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Critical Conditions of Wrinkling in Deep Drawing of Sheet Metals*

(1st Report, Fundamentals of Analysis and Results in Case where a Blank-Holder is not Used)

By Nozomu KAWAI**

In this work, examining actual phenomena of wrinkling in deep drawing operations, the mechanism of prevention of wrinkling are clarified. In addition, formulas are induced theoretically to predict the critical conditions of wrinkling in both cases where a blank-holder is not used and where a blank-holder is used.

As the first report, fundamentals of analytical method and results in case where a blank-holder is not used are reported.

1. Introduction

Flange wrinkling in the drawing of a cylindrical shell from a flat circular blank has been regarded as an axially unsymmetric buckling phenomenon of an annular flat plate corresponding to metal flange, but it has been even stated that it is impossible to solve theoretically this sort of buckling equation under boundary conditions fit for actual drawing process, especially under a form able to express quantitatively restraint of blank-holding⁽⁶⁾. In spite of earnest attempts by many researchers to explain theoretically the critical conditions of wrinkling^{(1)~(7)}, it has been hitherto confined to cases where a blank-holder was not used that the theoretical results are quantitatively satisfactory, and on the other side, in the case where a blank-holder is used they have only suggested the qualitative relations between various factors.

Moreover, as wrinkling is a sensitive phenomenon, it has not been easy to determine experimentally the effect of various factors on wrinkling. Hence, a considerable difference has often existed between the results reported by each of the workers⁽⁸⁾⁽⁹⁾.

For the reasons as mentioned above in both aspects of the theory and experiment, many obscure points still remain in the problem of prevention of wrinkling.

In view of the present circumstances as stated above, in this paper emphasis is laid on the next points, that is :

- (1) To carry out the experiments as precisely as possible and rightly evaluate the factors which exert influence on prevention of wrinkling.
- (2) To find the theoretical solution for predicting the critical conditions of wrinkling which are consistent with the experimental results.

For these objects, the orthodox methods for solving a buckling equation shall be not used, but an expedient analytical method shall be adopted in this analysis, that is, an equilibrium equation of moments acting on a segment of wrinkled flange is dealt with under assumptions fit for actual behaviours of wrinkles. Based on this equilibrium equation of moments, the critical conditions of wrinkling are examined separately for two cases, that is, where a blank-holder is not used and where a blank-holder is used.

In comparison with the experimental results, it has been found possible by the theory to predict with enough accuracy the critical conditions of wrinkling.

Above all, in the case where a blank-holder is used, by examining behaviours of metal during the drawing process, the mechanism of prevention of wrinkling and the properties of the critical blank-holding pressure are clarified⁽⁹⁾, and the analytical solution is put on a sound basis for applying it to the prediction formula of the critical blank-holding pressure.

In this paper, the subject is chiefly confined to flange wrinkling in a simple deep drawing

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operation.

As the first report, fundamentals of analysis and results in the case where a blank-holder is not used are reported.

2. Notations

Notations used in this paper are given in Fig. 1, whose (a) shows diagrammatically an intermediate stage in drawing and (b) the deflected form and the dimensions of a half-wave segment of the wrinkled flange. Where, N is the number of waves, r_0 the current radius of outer rim of blank, r_i the radius of inner edge of flange and r_m the mean radius of flange. Here, r_i is distinguishable from the inner radius r_3 of flat portion of die surface.

To arrange the results in order, the following non-dimension-quantities are used as notations which express blank size and tool size.

$B_0 = R_0/r_2$: Drawing ratio.

$\beta_0 = r_0/r_2$: Current drawing ratio.

$\beta_3 = r_3/r_2$, $\beta_i = r_i/r_2$

$\delta = 2r_2/t$: Specific die hole diameter.

3. Fundamentals of analysis

3.1 Moments acting on a half-wave segment of wrinkled flange

Fig. 2 shows a circumferential cross section at a certain radius r of wrinkled flange.

The moments about r -axis acting on a half-wave segment of $\theta = 0 \sim \pi/N$ in Fig. 2 consist mainly

(a) Dimensions of tools and blank

(b) Form and dimensions of a half-wave segment of wrinkled flange

Fig. 1

of next four terms.

M_θ : Buckling moment due to hoop stress σ_θ induced in deep-drawn flange.

M_B : Bending resistance moment induced in metal itself against bending deflection due to buckling.

M_H : Restraint moment due to blank-holding force.

M_D : Restraint moment induced at die radius.

These moments are expressed in absolute values.

Referring to Fig. 1 and Fig. 2 and assuming that the number of waves N is sufficiently large for the axis of a half wave segment to be considered straight, M_θ may be written as

$$M_\theta = \int_{r_i}^{r_0} |\sigma_\theta| t W dr \quad \dots\dots\dots (1)$$

The deflected form of wrinkled flange in Fig. 2 has a certain finite curvature in the circumferential direction at $\theta = 0$ and π/N , and so, therein the resistance moment M_B is induced in metal itself against the bending due to buckling. In evaluating M_B , it may be safely assumed that the deflected form in Fig. 2 can be expressed by

$$w = (W/2)(1 - \cos N\theta) \quad \dots\dots\dots (2)$$

Comparing the curved lines given by this equation with the experimental points measured by

Fig. 2 Circumferential cross section at a certain radius r

Drawing conditions:

Copper, $t = 0.3$ mm, $B_0 = 1.839$,

$\beta_3 = 1.129$ ($r_d = 2$ mm R),

Tallow lubricant, $H_0 = 20$ kg, $K = 2368$ kg/mm

Fig. 3 Circumferential deflected form at flange rim

dial gauge in Fig. 3, which shows how the circumferential deflected form at the outer rim of the blank varies with progress of drawing process, Eq. (2) is found accurate enough in the earlier stage of the process⁽⁹⁾.

The absolute value of the curvature $1/\rho$ at $\theta=0$ and π/N of the curve given by Eq. (2), the wave amplitude being small, becomes

$$\left| \frac{1}{\rho} \right|_{\theta=0, \pi/N} = \left| \frac{\partial^2 w}{r^2 \partial \theta^2} \right|_{\theta=0, \pi/N} = \frac{WN^2}{2r^2}$$

Considering that the cross section compressed plastically by hoop stress σ_θ suffers buckling deflection, a plastic buckling modulus E_θ is adopted after Kármán's concept. Then, M_B may be given by

$$M_B = \int_{r_i}^{r_0} \frac{WN^2}{2r^2} E_\theta \frac{t^3 dr}{12} = \frac{t^3}{24} N^2 \int_{r_i}^{r_0} \frac{E_\theta W}{r^2} dr \quad \dots\dots\dots (3)$$

where $E_\theta = 4EF_\theta/(\sqrt{E} + \sqrt{F_\theta})^2$, E is Young's modulus of metal and F_θ is the slope $\partial\sigma/\partial\varepsilon$ of the stress σ -strain ε curve of metal in simple tension at a particular value of strain. The influence of Poisson's ratio is neglected.

Assuming that the blank-holding force $H/(2N)$ acting on a half-wave segment is uniformly distributed along the radial lines of $\theta=0$ and π/N in Fig. 1 (b) and Fig. 2, M_H can be written as

$$M_H = \frac{H}{2N} r_m \theta_0 = \frac{\pi}{4} (r_0 + r_i) \frac{H}{N^2} \quad \dots\dots\dots (4)$$

3.2 Restraint moment induced at die radius

Observing practically a deflected form of a wave segment of wrinkled flange, we find that the next boundary conditions as shown in Fig. 4(a) are approximately satisfied.

$w=0$ along the inner edge r_i of flange.

$\partial w/\partial r=0$ or small finite value along the rim r_0 .

$\partial w/(r\partial\theta)=0$ along the radial line of $\theta=0$ and $2\theta_0$.

By actions of punching force working on the

metal portion in contact with the punch head, the normal pressure between die radius and metal and the bending of metal at die radius etc., the metal surface at r_i are applied against die radius; so that w at r_i becomes zero.

This fact proves existence of a restraint effect against flange wrinkling at die radius.

Now, it is assumed that this effect may be substituted by such a load q shown in Fig. 4(a) as makes just zero buckling deflection w at die radius. Then, the restraint moment about r -axis arises by the action of the load q and the reaction load q_r .

It is, however, not always easy to calculate strictly the value of q under the preceding boundary conditions. In addition, this restraint effect has a big influence only in the case where a blank-holder is not used and becomes negligibly small in the case where a blank-holder is used, as explained later.

For these reasons, with a chief aim to be compatible with the experimental results for the case where a blank-holder is not used, the next simple approximate method is used in this paper. For simplification, a half wave segment in Fig. 4 (a) is substituted by a strip in Fig. 4 (b) which is $(r_0 - r_i)$ in length and $r_m \theta_0$ in width. And, they are assumed that the uniformly distributed surface pressure q acts on the strip element of $r_m d\theta$ in width as a load condition to satisfy the preceding restraint boundary conditions $(\partial w/\partial r)_{r_0}=0$ and $(w)_{r_i}=0$, and that a reaction force against q appears along an end section r_0 of the strip corresponding to a fixed support. Then the relation between the restraint pressure q and the deflection w_m at the mean radius r_m may be expressed by the next equation which is derived easily from a formula of cantilever beam.

$$q = \frac{32}{31} \frac{E_r t^3}{(r_0 - r_i)^4} w_m \quad \dots\dots\dots (5)$$

Considering that the restraint pressure q

Fig. 4 Restraint at die radius

evaluated by Eq. (5) is distributed uniformly in the r -direction and proportionally to w_m in the θ -direction over the whole surface of the strip and that the reaction force against q is finally transmitted to die surface which supports the strip at the radial lines of $\theta=0$ and $2\theta_0$, the moment M_D produced under these circumstances may be given by

$$M_D = \int_0^{\theta_0} q(r_0 - r_i) r_m \theta r_m d\theta \\ = \frac{2(\pi^2 + 4)}{31} E_r t^3 \frac{(r_0 + r_i)^2}{(r_0 - r_i)^3} \frac{W_m}{N^2} \quad \dots\dots\dots (6)$$

Where a mean plastic buckling modulus in flange $E_r = 4EF_r/(\sqrt{E} + \sqrt{F_r})^2$ is used under consideration of the fact that a segment of wrinkled flange suffers the above restraint bending after deformed plastically by radial drawing stress σ_r . F_r is the slope of the stress-strain curve of metal in simple tension at a particular value of strain.

Comparing this method with Geckeler's⁽¹⁾, in which the restraint effect at die radius was evaluated by assuming the flange as a strut with one longitudinal edge under constraint, there is no considerable difference in the results.

Considering the acting direction of the above four moments, the equilibrium equation can be shown that

$$M_\theta \leq 2M_B + M_H + M_D \quad \dots\dots\dots (7)$$

Wrinkling arises as soon as the equality comes into existence.

3.3 Stresses and strains caused by drawing in sheet metal

In evaluating the stresses caused by drawing operation in a sheet metal, emphasis is laid on estimating them with good approximation to the experimental results and obtaining a simple analytical solution.

Assuming the condition of plane strain in which the thickness change of metal is neglected and the deformation resistance invariable across flange as yield stress of metal at the particular stage of drawing, the radial drawing stress σ_r and hoop stress σ_θ are given by

$$\sigma_r = 1.1\sigma_{eq} \ln \frac{r_0}{r} + \mu \frac{H}{\pi r_0 t} \quad \dots\dots\dots (8)$$

$$|\sigma_\theta| = 1.1\sigma_{eq} \left(1 - \ln \frac{r_0}{r}\right) - \mu \frac{H}{\pi r_0 t} \quad \dots\dots\dots (9)$$

where $\sigma_r - \sigma_\theta = 1.1\sigma_{eq}$ is used as yield criterion, and μ is the coefficient of friction between tool surfaces and metal surfaces.

It is assumed that the relation between the equivalent stress σ_{eq} and the equivalent strain ϵ_{eq} coincides with the stress-strain relation in tension such as expressed by an n -power law $\sigma = F\epsilon^n$ or a

linear relation $\sigma = \sigma_e + F_0\epsilon$.

For simplicity, adopting the value of ϵ_{eq} at the mean flange radius r_m as the mean equivalent strain $\bar{\epsilon}_{eq}$ in the flange and regarding the above yield criterion as Tresca's type, we have

$$\bar{\epsilon}_{eq} = (4/3)|\epsilon_\theta|_{r=r_m} \quad \dots\dots\dots (10)$$

where ϵ_θ is a circumferential strain expressed by the next form.

$$|\epsilon_\theta|_{r=r_m} = \frac{1}{2} \ln \frac{R_0^2 - r_0^2 + r_m^2}{r_m^2} \\ = \frac{1}{2} \ln \frac{B_0^2 - \beta_0^2 + \{(\beta_0 + \beta_i)/2\}^2}{\{(\beta_0 + \beta_i)/2\}^2}$$

Thus, assuming that the equivalent stress σ_{eq} corresponding to the above mean equivalent strain $\bar{\epsilon}_{eq}$ is uniformly distributed in the flange, σ_r and σ_θ may be calculated by Eqs. (8) and (9).

The values of the plastic buckling modulus E_θ and E_r are calculated as follows. Adopting the strains at the mean radius r_m as mean value $\bar{\epsilon}_\theta$ and $\bar{\epsilon}_r$ of ϵ_θ and ϵ_r in the flange, these become

$$\bar{\epsilon} = |\bar{\epsilon}_\theta| = \bar{\epsilon}_r = \frac{1}{2} \ln \frac{B_0^2 - \beta_0^2 + \{(\beta_0 + \beta_i)/2\}^2}{\{(\beta_0 + \beta_i)/2\}^2} \quad \dots\dots\dots (11)$$

Taking the values of F_θ and F_r at the mean strain $\bar{\epsilon}$, these become

$$F_0 = F_\theta = F_r = (\partial\sigma/\partial\epsilon)_{\bar{\epsilon}} = Fn\bar{\epsilon}^{n-1} \quad \dots\dots\dots (12)$$

Thus, we have

$$E_0 = E_\theta = E_r = 4EF_0/(\sqrt{E} + \sqrt{F_0})^2 \quad \dots\dots\dots (13)$$

3.4 Equilibrium equation of moments

In calculating the moments M_θ and M_B expressed by Eqs. (1) and (3), an expression of the variation of wave amplitude W along the radial direction is required. For simplicity, assuming that W is invariable in the radial direction and equal to wave amplitude W_m at the mean radius r_m , M_θ and M_B become

$$M_\theta = W_m \int_{r_i}^{r_0} |\sigma_\theta| t dr = W_m T_\theta \quad \dots\dots\dots (1)'$$

where T_θ is the stress resultant which appears circumferentially in the flange.

$$M_B = \frac{E_0 t^3}{24} \frac{r_0 - r_i}{r_0 r_i} W_m N^2 \quad \dots\dots\dots (3)'$$

Substituting Eqs. (1)', (3)', (4) and (6) into Eq. (7) and non-dimensionalizing, the moment equilibrium equation becomes

$$\omega \frac{\sigma_e}{E_0} = \frac{a_B}{3\beta_i \delta^2} \omega N^2 \\ + \frac{\pi \delta (B_0^2 - \beta_i^2)}{16\beta_i B_0} a_H \frac{p}{E_0} \frac{1}{N^2} \\ + \frac{8(\pi^2 + 4)}{31\beta_i \delta^2} a_D \omega \frac{1}{N^2} \quad \dots\dots\dots (14)$$

To arrange the experimental results in order, next notations are newly introduced in Eq. (14).

$$\left. \begin{aligned} \omega &= W_m / (2\pi R_0) : \text{Specific wave amplitude} \\ p &= H / \{\pi(R_0^2 - r_i^2)\} : \text{Blank-holding pressure} \\ &\quad \text{per unit area of initial flange} \\ \sigma_e &= \sigma_i - (\mu/2)\delta(B_0^2 - \beta_i^2)a_B p = T_0 / (r_i t) \\ \sigma_i &= 1.1\sigma_{eq} \ln(\beta_0/\beta_i) \end{aligned} \right\} \dots\dots\dots(15)$$

Being $a_B = (\beta_0 - \beta_i) / (\beta_0 \beta_i)$, $a_H = \beta_0 + \beta_i$ and $a_D = (\beta_0 + \beta_i)^2 / (\beta_0 - \beta_i)^3$, they are only functions of β_0 and β_i respectively.

The left side of Eq. (14) corresponds to the buckling moment M_θ , the first term in the right side the bending resistance moment M_B , the second the restraint moment M_H due to blank-holding pressure and the third the restraint moment M_D at die radius, respectively.

Practical value of β_i is within a range $\beta_3 \sim 1$, but it is comparatively near to 1 in the case where a blank-holder is not used, as mentioned later, and it is nearly equal to β_3 in the case where a blank-holder is used.

4. Results for case where a blank-holder is not used

4.1 Analysis

In the case where a blank-holder is not used, p is zero in Eq. (14). Then, as ω is attendant to all terms in the equation, next equation is obtained if $\omega \neq 0$.

$$\frac{\sigma_i}{E_0} = \frac{a_B}{3\beta_i\delta^2} N^2 + \frac{8(\pi^2+4)}{31\beta_i\delta^2} a_D \frac{1}{N^2} \dots\dots\dots(16)$$

This equation is now differentiated with respect to N to find the condition of minimum σ_i for the onset of buckling.

Then, the number of waves is given by

$$N = \sqrt[4]{\frac{24(\pi^2+4)}{31}} \sqrt[4]{\frac{a_D}{a_B}} = 1.81 \frac{\sqrt{\beta_0/\beta_i+1}}{\beta_0/\beta_i-1} \dots\dots\dots(17)$$

This equation states that N is dependent only on the geometrical conditions β_0 and β_i and independent of the blank thickness t , metal property and so on. This fact agrees qualitatively with Geckeler's equation⁽¹⁾.

Substituting Eq. (17) into Eq. (16) to eliminate N , the critical condition is given by

$$\frac{\sigma_i}{E_0} = \sqrt{\frac{32(\pi^2+4)}{93}} \frac{1}{\beta_i\delta^2} \sqrt{a_B a_D} \dots\dots\dots(18)$$

Wrinkling occurs at the drawing stage to satisfy this condition. Rearranging this to a form of critical thickness, it becomes

$$\left(\frac{1}{\delta}\right)_{cr} = \left(\frac{t}{2r_2}\right)_{cr} = 0.677 \frac{\sqrt{\beta_i}}{\sqrt{a_B a_D}} \sqrt{\frac{\sigma_i}{E_0}} \dots\dots\dots(19)$$

Giving the drawing ratio B_0 , β_i and the stress-strain relation of metal as the drawing conditions, the right side of this equation is only a function of β_0 . Hence, the critical blank thickness $(1/\delta)_{cr}$ may be evaluated in accordance to β_0 .

4.2 Experimental conditions

Table 1 shows the dimensions of the tools and the blank used in this experiment. The standard internal diameter $d_2 = 2r_2$ of die is 31 mm ϕ . In the case the blank thickness t is changed, the punch diameter d_1 is changed and the die clearances are put within the range $(1.25 \sim 2.0) \times t$.

4.3 Number of waves

A curve calculated by Eq. (17) is shown in Fig. 5 in comparison with the experimentally observed values of the number of waves formed in wrinkled flange.

In this experiment, blanks of half-hard copper, half-hard brass and mild steel of thickness $t = 0.3$ mm and external diameter $D_0 = 57$ mm ϕ were used in addition to the experimental conditions shown in Table 1.

In plotting the author's experimental values

Fig. 5 Number of waves

Table 1 Dimensions of tools and blank

Tools	Classification		Die		Punch	
	Diameter	mm	d_2	31	d_1	30, 29, 28, 27.5, 26, 25
	Radius	mm	r_d	1, 2, 3, 4, 5	r_p	4
Blank	Metals		Aluminum (Annealing 350°C×1 hr in salt bath half hard aluminum)			
	Dimensions	mm	t	0.3, 0.5, 0.8, 1.0, 1.2, 1.6, 2.0	D_0	60, 58, 57, 56, 54, 52, 50, 48, 45, 42, 40, 39, 38

in Fig. 5, the middle value $(\beta_3+1)/2$ of die radius is used as β_i . Then, its results agree generally better with the theoretical curve than in the cases where $\beta_i=\beta_3$ or $\beta_i=1$ is used, though the meaning of β_i is circumstantially explained later.

In the case where the blank thickness is very thin in comparison to internal diameter of die i.e. the specific die hole diameter $\delta=2r_2/t$ is relatively large, wrinkling occurs at the same time with beginning of the drawing process. Therefore, regarding β_0 as equal to the initial value B_0 , N is plotted at the position of B_0/β_i by the sign $\circ$. It is found from Fig. 5 that these plotted points, which cover all the above experimental conditions, agree well with the theoretical curve.

On the other hand, in the case where δ is relatively small, wrinkling occurs after some progress of the drawing process. It is not always easy to determine precisely the instantaneous stage β_{ocr} of wrinkling by observation, but searching out β_{ocr} by trial and error method, N is plotted at the position of β_{ocr}/β_i by the sign $\bullet$. These plotted points agree well also with the theoretical curve.

Comparing the author's curve with the theoretical curves of Geckeler⁽¹⁾, Kikuti⁽³⁾, Senior⁽⁵⁾ and Miyagawa⁽⁷⁾ in Fig. 5, it is found similar to Miyagawa's one.

4.4 Critical blank thickness

Fig. 6 shows the relations of $(1/\delta)_{cr}$ and β_0 calculated by Eq. (19) for four drawing ratios with aluminum*. In this calculation, the middle value $(\beta_3+1)/2=1.097$ of die radius used in the experiment is provisionally adopted as β_i . The maximum

Fig. 6 Critical thickness
Conditions: Aluminum, $\beta_i=1.097$

* The stress-strain relation of aluminum is shown in the second report.

values $(1/\delta)_x$ of $(1/\delta)_{cr}$ -curves indicate the critical blank thickness necessary for carrying out a drawing operation without wrinkling throughout the process.

Fig. 7 shows the relation of $(1/\delta)_x$ and B_0 . Where $\beta_3=1.193$ ($r_d=3$ mm) was used in this experiment, but $\beta_3=1.193$, $(\beta_3+1)/2=1.097$ and 1 are provisionally adopted as the value of β_i in this calculation and the calculated results are indicated separately by β_i .

The critical blank thickness found from the experimental results exists between the theoretical curves of $\beta_i=1$ and 1.097.

Now, it is found from observation of partially drawn cups that the flange becomes gradually conical and the value of β_i becomes nearer to 1, as β_0 approaches β_3 . In other words, the more β_0 deviates from β_3 , the nearer β_i comes to β_3 . On the other hand, the curves in Fig. 6 show that β_{ocr} in the case where a drawing process is performed with the critical blank thickness i.e. β_0 at the maximum point of the curve deviates further from β_3 as B_0 becomes larger. And, it is shown in Fig. 7 that the critical blank thickness obtained by the experiment comes nearer to the theoretical curve of larger β_i as B_0 becomes larger. Thus, the theory is compatible with the experimental results.

In the range of $B_0=1.25\sim 1.62$ used in this experiment, the critical blank thicknesses exist between two values of the critical blank thickness which were evaluated by adopting $\beta_i=1$ and $(\beta_3+1)/2$ in Eq. (19).

Fig. 7 shows also that the empirical formula

Fig. 7 Critical blank thickness

of Esser and Arend* agrees well with the author's result, the value calculated by Baldwin and Howald⁽²⁾ is rather larger and the theoretical curve of Miyagawa⁽⁷⁾ is slightly lower.

5. Conclusions

Considering an equilibrium of moments acting on a half-wave segment of wrinkled flange, its approximate expression has been induced in a relatively simple and non-dimensionalized form.

By using this expression as a starting point, an analysis has been performed and examined experimentally in the case where a blank-holder is not used. And, it has been confirmed that Eq.

* Rearranging this by notations used in this paper, it may be written.

$$\left(\frac{1}{\delta}\right)_x = \frac{1}{8.7}(B_0 - 1)$$

(17) which expresses the number of waves and Eq. (19) which expresses the critical blank thickness agree well with the experimental results.

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Critical Conditions of Wrinkling in Deep Drawing of Sheet Metals*

(2nd Report, Analysis and Considerations for Conditions of Blank-Holding)

By Nozomu KAWAI**

In this report, based on the equilibrium equation of moments induced in the first report, the critical conditions of wrinkling are considered and discussed for the case where a blank-holder is used.

In practice of deep drawing operation, we cannot but often allow arising of small wrinkles in a flange through various causes, but if the wave amplitude of wrinkles is lower than the allowable value, wrinkles do not practically remain in the product, since an effect of suppressing flange wrinkles exists in die radius.

Thus, the allowable specific wave amplitude of flange wrinkles is deduced, and by introducing this into the equilibrium equation, a formula for prediction of the critical blank-holding pressure is induced.

1. Introduction

In this report, analysis is first tried of the conditions of blank-holding on the basis of equilibrium Eq. (14) induced in the previous report. Then, what properties the critical blank-holding

pressure has essentially and by what conception the analysis and experimental examination should be attempted, are clarified.

2. Analysis of conditions of blank-holding

2.1 Number of waves

Dividing both sides of Eq. (14) in the first report by ω , it becomes

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