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On the Residual Stresses Due to Alternating Stressing of Annealed Steels*

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During alternating stressing of annealed steels in reversed bending, residual compressive stress is introduced near the surface of specimen, and its magnitude changes with the number of stress cycles. Residual stress appears from the very early period of stress repetitions, and the maximum value occurs at about 10^5 stress cycles, thereafter it fades gradually. Residual stress was found to vanish approximately immediately before fracture. The higher the carbon content or the yield point of the steel, the larger is the residual stress produced. The change in hardness due to alternating stressing is quite similar to that in residual stress. In the case of ascending stressing, both residual stress and hardness were found to increase more than in the case of stressing at a primary stress amplitude. By numerical calculations, the contribution of residual stresses to the fatigue strength at a fixed stress amplitude was studied. On the basis of this ideal *S-N* curve, the role of residual stresses in the coxing phenomenon was also made clear.

1. Introduction

In the ordinary fatigue tests, specimens are tested in the annealed condition to eliminate the effect of residual stresses set up by machining or grinding operations. If residual compressive stresses were introduced at the surface layers of annealed specimens during fatigue stressing, these residual stresses might act favorably on the fatigue strength⁽¹⁾, and the *S-N* curve obtained may be regarded to include the contribution of residual stresses. Concerning this subject, Bühler and Buchholtz have reported that longitudinal residual compressive stresses were introduced after stressing alternately the annealed or normalized specimens of carbon steels at the stress amplitudes near the fatigue limits in rotary bending⁽²⁾. The authors, to confirm this phenomenon in the case of fatigue stressing in reversed bending, carried out experiments with plate specimens of low and medium carbon steels. Among the fatigue tests with varying stress amplitudes, the one in the ascending stressing with small stress increments is of interest, which results in the improvement in fatigue strength, known as the coxing phenomenon. The authors, in expectation that the residual stresses introduced by alternating stressing

would have some connection with coxing, examined the residual stresses after an alternating stressing. By introducing some assumptions, analysis was made on the contribution of residual stresses to the fatigue strength at a fixed stress amplitude, and the ideal *S-N* curve without the influence of residual stresses was established. On the basis of this ideal *S-N* curve, the contribution of residual stresses to coxing was also made clear.

2. Material tested and testing apparatus

The materials tested in this study were a low carbon steel S15C and a medium carbon steel S40C, and their chemical compositions and mechanical properties are given in Table 1 and 2, respectively. The specimens used were of plate form with a thickness of 3.2 mm. Residual stresses were measured by the etching method in the apparatus reported previously⁽³⁾. Fatigue tests were made in the Nishihara's plate bending fatigue testing machine with a frequency of 2000 c.p.m.

3. Experimental results

(1) Fatigue tests

All specimens of both steels were fully annealed and then low-temperature annealed in a vacuum furnace. Fatigue tests were made in reversed bending, and the fatigue limit obtained was 20 kg/mm² for S15C steel and 22 kg/mm² for

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S40C steel.

(2) Change in residual stress and hardness due to alternating stressing

Specimens after annealing were nearly stress free. Residual stresses were measured after several numbers of stress cycles at the stress amplitudes below and above the fatigue limit. One example of the result of residual stress measurement is shown in Fig. 1, indicating the introduction of residual compressive stresses near the surface layers, similarly to the experimental result reported by Bühler et al. Fig. 2 shows the surface residual stress versus the number of stress cycles in the four cases of residual stress measurement. From this diagram, it is seen that the residual stresses introduced are small in the early period of stress repetitions, and reach a maximum value at 10^5 stress cycles, thereafter fading gradually. As to the effect of stress amplitude, the residual stress introduced is the larger and its fading is also the greater, the higher the stress amplitude. This trend was seen in the behavior of residual stresses produced by carburizing and quenching⁽³⁾ and also by low-temperature quenching⁽⁴⁾. With respect to the effect of material, the residual stress introduced is larger in the hard S40C steel than in the soft S15C steel. This is in agreement with the experimental result obtained by Bühler et al. Extending the fading line at the stress amplitude above the fatigue limit to its corresponding number of stress cycles to fracture, it may be said that residual stress would vanish approximately immediately before fracture.

Hardness was measured by a micro-Vickers hardness tester with a 200 g load after each repetition of stress cycles. Hardness showed a maximum value at 10^5 stress cycles, too. It is of interest that both residual stress and hardness showed a maximum value at the same number of stress cycles.

(3) Change in residual stress and hardness due to ascending stressing

Table 1 Chemical compositions of materials

Symbol	C	Si	Mn	P	S
S15C	0.16	0.22	0.50	0.021	0.015
S40C	0.44	0.26	0.47	0.014	0.015

Table 2 Mechanical properties of materials

Symbol	Lower yield point σ_s kg/mm ²	Tensile strength σ_B kg/mm ²	Actual breaking stress σ_T kg/mm ²	Elongation φ %	Reduction of area ψ %
S15C	24.2	44.6	84.7	36.0	63.7
S40C	38.2	63.0	98.9	25.9	47.3

In expectation that the residual compressive stress introduced might have some connection with the coxing phenomenon, the following experiment was made with S 40 C steel, which produced higher residual stress. First, an ascending stressing of 18 $\rightarrow$ 20 $\rightarrow$ 22 kg/mm², each up to 10^5 stress cycles, was given to a specimen. The change in surface residual stress due to this ascending stressing was obtained as shown by the curve (I), Fig. 3. On the other hand, the change in surface residual stress at a fixed stress amplitude of 18 kg/mm² is shown by the curve (III). By comparing these two curves, it is realized that the surface residual stress increases by raising the stress level stepwise in contrast to its fading at a fixed stress amplitude. This suggests that residual stresses may contribute to coxing qualitatively. Next, another ascending stressing of 18 $\rightarrow$ 20 $\rightarrow$ 22 kg/mm², each up to 2×10^5 stress cycles was given to another specimen. The result is shown

Fig. 1 Residual stress distribution introduced by alternating stressing (S15C)

Fig. 2 Change in surface residual stress due to alternating stressing

by the curve (II) in comparison with the curve (IV) at a fixed stress amplitude of 18 kg/mm². The change in hardness due to the ascending stressing above mentioned is also indicated in Fig. 3, showing a similar trend to that in residual stress.

4. Analysis on the contribution of residual stresses to the fatigue strength

From the experimental result above mentioned, it is established that residual stresses contribute to coxing qualitatively. Consequently, in order to estimate the degree of contribution quantitatively, two stress levels of 23 and 24 kg/mm², slightly above the fatigue limit, were chosen and the changes in residual stresses due to alternating stressing at each stress level and in ascending stressing were investigated, as shown in Fig. 4. In this figure the change in hardness is also indicated, showing the same trend as that in residual stress. On the basis of these changes in residual stresses, analysis will be conducted.

Generally, a material will sustain damage under an alternating stressing, and it is assumed that fracture may occur when the damage reaches unity. According to Corten and Dolan⁽⁵⁾, the progress of damage with the number of stress cycles may be assumed to follow a certain power function, namely,

$$D = r \cdot N^\alpha, \quad (1)$$

where r represents the rate of damage propagation, which depends on stress levels. The power index α may be assumed to be approximately equal for stress levels which are close together. Differentiating both sides of Eq. (1), the increment of damage during an infinitesimal number of stress cycles ΔN is obtained as

$$\Delta D = r \alpha N^{\alpha-1} \Delta N. \quad (2)$$

Generally, the finite-life part of the S-N curve plotted in a semi-logarithmic scale may be expressed

Fig. 3 Change in residual stress and hardness due to ascending stressing in three stress levels

as $\log N = a - b\sigma$ (a and b are constants). Since the number of stress cycles to fracture N_1 at a stress level of σ_1 is assumed to include the contribution of residual stresses, that without the effect of residual stresses N_{10} may be plotted at certain stress cycles in front of N_1 , as shown in Fig. 5. Similarly, at a stress level of σ_2 , N_{20} may be plotted and so on. Thus, the finite-life part of the ideal S-N curve without the influence of residual stresses may be expressed as $\log N_0 = a_0 - b_0\sigma$. The contribution of residual stresses to the fatigue life can be known, if the unknowns a_0 and b_0 are determined. Similarly, since the fatigue limit σ_w may be presumed as being improved by the effect of residual stresses, the net fatigue limit σ_{w0} without the contribution of residual stresses is assumed slightly below σ_w , as seen in Fig. 5. Generally, residual stress may be assumed to act as a mean stress under an alternating stressing. With the existence of residual compressive stress σ_r , the stress amplitude σ_a in an alternating stressing may be assumed to be equivalent to the following stress amplitude $\sigma_e^{(6)}$ (hereafter will be noted as an equivalent stress amplitude),

$$\sigma_e = \frac{\sigma_T}{\sigma_T + \sigma_r} \sigma_a, \quad (3)$$

where σ_T is the actual breaking stress of the material.

(1) Contribution of residual stresses to the fatigue limit

Eq. (3) leads to the following relation.

$$\sigma_{w0} = \frac{\sigma_w}{1 + \sigma_r/\sigma_T} \quad (4)$$

Fig. 4 Change in residual stress and hardness due to ascending stressing in two stress levels

From Figs. 2 and 4 the change in residual stresses at the fatigue limit of 22 kg/mm² can be presumed by interpolation. Taking 10⁴ cycles as a unit of the number of stress cycles for convenience's sake, the change in residual stresses at each interval (between the number of stress cycles n at which the residual stress measurement was made) can be expressed as $\sigma_r = k + m \cdot n$ (k and m are constants). Then, Eq. (4) will be written as

$$\sigma_{w0} = \frac{n_n}{\sum_{n_i} \int_{n_i}^{n_{i+1}} [1 + (k_i + m_i n) / \sigma_T] dn} \cdot \sigma_w, \dots (5)$$

where k_i and m_i are the constants in the expression for the change in residual stresses at any interval between n_i and n_{i+1} .

Taking n_n as 10³ (10⁷ in the actual number of stress cycles), the calculation according to Eq. (5) gave the result of $\sigma_{w0} = 21.1$ kg/mm². Therefore, the increase in fatigue limit due to the contribution of residual stresses becomes 4.3 %.

(2) Contribution of residual stresses to the fatigue life at a fixed stress amplitude

From Fig. 5, taking the stress level of 10 kg/mm² as a base line, the finite-life part of the S-N curve is expressed as

$$\log N = 7.98 - 0.1 \sigma. \dots (6)$$

Let the equivalent stress amplitude in any infinitesimal number of stress cycles ΔN be σ_e , the ideal number of stress cycles to fracture N_0 can be obtained from the following equation.

$$\log N_0 = a_0 - b_0(\sigma_e - 10) \text{ or } N_0 = 10^{a_0 - b_0(\sigma_e - 10)} \dots (7)$$

Accordingly, the increment of damage during an infinitesimal number of stress cycles ΔN is expressed as

$$\Delta D = \frac{\alpha}{10^{\alpha[a_0 - b_0(\sigma_e - 10)]}} N^{\alpha-1} \Delta N. \dots (8)$$

To calculate the number of stress cycles N_{10} at the stress level of σ_1 , the following condition may be used.

$$\sum_0^{N_1} \Delta D = \sum_0^{N_1} \frac{\alpha}{10^{\alpha[a_0 - b_0(\sigma_e - 10)]}} N^{\alpha-1} \Delta N = 1 \dots (9)$$

Dividing the interval between zero and N_1 cycles into several small ones, and calculating the equivalent stress amplitude from the mean residual stress at each interval, the unknowns a_0 and b_0 can be determined by the method of successive approximation. The value of power index α was taken as 2.5 from the crack growth curve in plate specimens investigated by Frost and Phillips⁽⁷⁾. The results of calculation at the three stress levels of 23, 24 and 25 kg/mm² gave the following values.

$$\begin{aligned} N_{10} &= 3.784 \times 10^5 \quad (N_1 = 4.8 \times 10^5), \\ N_{20} &= 2.805 \times 10^5 \quad (N_2 = 3.8 \times 10^5), \\ N_{30} &= 2.080 \times 10^5 \quad (N_3 = 3.0 \times 10^5). \end{aligned}$$

Therefore, the increase in fatigue life due to the contribution of residual stresses at each stress level becomes $\psi_1 = 26.8$ %, $\psi_2 = 35.5$ % and $\psi_3 = 44.2$ %. From the above results, the ideal S-N curve without the influence of residual stresses can be drawn as shown in Fig. 5.

(3) Contribution of residual stresses to the fatigue life in the case of coaxing

Fatigue test was carried out in an ascending stressing, that is, specimens were stressed alternately up to 10⁵ cycles at the primary stress level of 23 kg/mm², and then to fracture at the secondary stress level of 24 kg/mm². As the result, 4.16×10^5 cycles (mean value of three tests) at the stress level of 24 kg/mm² was obtained. From Miner's linear law⁽⁸⁾, the cumulative cycle ratio $\Sigma(n/N)$ was calculated as 1.303, indicating a considerable amount of coaxing effect. In this case, the number of stress cycles of 10⁵ under the primary stress level of 23 kg/mm² was transferred to that under the secondary stress level of 24 kg/mm² under the condition of equal damage. The result of calculation gave the value of 8.1×10^4 cycles, a little larger than 7.92×10^4 cycles calculated from Miner's linear law. Consequently, the number of stress cycles to fracture in the case of coaxing transferred to the stress level of 24 kg/mm² is 4.97×10^5 . Taking a small interval of stress cycles, numerical calculation of damage during each interval was made progressively, which gave the total damage of $\Sigma \Delta D = 1.006$ at 4.42×10^5 cycles. Taking $N_{20} = 2.805 \times 10^5$ as a basis, the increase in fatigue life in the case of coaxing ψ_c and that due to the contribution of residual stresses ψ_{er} were calculated as $\psi_c = 77.2$ % and $\psi_{er} = 57.6$ %. The difference between these two values 19.6 % may be attributed to strain aging⁽⁹⁾ and others. According to the above study, the contribution of residual stresses is about three times as large as that of strain aging, etc. Vitovec made fatigue tests using the stretched and compressed specimens with V notches, and attributed the

Fig. 5 Ordinary and ideal S-N curves (S40C)

improvement in fatigue strength mainly to residual stresses rather than to strain aging⁽¹⁰⁾.

5. Verification of analysis with experiment

To verify the results of analysis, the following experiment was made. Specimens were annealed at 500°C for 1 hour at every alternating stressing of 10^5 cycles until fracture both at a fixed stress amplitude and in an ascending stressing. The annealing temperature of 500°C was chosen in order to remove residual stresses without affecting the damaged structure, avoiding recrystallization. The numbers of stress cycles to fracture obtained (mean value of three tests) were $N_{10}=3.90 \times 10^5$ and $N_{20}=2.94 \times 10^5$, respectively. Comparing these values with those of analysis, $N_{10}=3.784 \times 10^5$ and $N_{20}=2.805 \times 10^5$, though the former is a little larger, it may be regarded that both are in fairly good agreement. In the case of coxing, the number of stress cycles to fracture transferred to the stress level of 24 kg/mm² was obtained as 3.51×10^5 (mean value of three tests). This value, compared with the analytical result of $2.805 \times 10^5 \times 1.196 = 3.355 \times 10^5$, gives a little higher value, but may be considered to be in fairly good coincidence.

6. Conclusions

From the above experimental results, the following conclusions may be drawn.

(1) Residual compressive stresses are introduced at the surface layers of annealed specimens of carbon steels by an alternating stressing in reversed bending. Their maximum values appear at about 10^5 stress cycles, thereafter they fade gradually, and they nearly vanish immediately before fracture.

(2) The residual stress introduced is the larger, the higher the carbon content or the yield

point of the material.

(3) The change in hardness due to alternating stressing shows nearly the same trend as that in residual stress, and the maximum values of both residual stress and hardness appear at the same number of stress cycles.

(4) It was found that both residual stress and hardness increase progressively by raising the stress level stepwise. Consequently, residual stresses may be assumed to contribute to the coxing qualitatively.

(5) On the assumption of an equivalent stress amplitude and the damage curve with the number of stress cycles, analysis was made on the contribution of residual stresses to the fatigue limit and life at a fixed stress amplitude, and the ideal S-N curve without the influence of residual stresses was established. Based on this ideal S-N curve, the degree of contribution of residual stresses to coxing in the case of two stress levels was calculated. Experimental results proved the validity of this analysis.

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