating \$K_{CL}'\$ from the following Eqs. (5) and (6). The relations between \$K\$, \$KL\$, \$KAL\$ and \$\Delta L\$ are shown in Fig. 5 and that of \$KL\$ and \$PcL\$ for \$CO_2\$-gas in Fig. 6.

(3) Approximate formulae As, in general, it is possible to assume \$T_P \approx T_G = T_{G'}\$, Eq. (1) will be reduced to Eq. (2).

$$\Delta\epsilon_{Ga'} = \Delta\epsilon_{Ga} + (1 - \Delta\epsilon_{Ga}) (\epsilon_P + \Delta\epsilon_{Ga} - \epsilon_P \Delta\epsilon_{Ga}) \frac{A_P}{A_B} \dots (2)$$

When the diameter of a particle is small, \$\Delta\epsilon_{Ga}\$ and \$\Delta\epsilon_{Ga}'\$ are also small and so Eq. (3) is obtained. As \$KAL\$ and \$KAL'\$ are small and it is possible to neglect the higher order of these values, Eq. (4) is obtained.

$$\Delta\epsilon_{Ga} = 1 - e^{-(K_{GAL} \Delta L)} \approx K_{GAL} \Delta L$$

$$\Delta\epsilon_{Ga'} = 1 - e^{-(K_{GAL}' \Delta L)} \approx K_{GAL}' \Delta L \dots (3)$$

$$\frac{K_{GAL}'}{K_{GAL}} = 1 + \frac{1}{(K_{GAL} \Delta L)} \cdot \epsilon_P \cdot \frac{A_P}{A_B} \dots (4)$$

Above calculations are deduced for small cubes (edge length is \$\Delta L\$). While it is possible to assume that the small unit cube is filled with a gas of emissivity \$\Delta\epsilon_{Ga}'\$ instead of a gas of emissivity \$\Delta\epsilon_{Ga}\$ and a particle \$P\$, it is also rigorous to think that the whole space under consideration is filled with a gas of emissivity

ε_{Ga}' in place of a gas of emissivity ε_{Ga} and particles P . And the following equations are deduced.

$$\frac{\varepsilon_{Ga}'}{\varepsilon_{Ga}} = \frac{1 - e^{-K_{GL}L}}{1 - e^{-K_{GGL}L}} \quad \dots\dots\dots (5)$$

$$\frac{K_{GL}'}{K_{GL}} = \frac{K_{GAL}'}{K_{GAL}} = 1 + \frac{1}{(K_{GAL}\Delta L)} \cdot \varepsilon_P \frac{A_P}{A_B} \quad \dots\dots\dots (6)$$

$$K_{GL}' = K_{GL} + \frac{K_{GL}}{K_{GAL}} \cdot \frac{\varepsilon_P}{\Delta L} \cdot \frac{A_P}{A_B} \quad \dots\dots\dots (7)$$

$$\equiv K_{GL} \cdot (1 + \gamma) \quad \dots\dots\dots (8)$$

$$\gamma = \frac{1}{K_{GAL}} \cdot \frac{\varepsilon_P}{\Delta L} \cdot \frac{A_P}{A_B} \quad \dots\dots\dots (9)$$

$$\begin{aligned} \varepsilon_{Ga}' &= 1 - e^{-K_{GL}L} \\ &= 1 - e^{-\{K_{GGL} + (K_{GL}/K_{GAL}) \cdot (\varepsilon_P/\Delta L) \cdot (A_P/A_B) \cdot L\}} \quad \dots\dots\dots (10) \end{aligned}$$

The term (K_{GGL}) or absorption strength in the Eq. (7) corresponds to gas alone and the following term $(K_{GL}/K_{GAL}) \cdot (\varepsilon_P/\Delta L) \cdot (A_P/A_B) \cdot L$ is the value for the increment of emissivity due to particles in the gas.

This second term is also a function of K_{GL} and K_{GAL} of the gas alone. It is also possible to formulate by the following equations.

$$\varepsilon_{Ga}' = 1 - e^{-\{K_{GGL} + (K_{GL}/K_{GAL}) \cdot (\varepsilon_P/\Delta L) \cdot (A_P/A_B) \cdot L\}} \quad \dots\dots\dots (11)$$

$$\left. \begin{aligned} K_P &= \frac{K_{GL}}{K_{GAL}} \cdot \frac{\varepsilon_P}{\Delta L} \cdot \frac{A_P}{A_B} \equiv \frac{K_{GL}}{K_{GAL}} \cdot K_{P0} \\ K_{P0} &= \frac{\varepsilon_P}{\Delta L} \cdot \frac{A_P}{A_B} \end{aligned} \right\} \quad \dots\dots\dots (12)$$

2.4.2 Equations in the case of coexistence of many particle groups When there are two groups of particles in gas, l_1 and l_2 , assume that the number of particles of l_2 per unit volume is greater than that of l_1 . Imagine that there exists only one group l_2 . Then the following equations are obtained according to Eq. (9).

$$K_{GL}'_{l_2} = K_{GL}(1 + \gamma_{l_2}) \quad \dots\dots\dots (13)$$

where K_{GL} is an exponent of emissivity equation for particleless gas according to Eq. (3); $K_{GL}'_{l_2}$ is an exponent of emissivity equation for an imaginary particleless gas taking account of the group l_2 ; γ_{l_2} is an increment of K_{GL} due to particle group l_2 [c.f. Eq. (9)].

Then it is supposed to add another group l_1 to the imaginary particleless gas considered above.

$$K_{GL}'_{l_1 l_2} = K_{GL}'_{l_2} \cdot (1 + \gamma_{l_1}) = K_{GL}(1 + \gamma_{l_1})(1 + \gamma_{l_2}) \quad \dots\dots\dots (14)$$

In this way, it is possible to calculate for two coexisting particle groups. By repeating the above calculation, Eq. (15) is obtained for coexistence of i -groups of particles.

$$K_{GL}' = K_{GL}(1 + \gamma_{l_1})(1 + \gamma_{l_2}) \dots\dots (1 + \gamma_{l_i}) \quad \dots\dots\dots (15)$$

When it is possible to discriminate between luminous and nonluminous gas for particleless gas,

the following equations are deduced in general, where (K_{GGL}) is an exponent for nonluminous gas and $(1 + \delta)(K_{GGL})$ is that for the mixture of nonluminous & luminous gas.

$$(K_{GGL})' = (K_{GGL})(1 + \delta)(1 + \gamma_{l_1})(1 + \gamma_{l_2}) \dots\dots \dots (1 + \gamma_{l_i}) \quad \dots\dots\dots (16)$$

δ is relatively large in comparison to δ_{l_n} and γ_{l_n} is small compared to unity. It is possible to deduce Eq. (17) from Eq. (16)

$$(K_{GGL})' = (K_{GGL})(1 + \delta)(1 + \sum_{n=1}^i \gamma_{l_n}) \quad \dots\dots\dots (17)$$

3. Comparison between the author's theory and his experiment

3.1 The apparatus and the method of the author's experiment

It is the aim of the experiment to prove that the author's theory agrees well with the results of experiment.

High temperature combustion gas fills a furnace and the particles of the same temperature with that of the gas fall in the gas. The emissivities of the gas, with and without the particles, are compared.

The furnace for the experiments is shown in Fig. 7. Its main part consists of a firebrick cylinder, whose inside diameter is 105 mm and the height is 600 mm, and the gas burner at the bottom of cylinder, which burns and produces high temperature combustion gas. At the middle height there are two horizontal radial holes, one for radiation pyrometer and another for thermo-couples. Upon the main furnace cylinder, there is another furnace which serves to heat the particle. The particles are contained in a vessel in the upper furnace and heated by another gas burner. The vessel can rotate round the horizontal shaft. Particles fall downward from the vessel, when the latter is turned over round the axis, after their temperature has risen to that of the gas in the furnace.

3.2 Method of measurement and calculation

3.2.1 Gases under test In order to get high temperature gas, it is possible to heat by electrically heated elements. But, as shown in the following analysis for emissivity calculation, it is better to heat the gas hotter than the furnace wall. So town gas is used to supply combustion gas.

Gas temperature along the vertical center line of furnace and that along the line of radiation pyrometer are measured by traversing the thermocouple in sheath to prevent radiation. Temperatures at many other points are also measured by the same thermocouple to confirm that the temperatures in the gas are uniform except the very thin layers

in the bottom of the furnace. This uniform temperature is the temperature of the gas under test.

3.2.2 Particles used in the test Pebbles for sandblasting are used as particles in the test. Powders of brick and steel balls are rejected on account of the change of state due to high temperature. Sizes of particles are shown in Table 2. Several sizes of particles are used. Temperatures of the particles are measured by thermocouples inserted in the group of particles.

In order to assure the uniform distribution of particles in gas, particles are dropped in the same test condition and received on a rotating disc. In the test, time Z , in which they traverse the field of radiation pyrometer, is measured. From the time Z and the falling velocity of particles, the imaginary cylindrical volume V ,—in that volume the particles are supposed to suspend uniformly,—is calculated.

3.2.3 Emissivity The emissivity is calculated as below. At first the indication of photoelectric radiation pyrometer is recorded. The indication is proportional to the heat received on the plane C in Fig. 8. There exist the following equations between the received heat quantity q (kcal/hr), emissivity of gas in the furnace and the temperatures of surrounding surfaces (see Fig. 8).

$$q = A_C \phi E_G \quad \dots\dots\dots (18)$$

$$E_G = 4.88 \left\{ \left(\frac{T_G}{100} \right)^4 - \left(\frac{T_C}{100} \right)^4 \right\} \quad \dots\dots\dots (19)$$

$$\phi = \frac{T_H^4 - T_C^4}{T_G^4 - T_C^4} \cdot \phi_{HC} + \frac{T_B^4 - T_C^4}{T_G^4 - T_C^4} \cdot \phi_{BC} + \phi_{CG} \quad \dots\dots\dots (20)$$

where the notations are as follows:-

A_C : Area of port opening for pyrometer (plane- C) (cm²),

T_G : Temperature of gas in furnace (°K),

Fig. 7 Experimental equipment

Table 2 Size of pebbles under test

No. of sieve mesh (ASTM)	Size of mesh openings (cm)	Range in mesh	Mean pebble diameter d_m (cm)	Weight of a pebble w_0 (g)	Surface area of a pebble A_P (cm ²)
8	0.238	8—10	0.219	12.20×10^{-3}	0.150 7
10	0.200	10—12	0.184	7.24	0.106 4
12	0.168	12—20	0.126	2.32	0.049 9
20	0.084	20—30	0.072	0.434	0.016 3
30	0.059				

T_C : Temperature at port opening for
pyrometer (plane-C) ($^{\circ}\text{K}$),

T_H : Temperature of furnace wall (plane-H)
($^{\circ}\text{K}$)

$\phi_{HC}, \phi_{BC}, \phi_{CG}$: The coefficient ϕ_{ij} is a modified geometrical factor from plane i to plane j , considering all the surfaces in the system.

Eq. (21) is obtained by putting the numerical value for the furnace, used in the experiment, in above equations (see Fig. 9). In order to prevent the hot gas from reaching the glass of radiation pyrometer, a small amount of room air is blown to the opening hole. Due to this cold air, air temperature T_C at surface C has a relation expressed in Eq. (22).

$$\left. \begin{aligned} \phi_{HC} \cdot M &= 0.750 - 0.675 \varepsilon_G \\ \phi_{BC} \cdot M &= -0.098 + 0.275 \varepsilon_G - 0.168 \varepsilon_G^2 \\ \phi_{CG} \cdot M &= +0.900 \varepsilon_G + 0.001 \varepsilon_G^2 \\ M &= 0.829 + 0.163 \varepsilon_G \end{aligned} \right\} \dots (21)$$

$$q = c_p \rho v (T_C - T_0) A_C \cdot 3600 \dots (22)$$

where c_p is a specific heat of air in kcal/g $^{\circ}\text{C}$ or kcal/g $^{\circ}\text{K}$, ρ is density of air in g/cm 3 and v is air velocity in cm/sec.

By equating two Eqs. (18) and (22), Eq. (23) is obtained. In the equation $t_{\bar{C}}^{\circ}\text{C}$ or $T_{\bar{C}}^{\circ}\text{K}$ is the temperature indication for gas alone and $t_{\bar{C}}^{\circ}\text{C}$ or

$T_{\bar{C}}^{\circ}\text{K}$ is that of gas and particles. $T_{\bar{C}}$, temperature of surface c , is expressed in terms of $T_{\bar{C}}$ as follows.

$$K = T_{\bar{C}} / T_{\bar{C}}$$

$$\frac{T_{\bar{C}}' - T_{\bar{C}}}{T_{\bar{C}} - (T_0/K)} = \frac{M(\varepsilon_G')\phi(\varepsilon_G') - M(\varepsilon_G)\phi(\varepsilon_G)}{M(\varepsilon_G)\phi(\varepsilon_G)} \dots (23)$$

As a numerical example, put $t_G = t_P = 935^{\circ}\text{C}$, $t_{\bar{C}} = 630^{\circ}\text{C}$, $t_{\bar{C}} = 750^{\circ}\text{C}$, then Eq. (24) is obtained. It is shown in Fig. 10.

$$\phi M = 0.243 + 1.543 \varepsilon_G - 0.167 \varepsilon_G^2 \dots (24)$$

From Eqs. (23) and (24), Eq. (25) is deduced.

$$\varepsilon_G' - \varepsilon_G = 0.000532 \dots (25)$$

where $K = 0.883$ and $\Delta T \equiv T_{\bar{C}}' - T_{\bar{C}}$

According to Eq. (25), change of emissivity ($\Delta_2 \varepsilon_{Ga} = \varepsilon_{Ga}' - \varepsilon_{Ga}$) is obtained by measuring the difference of temperature indication ΔT . For the thickness of gas layers and gas composition in test furnace, PL is nearly 1 cm.at and $P\Delta L$ is nearly 0.1 cm.at. In order to calculate KL or $K\Delta L$ from PL or $P\Delta L$ respectively, diagrams by Hottel and others are used. Fig. 6 shows one example of the diagrams and it is obtained from the original curve by Hottel.

3.3 Results of experiments

The experimental value $\Delta_2 \varepsilon_{Ga}$ for the emissivity change is calculated from the difference of temperature of radiation pyrometer. The theoretical value $\Delta_1 \varepsilon_{Ga}$ is obtained from the author's formulae by using the condition of particles in gas. The two values, $\Delta_1 \varepsilon_{Ga}$ and $\Delta_2 \varepsilon_{Ga}$, are compared for each experiment.

The relation between two values, $\Delta_1 \varepsilon_{Ga}$ and $\Delta_2 \varepsilon_{Ga}$, is expressed in Fig. 11. The direction coefficient, $\Delta_1 \varepsilon_{Ga} / \Delta_2 \varepsilon_{Ga}$, is 0.978 by regression method. Standard deviation, σ , is 0.0026. The deviation of the coefficient is ± 0.078 and unity of the ratio, $\Delta_1 \varepsilon_{Ga} / \Delta_2 \varepsilon_{Ga}$ lies in the range 0.978 ± 0.078 . Thus it is proved that the theory holds for the experiment. No difference is found arising from

Fig. 8 Relative position of surfaces in the furnace

Fig. 9 Relation between ϕ and emissivity of gas ε_G

(1) $t_{\bar{C}} = 800^{\circ}\text{C}$
(2) $t_{\bar{C}} = 750^{\circ}\text{C}$
Fig. 10 ϕM and ε_G

the particle sizes, the temperature of particles and gas.

3.4 Some considerations on the experiments

3.4.1 Indication of radiation pyrometer

Indication of the temperature of radiation pyrometer is recorded by electronic recorder. As the recorder has time lag and cannot follow the rapid change of temperature, the indication on the chart $\Delta T'$ does not show the true temperature rise ΔT . By examining the characteristics of the meter, the relation between the coefficient $f = \Delta T' / \Delta T$ and time Z is obtained as shown by Fig. 12; ΔT is calculated by measuring two quantities, Z and $\Delta T'$. The relation is shown in Fig. 13. An example of the recording of pyrometer is shown in Fig. 14.

3.4.2 Accuracy of the experiment Theoretical values of ϵ_{Ga} are calculated from the conditions of particles and the amount of error arising from the conditions is less than 5.5 percent. The experimental values are calculated from the temperature change ΔT . In the test, amount of error due to measurement of temperature is less

Fig. 11 Comparison between theoretical and experimental value of change of gas emissivity

Fig. 12 Relation between f and Z

than 2.5 percent, that due to the time interval Z is less than 6.5 percent, and that due to the unevenness of furnace temperature is less than 1 percent. So the total amount of errors is less than 10 percent.

3.5 Conclusions

By regression analysis of the test data and the theoretical value, the ratio $\Delta_2 \epsilon_{Ga} / \Delta_1 \epsilon_{Ga}$ is 0.98. It shows that the theory agrees well with the experimental results. The author's theory holds good in the experiment.

4. Comparison between the results of the author's theory and experimental results by others

4.1 Remarks

In this section, the author compares the result of his theory with the experimental results obtained and published by other authors. There are only a few reports which treat theoretically the radiant heat transfer in gas which contains group of particles. Comparison will be made successively with the experiment by Sherman⁽²⁾ in (4.2), with that by Lindmark (4.3) and with that by Wohlenberg (4.4).

4.2 Comparison between the results of the author's theory and the Sherman's experiment

4.2.1 Assumption for calculation

(1) Estimation of the diameter of coal particles is based on the assumed state of progress rate of combustion. Percentage of unburnt carbon is given in Sherman's paper and the author of the present report assumes that volatile matters, hydrogens in coal and all the moisture evaporate before the gas reaches 0.5 ft from the burner mouth. (2) Coal

Fig. 13 Assumed change of temperature (ΔT)

Fig. 14 Example of a record of pyrometer

particles are actually different in size. But, for simplicity, it is assumed that they are of only one size. With the progress of combustion, the diameter of particle reduces in accordance with the residual weight of unburnt part. And so, under the assumption, the number of the particles does not change throughout the progress of combustion in the case, i.e. particles do neither separate nor segregate from each other. (3) In the example cited, the excess air is 20 percent and the heat

Fig. 15 Gas absorption strength KL and increment γ of KL due to particles

input is 2.7×10^6 Btu/hr. (4) Distribution of coal particle size follows the law of Rosin and Rammler⁽³⁾. It is expressed by the following equation,

$$y = 1 - e^{-bx^n} \dots \dots \dots (26)$$

where x is the edge length of square hole of sieve in mm and y is a percentage in weight of the coal which passes through the sieve openings to the total amount of coal; b and n are constants for coal grades and for coals under test they are assumed as in Table 3. (5) Coal particles are divided into several groups and the mean diameter of coal for each group is calculated. For mean diameter of one group, increment γ_{ln} of the exponent (K_{GLL}) of the emissivity is calculated. (6) (K_{GLL}) for non-luminous gas is obtained by the chart given by Hottel⁽⁴⁾. (7) δ for the increment of the exponent of luminous gas is assumed by using the corresponding chart.

4.2.2 Comparison of the experimental results with theoretical calculation Based on Sherman's experimental data, Table 4 is compiled. An example of the change of the exponent of emissivity is shown in Fig. 15. Following conclusions are deduced from these results.

(1) The exponents K_{GLL} for non-luminous gas

Table 3 Value of b and y

Coal name	Hocking coal	Pocahontas C	Illinois C	Pittsburgh C
b	22.0	21.1	19.0	26.0
y ($x=0.074$)	0.802	0.790	0.760	0.852

Table 4 Gas absorption strength

Items	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Position	For gas A without particles	Increment of absorption strength due to particles	For gas A and particles	For gas and particles	For gas A and B without particles	Abs. strength due to particles in gas
		K_{GLuL}	$1+\gamma$	$K_{GLuL}(1+\gamma)$	$K_{GL'L}$	$\frac{K_{GLL}}{K_{GL'L}} = \frac{K_{GLL}}{K_{GL'L}(1+\gamma)}$	$\frac{K_{GL'L} - K_{GLL}}{K_{GL'L} \cdot \gamma}$
Coal		Calculated value	Theoretical value	(2) × (3)	Sherman's experimental value	(5)/(3)	(5) - (6)
1 Hocking Coal	1	0.111	1.525	0.169	0.964	0.630	0.334
	2	0.145	1.11	0.161	0.612	0.550	0.062
	3	0.166	1.04	0.173	0.435	0.419	0.016
	4	0.176	1.04	0.183	0.408	0.395	0.013
2 Pocahontas Coal	1	0.084	2.35	0.198	1.276	0.543	0.773
	2	0.124	1.26	0.156	0.839	0.666	0.173
	3	0.152	1.03	0.157	0.409	0.396	0.013
	4	0.166	1.02	0.169	0.478	0.469	0.009
3 Illinois Coal	1	0.114	1.67	0.190	1.266	0.760	0.506
	2	0.124	1.18	0.147	0.919	0.780	0.139
	3	0.142	1.09	0.155	0.571	0.526	0.045
	4	0.150	1.05	0.158	0.521	0.495	0.026
4 Pittsburgh Coal	1	0.129	2.15	0.277	1.008	0.468	0.540
	2	0.178	1.11	0.197	0.701	0.630	0.071
	3	0.192	1.05	0.201	0.383	0.366	0.017
	4	0.201	1.04	0.209	0.386	0.371	0.015

(CO₂, H₂O) lie between 0.15 and 0.17 and do not change greatly by the gas composition and gas temperature. As the combustion proceeds, K_{GLL} increases but its rate of increase is very small. (2) For the coexistence of luminous and non-luminous gas, which contains no particles, the exponent of gas emissivity, expressed as (K_{GLL}) in Table 4, lies between 0.36 and 0.78. The ratio of these values to those of the non-luminous gas varies from 1.8 to 7. It is possible to confirm that the existence of luminous gas increases the emissivity greatly. The reasons for the increase in the case, it is not possible to explain. But it may be possible to explain by the existence of particles, which are smaller in size than 10- μ diameter and neglected in author's analysis. (3) For the coexistence of luminous and non-luminous gas, the exponent of emissivity decreases with the progress of combustion as it travels far from the burner mouth. (4) The exponent (K_{GLL}) of emissivity for a gas, which contains particles, is great at the beginning of combustion but decreases as the combustion goes on. (5) The rate of increase in the exponent due to the existence of a group of particles to that of a gas without particles is 1.0 at the beginning of combustion, decreasing to 0.02 at the end. This is ascribed to the fact that the diameter of coal particles becomes smaller as combustion proceeds.

4.2.3 Summaries The Sherman's experiment gives the emissivity of gas. These values are for the cases of coexisting particles in luminous and non-luminous gas. It is possible to calculate emissivity for non-luminous gas. In the remainder, the parts due to the particles are calculated by the author's method. And the parts, which is attributable to the luminous gas, remains as positive value. And so, there is no contradiction between the theory and Sherman's experiments.

4.3 Comparison with Lindmark's equation

Lindmark⁽⁴⁾ deduced the following empirical formulae:-

$$\varepsilon_{GL} = 1 - e^{-(3/2) \cdot (v/V) \cdot (L/d)} \equiv 1 - e^{-K_L L} \quad (27)$$

where,

d : diameter of fuel sphere

L : thickness of luminous gas

v : total volume of fuel particles in gas, whose volume is V .

In comparing this with author's equation, it is found that Lindmark's coefficient K_L corresponds to author's coefficient [Eq. (29)].

$$K_L = \frac{\pi}{4} \cdot \frac{d^2}{(\Delta L)^3} \quad (28)$$

$$(1+\delta) \frac{K_{GL}}{K_{GAL}} \varepsilon_P \frac{\pi}{6} \cdot \frac{d^2}{(\Delta L)^3} \quad (29)$$

As ε_P is always smaller than unity, the relation $K_L > K_P$ is always true. And the difference between the two corresponds to that due to luminous gas.

Two coefficients are equal when Eq. (30) holds. The Lindmark's equation is a special case of the author's formulae.

$$(1+\delta) \frac{K_{GL}}{K_{GAL}} \varepsilon_P = \frac{3}{2} \quad (30)$$

4.4 Comparison with Wohlenberg's empirical equation

The following equations are Wohlenberg's empirical ones⁽⁵⁾:

$$\varepsilon_d = 1 - e^{-\alpha L/T_F} \quad (31)$$

$$\alpha = \frac{5.48 \times 10^2}{G_B \rho_0 d_0} [(1-v) \rho_0 / \rho_1]^{2/3} \quad (32)$$

where

d_0 : diameter of coal particle, cm

G_B : weight of product of combustion from 1 kg coal, kg

v : volatile matter in coal

ρ_0, ρ_1 : specific weight of coal dust and cokes respectively, kg/m³

L : effective thickness of gas layer, cm

T_F : temperature of flame

The value α/T_F corresponds to author's coefficient. And these two coefficients are expressed as follows:

$$\frac{\alpha}{T_F} = 1.21 \cdot \frac{\pi}{6} \cdot \frac{d^2}{(\Delta L)^3} \cdot \frac{273}{273+t} \quad (33)$$

$$(1+\delta) \frac{K_{GL}}{K_{GAL}} \cdot \varepsilon_P \cdot \frac{\pi}{6} \cdot \frac{d^2}{(\Delta L_0)^3} \cdot \frac{273}{273+t} \quad (34)$$

Since ε_P is always smaller than unity, α/T_F is greater than the value of Eq. (34). Thus, two equations coincide when Eq. (35) holds.

$$(1+\delta) \frac{K_{GL}}{K_{GAL}} \varepsilon_P = 1.21 \quad (35)$$

The Wohlenberg's equation is also a special case of the author's formulae.

4.5 Summaries of comparison

As seen from the above statements, the author's theoretical formulae agree well with the previous experiments and equations.

5. Scope of applicability of the author's formulae

The scope of applicability of the formulae is decided from the assumptions which are made in deducing the theory and in the approximation in the course of deduction.

In formulae (8) the term, which gives the effect of particles, is the non-dimensional value γ in Eq. (8); γ is expressed by Eq. (36).

$$\gamma_i = \frac{1}{K_{G\Delta L}} \cdot \frac{\varepsilon_P}{\Delta L} \cdot \frac{A_P}{A_B} = \frac{\pi}{6} \cdot \frac{\varepsilon_P}{K_{G\Delta L}} \cdot \frac{d^2}{(\Delta L)^3} \quad \dots\dots\dots (36)$$

In the assumption, the diameter d of particle is relatively small compared with the edge length, ΔL , of an imaginary unit cube. When it is assumed that the volume of a droplet is less than one percent to that of corresponding unit cube, $d/\Delta L$ is not greater than 0.267. On the other hand the wave length for radiation lies in the range $(1 \sim 10) \mu$ and the diameter of particle should be greater than the wave length, so d is greater than 10μ . In the approximation, it is assumed that Eq. (37) is accepted. From this point, $K_{G\Delta L}\Delta L$ should be less than 0.05.

$$1 - e^{-K_{G\Delta L}\Delta L} \approx K_{G\Delta L} \cdot \Delta L \quad \dots\dots\dots (37)$$

From Eq. (36), Eq. (38) is deduced.

$$\frac{\gamma_i K_{G\Delta L}}{\varepsilon_P} = \frac{\pi}{6} \cdot \frac{d^2}{(\Delta L)^3} \quad \dots\dots\dots (38)$$

Area ABCD in Fig. 16 shows the range, within which the formulae are applicable. In the figure the relation between $\gamma_i K_{G\Delta L}/\varepsilon_P$ and d , with $d/(\Delta L)$ as a parameter, is shown. The largest size for diameter of particle is here 0.6 cm. In the same figure the author's experimental region and Sherman's one are also shown.

6. Method of calculation of the gas suspended with small particles

From the above statements, the author advocates the following method of calculation:

(1) Assume or measure the diameter of suspended particles. Calculate the volume of unit cube, which corresponds to a single particle. Then, ΔL , length of one edge of the unit cube, is calculated.

(2) Calculate the coefficient γ_i by Eq. (36). When there are some (i) groups of particles, calculate γ_i and denote them $\gamma_{i1}, \gamma_{i2}, \dots, \gamma_{in}$.

(3) Calculate the emissivity $\varepsilon_{Ga}, \Delta\varepsilon_{Ga}, (K_{G\Delta L})$ and $(K_{G\Delta L}\Delta L)$ for non-luminous gas in the parts in question. They are obtainable from diagrams according to the kind and conditions of gas. The diagrams are given by Hottel and others. And the following equations hold.

$$\left. \begin{aligned} \varepsilon_{Ga} &= 1 - e^{-(K_{G\Delta L})} \\ \Delta\varepsilon_{Ga} &= 1 - e^{-(K_{G\Delta L}\Delta L)} \end{aligned} \right\} \quad \dots\dots\dots (39)$$

(4) Calculate the emissivity of luminous gas which coexists with non-luminous gas and express it in the form $(1 + \delta_i')K_{G\Delta L}$.

(5) Calculate the emissivity of gas, suspended with particles, in the following way.

$$\left. \begin{aligned} \varepsilon_{Ga}' &= 1 - e^{-(K_{G\Delta L})'} \\ (K_{G\Delta L})' &= (K_{G\Delta L})(1 + \delta_i')(1 + \gamma_{i1}) \\ &\quad \times (1 + \gamma_{i2}) \dots (1 + \gamma_{in}) \end{aligned} \right\} \quad \dots\dots\dots (40)$$

7. Conclusions

The conclusions of the study are summarised as follows:-

(1) There are some ambiguous points in the way to calculate the emissivity of gas, which is suspended with a group of particles. The author advocates to calculate in three ways, emissivity due to non-luminous gas, due to luminous gas and due to suspended particles. Hitherto it is treated as one mixture of luminous gas and particles and there has been no way before to solve the problem theoretically. The author has found a new theoretical way of calculating the increment of emissivity of gas due to suspended particles. It is possible to calculate the above value by distribution, diameter and surface emissivity of particles in gas. By comparing the theoretical calculation with his experimental results, he shows they agree very well with each other. Then he compares his theory with the experiments of Sherman and with the experimental formulae of Wohlenberg and others. In the case the theory holds good with these.

(2) Conclusions deduced from the author's theory are as follows. With the particles, which are suspended in gas and whose temperature is not less than that of gas, the emissivity of gas increases. For the same volumetric content of particles in gas, the amount of increment of emissivity is greater for smaller particle diameter or larger number of particles in a certain volume.

(3) By the author's theory, it is possible to calculate the emissivity of a gas with particles in the way described in the foregoing chapter.

(4) Rate of increase of gas emissivity due to suspended particles in a combustion chamber is large at the beginning of combustion and decreases

Fig. 16 Relation between $\gamma_i K_{G\Delta L}/\varepsilon_P$ and d

gradually as combustion goes on. But this rate is relatively small compared to that due to luminous gas.

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References

- (1) For example, McAdams: *Heat Transmission* (1954), Chapt. 4 (by H.C. Hottel), p. 55.
- (2) Sherman: *Trans. ASME*, Vol. 56 (1934), p.401.
- (3) British National Committee: *World Power Conference, Technical Data on Fuel* (1952), p.408.
- (4) F.M. Comerford: *Review of previous work on luminous radiation, Supplement to the Jour. Inst. Fuel* (London) (1951-11) p.S. 4.
- (5) Wohlenberg and Morrow: *Trans. ASME*, Vol. 47, (1925), p. 127; Wohlenberg and Lindseth: *Trans. ASME*, Vol. 48 (1926), p.849; Wohlenberg: *Trans. ASME*, Vol. 51, (1927), (FSP 51-36).

621. 43-214:539. 384. 6:621-233. 13

Bending Deflection of a Crankcase of High Speed Internal Combustion Engines*

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Bending rigidity of a crankcase of high speed internal combustion engines is usually smaller in the horizontal-transverse direction of the engine, because of its common hanger-bearing-type construction.

For this reason, the elastic vibration appears mainly in the horizontal direction.

In our research the bending deflection of a crankcase was measured under such conditions that the load was applied to the middle point of a crankcase in the horizontal-transverse direction being simply supported at the both end-bearing centres.

In analysing the results of bending tests with several models of crankcase, the calculating equations for the bending deflection of a crankcase were induced from "Torsion Theory of Thin Sections" with some modifications.

The results of bending tests on regular engines show that those equations are practically adequate to calculate the deflection of a crankcase.

1. Method of experiment

1.1 Test with model crankcases (Fig. 1)

The model crankcases are combinations of each type of the crankcases and the oil pans shown in Figs. 6 to 10. The model is placed on the Amsler testing machine with the center bearing part loaded. Deflection is measured at the center and ending parts with dial gauges of 1/100 th scale.

1.2 Test with actual crankcases (Fig. 2)

The crankcase is placed at its side on the fixed foundation plate. The crankcase is pulled

downward with a bolt attached to the iron bar of the same diameter as the crankshaft which is inserted in the center bearing part. The pull force is measured with a strain gauge attached to the bolt. The displacement of both shells of the crankcase is measured with several dial gauges fixed on the channel in parallel to the engine.

2. Formulae for computing the horizontal deflection of the crankcases

Deflection of the crankcase in the horizontal direction, which is supported at its both ends within the horizontal level and bears the load at its center part, is the added sum of bending torsion and simple bending. But the simple bending is far

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