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A Mechanism of Deformation of Metals at High Temperatures with Special Reference to the Creep after a Sudden Change in Stress*

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Mechanisms of work hardening and recovery following an abrupt change in stress were discussed on the basis of the behavior of dislocations. It was suggested that there would be a mechanism, which might act as a mechanism of recovery when the applied stress was suddenly decreased, while it might act as a mechanism of hardening when the applied stress was abruptly increased. The effect of this hardening and recovery was analytically expressed using the data of creep tests with sudden change in stress. It was shown that the behavior of mild steel after a sudden change in stress in primary or secondary stage of creep could be predicted well by the authors' theory, using the characteristic functions just and previously determined.

1. Introduction

In the previous papers⁽¹⁾⁽²⁾, the authors have determined the forms of the three kinds of characteristic functions, i. e. the strain rate function $f(\bar{\sigma}, \sigma, T)$, the work hardening function $(\partial \bar{\sigma} / \partial \epsilon)'$ and the rate of recovery $(\partial \bar{\sigma} / \partial t)'$, which are contained in the deformation theory proposed by the authors. They have also discussed a probable mechanism of the processes during deformation, especially of the process of recovery. From the discussion, it was found that the mechanism of recovery could predict the existence of the transient rate of recovery following an abrupt change in stress or strain rate. An analytical expression of the transient rate of recovery was determined⁽²⁾ from the experimental result of the creep with a period of stress removal.

In those papers, the process underlying the transient phenomena of the recovery was interpreted as the climbing process of piled-up dislocations from the active slip plane, although it was pointed out that a contribution of the recovery due to the movement of dislocations on the identical slip plane

to the total rate of recovery was present.

In this paper it is intended to discuss in detail the mechanism of recovery due to the movement of dislocations on the slip plane, and to express in an analytical form the effect of this process on the recovery and the hardening. Then the behavior of mild steel after a sudden change in stress will be predicted by the deformation theory using this result together with the three kinds of characteristic functions determined previously⁽¹⁾⁽²⁾.

2. Mechanism of recovery

As described in the previous paper⁽²⁾, at least two sorts of prevailing processes may play important roles on recovery of the internal yield stress. One is the process, in which some of the dislocations piled up at obstacles on the active slip planes annihilate by climbing up or down from the slip planes. Another is the process that some of the piled-up dislocations spring back instantaneously or time-dependently to some other places on the same slip plane where the potential energy is lower than at obstacles, when the applied stress is relaxed.

In the second process, however, the sprung-back dislocations may be pushed forward instantaneously or time-dependently to the initial positions, when the stress is increased again to the original value. Therefore this process may be reversible when the applied stress is reversed, where other state values such as temperature and strain are unchanged.

The authors have interpreted the work

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hardening as induced by the backward pressure of the piled-up group of dislocations at the obstacles. Then the yield stress may be expressed as a function of the number of dislocation piled up at the obstacles, if the other conditions remain unchanged. It follows from these considerations that the yield stress immediately after the stress change is no more the same as that just before the stress change.

This type of recovery or hardening is not so important in the case of creep with a period of complete removal of stress, because the process of recovery and hardening may be considered as a reversible one and the behavior under zero stress is not so significant⁽²⁾. In order to predict the behavior after a partial removal or an addition of load, however, consideration of this type of recovery or hardening is necessary.

This mechanism may act as a mechanism of recovery when stress is removed, and as a mechanism of work hardening when stress is reapplied. The duplicated roles of this mechanism would not be confined to the case where the applied stress is completely removed for some period and then the stress of the original magnitude is reapplied, but would be extended to the case where the stress is varied, that is, increased or decreased.

The yield stress of material may recover by some amount according to the magnitude of stress removed, and may harden by the same amount as the recovered portion when the stress is again increased to the original value. If the applied stress is increased still further than the original value, the magnitude of hardening would be greater than the magnitude of recovery, which has been caused by the partial or the complete removal of stress.

On the magnitude of recovery or hardening by this mechanism, it is natural to consider that they are closely related with the state at and the condition of the stress change. The most influential factors on these magnitudes may be the magnitude of stress change $\Delta\sigma$ and the strain at the moment of stress change. As a rough approximation, the magnitude of recovery or hardening may be expressed to be proportional to $\Delta\sigma$. Concerning the influence of strain on the recovery or the hardening, it is considered that this reversible hardening would be less significant than the ordinary work hardening, if the total strain ϵ_1 just before the stress change were smaller than the initial instantaneous strain ϵ_{01} , which appears in the creep test under a constant stress σ_1 , the magnitude of which is equal to the one after the stress change. The role of reversible hardening would have a meaning when ϵ_1 is greater than ϵ_{01} . Roughly

speaking, the magnitude of recovery or hardening, i.e., the change in internal yield stress $(\delta\bar{\sigma})_0$, for a given $\Delta\sigma$ may be considered to increase with $(\epsilon_1 - \epsilon_{01})$ and to converge to a certain value.

Then the approximate expression of $(\delta\bar{\sigma})_0$ may be assumed as

$$\left. \begin{aligned} (\delta\bar{\sigma})_0 &= K_1 \Delta\sigma \cdot [1 - \exp\{-K_2(\epsilon_1 - \epsilon_{01})\}], \\ &\quad \epsilon_1 - \epsilon_{01} \geq 0 \\ &= 0, \quad \epsilon_1 - \epsilon_{01} < 0 \end{aligned} \right\} \dots\dots\dots (1)$$

where K_1 and K_2 are constants.

3. Determination of the constants K_1 and K_2

For the determination of K_1 and K_2 , the internal yield stress $\bar{\sigma}$ just before and immediately after the stress change should be given for several values of $\Delta\sigma$ and $(\epsilon_1 - \epsilon_{01})$. Concerning the stress change, stepwise increase and decrease are possible for this purpose. However, the strain rate after the increase in stress becomes so high⁽³⁾ that the effects of work hardening as well as recovery can not be neglected in determining the magnitude of $\bar{\sigma}$ after the stress change, while in the case of decrease in stress the effect of work hardening may be neglected as compared with that of recovery because of diminished rate of strain following the stress change⁽³⁾. Furthermore some other transient phenomena such as rapid work hardening and diminishing rate of recovery⁽¹⁾⁽²⁾, which are not taken into consideration in this paper, would affect much the strain rate after the increase in stress. Therefore the results of creep tests with sudden decrease in stress will be used for determining the values of K_1 and K_2 . In the previous paper⁽¹⁾ the authors have determined the functions $f(\bar{\sigma}, \sigma, T)$, $(\partial\bar{\sigma}/\partial\epsilon)'$ and $(\partial\bar{\sigma}/\partial t)'$ for 0.14 percent carbon steel. Then the values of K_1 and K_2 will be determined for the material having the same nominal chemical composition.

3.1 Method of determination of K_1 and K_2

The results of the creep test, in which the stress σ_0 is decreased stepwise to σ_1 at a strain ϵ_{t1} , are used. The creep test under constant stresses σ_0 and σ_1 are assumed to be available.

If the strain rate function $f(\bar{\sigma}, \sigma, T)$ of the material is known as

$$d\epsilon/dt = f(\bar{\sigma}, \sigma, T) \dots\dots\dots (2)$$

the relations between $\bar{\sigma}$ and ϵ , between $\bar{\sigma}$ and t , and between the function of "effective work hardening" $1/F^{(4)}$ and $\bar{\sigma}$ under a constant stress at a given temperature can be obtained from the results of a constant stress creep test. Then the value of $\bar{\sigma}$ just before the stress change, that is, the value of $\bar{\sigma}$ under σ_0 at ϵ_{t1} can be easily determined from

these relations. By the similar procedure, the relations between $\bar{\sigma}$ and ϵ , $\bar{\sigma}$ and t , and so on under stresses changing stepwise, and therefore the value of $\bar{\sigma}$ immediately after the stress change, i.e., $\bar{\sigma}$ under σ_1 at ϵ_{t1} , can be determined, if the results of the creep tests under changing stress are used instead of the results of constant stress creep test. The difference of the values of $\bar{\sigma}$ just before and immediately after the stress change gives $(\partial\bar{\sigma})_0$ in Eq. (1).

Other quantities in Eq. (1) are expressed as

$$\left. \begin{aligned} \Delta\sigma &= \sigma_1 - \sigma_0, \\ \epsilon_1 &= \epsilon_{t1}, \\ \epsilon_{01} &= \epsilon_0|_{\sigma=\sigma_1} \end{aligned} \right\} \dots\dots\dots (3)$$

Then K_1 and K_2 can be determined by using the values of $(\partial\bar{\sigma})_0$ for several sets of σ_0 , σ_1 and ϵ_{t1} .

To determine K_1 and K_2 only from the results just before and immediately after the stress change is however apt to introduce an error in the values of K_1 and K_2 , because of the transient behavior which is often observed just after the stress change. In this study, therefore, the results during some period after the stress change are used for determining K_1 and K_2 .

From the results of the creep test under the constant stress σ_1 , the magnitude of which is the same as that after the stress change, the relation between $1/F$ and $\bar{\sigma}$ can be obtained⁽¹⁾. Since $1/F$ is expressed as⁽⁴⁾

$$1/F \equiv d\bar{\sigma}/d\epsilon = (\partial\bar{\sigma}/\partial\epsilon)' + (\partial\bar{\sigma}/\partial t)' / f(\bar{\sigma}, \sigma, T) \dots\dots\dots (4)$$

then

$$(1/F) \cdot f(\bar{\sigma}, \sigma, T) = (\partial\bar{\sigma}/\partial\epsilon)' \cdot f(\bar{\sigma}, \sigma, T) + (\partial\bar{\sigma}/\partial t)' \dots\dots\dots (5)$$

As $f(\bar{\sigma}, \sigma, T)$ represents the strain rate, the value of the left-hand side of Eq. (5) can be determined experimentally. This value is for the constant stress of σ_1 , and it can not be used for prediction of the deformation after the stress change from σ_0 to σ_1 , since the transient component of rate of recovery $(\partial\bar{\sigma}/\partial t)'$, which appears on abrupt decrease in stress (or strain rate), is not considered in $(\partial\bar{\sigma}/\partial t)'$ of Eq. (5). Then if $(\partial\bar{\sigma}/\partial t)'$ is added to $(1/F) \cdot f(\bar{\sigma}, \sigma, T)$ and the sum is divided by $f(\bar{\sigma}, \sigma, T)$, the value of $1/F$ after the stress change can be determined. Adopting this value $\bar{\sigma}$ may be approximately predicted⁽¹⁾. Comparing this predicted value of $\bar{\sigma}$ with the experimental value of $\bar{\sigma}$, the proper value of $(\partial\bar{\sigma})_0$ may be obtained, and thus the values of K_1 and K_2 may also be determined.

3.2 The values of K_1 and K_2

Concerning the experimental data for determining K_1 and K_2 , the results of the creep tests with a sudden decrease in stress, which were made on

0.14 percent carbon steel⁽³⁾ (the symbol SC-14-2 will be used), are available. The correct forms of the characteristic functions of this material, which are necessary to determine K_1 and K_2 , are not obtained, although those functions were determined for a material having the same nominal chemical composition⁽¹⁾ (the symbol SC-14-1 will be used) as SC-14-2. Therefore the characteristic functions of SC-14-2 will be estimated under suitable assumptions from those of the similar materials.

For the strain rate function $f(\bar{\sigma}, \sigma, T)$ of SC-14-2, the expression for SC-14-1 is adopted in its original form. Then strain rate in %/min is expressed as⁽¹⁾

$$f(\bar{\sigma}, \sigma, T) = \frac{1.42 \times 10^{-5}}{\bar{\sigma} - \sigma} \times \sigma^{5.0} \cdot \exp \left[-\frac{8.53 \times 10^2}{RT} (\bar{\sigma} - \sigma)^2 \right] \dots\dots (6)$$

where σ is the applied stress in kg/mm², T the temperature in °K, and R the gas constant having the value of 1.987 cal/°K mol.

The transient rate of recovery $(\partial\bar{\sigma}/\partial t)'$ was expressed as⁽²⁾

$$(\partial\bar{\sigma}/\partial t)' = (\partial\bar{\sigma}/\partial t)_1' \cdot b' / (1 + c_1 t) \dots\dots\dots (7)$$

where $(\partial\bar{\sigma}/\partial t)_1'$ was the rate of recovery just before the stress reduction, t_1 the time elapsed after the stress reduction, c_1 a constant, and b' a coefficient which varied with the condition of stress change. b' was determined for 0.12 percent carbon steel as⁽²⁾

$$b' = 3.70 \times 10^{16} \cdot \log e \cdot \epsilon_c \cdot \sigma_0^{-12.8} \dots\dots\dots (8)$$

where ϵ_c in % represented the creep strain under the stress σ_0 , which was the stress before the stress change.

Using Eq. (8), we shall estimate a probable expression of b' for SC-14-2. As previously discussed⁽¹⁾⁽²⁾, the most important factor affecting the rate of recovery at a constant temperature is the strain rate. Then it is assumed that the values of b'/ϵ_c are the same for the two sorts of materials having similar properties regarding deformation, when the strain rates at any time in constant stress creep test are the same for the two materials. Following this assumption, b' for SC-14-2 is determined from the results of the constant stress creep tests for both materials as

$$b' = 3.15 \times 10^{17} \cdot \log e \cdot \epsilon_c \cdot \sigma_0^{-12.8} \dots\dots\dots (9)$$

If we use the expression of $(\partial\bar{\sigma}/\partial t)'$ for SC-14-1⁽¹⁾ as $(\partial\bar{\sigma}/\partial t)_1'$ in Eq. (7), $(\partial\bar{\sigma}/\partial t)'$ for SC-14-2 is written as

$$(\partial\bar{\sigma}/\partial t)' = - \{1 + (10^7 \cdot \dot{\epsilon}_1)^{0.85}\} \times 10^{-6.01 - 0.004\bar{\sigma}_1} \times \frac{3.15 \times 10^{17} \cdot \log e}{1 + 10t_1} \cdot \frac{\epsilon_c}{\sigma_0^{12.8}} \dots\dots (10)$$

where ϵ_1 and $\bar{\sigma}_1$ represent the strain rate and the internal yield stress just before the stress reduction,

respectively.

Using Eqs. (6) and (10) together with the results of creep tests under constant stress and under varying stress, K_1 and K_2 are determined as about

$$K_1 = 3/4, \quad K_2 = 1.5 (\%)^{-1} \quad \dots\dots\dots (11)$$

4. Prediction of the creep deformation after the increase or the decrease in stress

In this section, we shall predict the behavior of mild steel after a sudden change in stress, using the characteristic function just determined as well as those previously determined⁽¹⁾⁽²⁾.

4.1 Creep behavior after a stress change given in the primary stage of creep

4.1.1 Behavior after the increase in stress
We have assumed⁽²⁾ that the effect of the transient rate of recovery on the change in internal yield stress could be neglected in the case of stress increase, as compared with the effect of the stationary rate of recovery, which was mainly affected by the current value of strain rate. Under this assumption, the behavior after the stress increase can be easily predicted using the results of constant stress creep test together with Eqs. (6) and (1).

From the results of constant stress creep tests and Eq. (6), the relations between $F(\bar{\sigma}, \sigma, T)$ and $\bar{\sigma}$ under constant stresses can be obtained. Then the relation between strain ϵ and $\bar{\sigma}$ can be determined as⁽⁴⁾

$$\begin{aligned} \epsilon &= \int_{\bar{\sigma}_0}^{\bar{\sigma}_1} F(\bar{\sigma}, \bar{\sigma}_0, T) d\bar{\sigma} + \int_{\bar{\sigma}_1 + (\partial\bar{\sigma})_0}^{\bar{\sigma}} F(\bar{\sigma}, \sigma_1, T) d\bar{\sigma} \\ &= \epsilon_1 + \int_{\bar{\sigma}_0 + (\partial\bar{\sigma})_0}^{\bar{\sigma}} F(\bar{\sigma}, \sigma_1, T) d\bar{\sigma} \quad \dots\dots\dots (12) \end{aligned}$$

where $\bar{\sigma}_0$ is the initial internal yield stress of material. The relation of strain rate $d\epsilon/dt$ to $\bar{\sigma}$ can be computed by Eq. (6). From these two relations, the relation between $d\epsilon/dt$ and ϵ can also be obtained.

The chain lines in Fig. 1 illustrate the predicted relations of $d\epsilon/dt$ to ϵ for the four cases of stress change, i.e., a, b, c and d, which are shown in Fig. 1. The predicted curves represent considerably well the experimental results⁽³⁾, which are shown by the circles, except the behavior in some period after the stress change.

4.1.2 Behaviors after the reduction and the reapplication of stress
As described in section 3.1, the effect of the transient rate of recovery on the change in internal yield stress should be taken into account for predicting the behavior after the stress reduction. Therefore the relation between F and $\bar{\sigma}$ after the stress reduction was first determined following the method described in 3.1. Then the relation between $d\epsilon/dt$ and ϵ after the stress reduction was computed in the way similar to 4.1.1.

The behavior after the reapplication of stress can be obtained in quite the same manner as in 4.1.1.

Fig. 2 shows the relation of $d\epsilon/dt$ to ϵ , when the stress was decreased from 22.0 kg/mm² to 20.0 kg/mm² at the strain of 5.0 % and then was increased again to 22.0 kg/mm² at the strain of 8.0 %. The predicted curve represents well the experimental results⁽³⁾, especially in the case of stress reduction.

4.2 Creep behavior after a stress decrease given in the secondary stage of creep

As previously reported⁽⁵⁾, the value of F after the stress reduction becomes generally a negative value, when the stress reduction is given in the secondary stage of creep. In the constant stress creep test, however, the value of F remains always positive⁽⁴⁾⁽⁵⁾. Therefore it is impossible to determine the relation of F in the negative range to $\bar{\sigma}$, which is necessary to predict the behavior of deformation after a stress decrease in the secondary stage of creep, from the results of the constant stress creep tests. Then for the prediction, the three sorts of characteristic functions such as strain rate function, work hardening function and rate of recovery should be known.

The experiments, whose results are available

Fig. 1 Relations of strain rate to strain in the creep tests with sudden increase in stress in the primary creep

for comparison with the prediction, were conducted with 0.14 percent carbon steel⁽⁵⁾ (symbol SC-14-3), which had the same nominal chemical composition as SC-14-1 and SC-14-2. The correct forms of the characteristic functions for SC-14-3, however, have not been determined yet. Then we shall estimate their approximate forms from the expressions for SC-14-1 and SC-14-2, and shall predict the behavior of SC-14-3.

For the strain rate function $f(\bar{\sigma}, \sigma, T)$, the expression for SC-14-1, i.e., Eq. (6), is used as in the case for SC-14-2.

The work hardening function $(\partial\bar{\sigma}/\partial\epsilon)'$ is obtained from the relation of initial instantaneous strain ϵ_0 to applied stress σ in the constant stress creep tests as⁽¹⁾

$$(\partial\bar{\sigma}/\partial\epsilon)' = 1/(\partial\epsilon_0/\partial\sigma)|_{\sigma=\bar{\sigma}} \dots\dots\dots (13)$$

ϵ_0 for SC-14-3 was expressed as⁽⁵⁾

$$\epsilon_0 = 2.07 \times 10^{-4} \cdot \sigma^{3.0} \dots\dots\dots (14)$$

Then $(\partial\bar{\sigma}/\partial\epsilon)'$ is given as

$$(\partial\bar{\sigma}/\partial\epsilon)' = 1.61 \times 10^3 \cdot \bar{\sigma}^{-2.0} \dots\dots\dots (15)$$

Concerning the rate of recovery, the expression for SC-14-1 such as⁽¹⁾

$$(\partial\bar{\sigma}/\partial t)'' = -\{1 + (10^7 \cdot \dot{\epsilon})^{0.85}\} \times 10^{-6.01-0.004\sigma} \dots\dots\dots (16)$$

is used as the stationary rate of recovery under a constant stress. The transient component of the rate of recovery $(\partial\bar{\sigma}/\partial t)'$, which appears on sudden decrease in stress or strain rate, is determined from

the results of 0.12 percent carbon steel⁽²⁾ by the assumption that "the values of b'/ϵ_0 are the same for the two sorts of material when the strain rates at any time in constant stress creep tests are the same for the two materials", as was used for SC-14-2. Then

$$(\partial\bar{\sigma}/\partial t)' = -\{1 + (10^7 \cdot \dot{\epsilon})^{0.85}\} \times 10^{-6.01-0.004\sigma} \times \frac{2.53 \times 10^{17} \cdot \log e}{1 + 10t_l} \cdot \frac{\epsilon_0}{\sigma_0^{12.8}} \dots\dots (17)$$

For $(\partial\bar{\sigma})_0$, Eq. (1) with substitution of the values of Eq. (11) is adopted for SC-14-3.

Using the relation of F to $\bar{\sigma}$ calculated from Eqs. (6), (15), (16) and (17), the behaviors of SC-14-3 after the stress decrease for the three cases, i.e., i, j and k, which are tabulated in Fig. 3, were predicted as shown by chain lines in Fig. 3. The prediction shows qualitatively the tendency of the experimental results⁽⁵⁾, although some discrepancies exist. The discrepancies are considered to be caused by the inaccuracy in the estimated expression of the characteristic functions.

5. Concluding remarks

It was suggested on the mechanism of deformation of mild steel that there would be a mechanism, which could cause hardening when the stress was increased and recovery when the stress was decreased. This mechanism was interpreted from the viewpoint of the movement of dislocations on the slip plane. An approximate expression of this

Fig. 2 Relations of strain rate to strain in the creep tests with sudden decrease and increase in stress in the primary creep

Fig. 3 Relations of strain rate to time in the creep tests with sudden decrease in stress in the secondary creep

hardening and recovery was determined for 0.14 percent carbon steel.

The behavior of 0.14 percent carbon steel under stepwise changing stress was predicted by the authors' theory, using the characteristic functions just and previously determined. From the comparison of the prediction with the experiment, it was found that the deformation under a varying stress could be predicted considerably well by the authors' theory.

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On Sub-Harmonic and "Summed and Differential Harmonic" Oscillations of Rotating Shaft*

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The author has found experimentally that sub-harmonic oscillations and "summed and differential harmonic" oscillations take place in a shaft system with non-linear spring characteristics and multiple degrees of freedom. In the present paper, these vibrations are treated mathematically and analytically. Conclusions arrived at may be summarized as follows:

(1) The equation of response curves of sub-harmonic oscillations of order 1/2 has qualitatively the same construction as that of rectilinear vibratory system. The hard or soft spring characteristics result in response curves of both hard and soft spring types.

(2) When two vibrations taking place at the critical speed of "summed and differential harmonic" oscillations are both forward or both backward precessional whirling motions, only the summed harmonic type can occur and the differential type does not take place.

When "summed and differential harmonic" oscillation consists of forward and backward precessional motions, only differential type has possibility of occurrence.

1. Introduction

When a rotating body is supported by single-row radial ball bearings, the non-linear spring characteristics appears in the elasticity of the shaft supporting the rotating body. This non-linearity of spring characteristics results in occurrence of sub-harmonic and "summed and differential harmonic" oscillations of the rotating shaft^{(1)~(4)}. In the present paper, the author will discuss analytically conditions for the possibility of occurrence of "summed and differential harmonic" oscillation, and he will determine the modes of vibration which can take place, equations of

response curves. Further the author will compare the conclusions obtained with the experimental results.

2. Transformation into normal coordinates

The differential equations of a rotating shaft consisting of a light shaft and a rotating body are⁽⁵⁾

$$\left. \begin{aligned} m\ddot{x} + c_1\dot{x} + \alpha_0x + \gamma_0\theta_x + \Phi_1 &= me\omega^2 \cos \omega t, \\ m\ddot{y} + c_2\dot{y} + \alpha_0y + \gamma_0\theta_y + \Phi_2 &= me\omega^2 \sin \omega t, \\ I\ddot{\theta}_x + I_p\omega\dot{\theta}_y + c_3\dot{\theta}_x + \gamma_0x + \delta_0\theta_x + \Phi_3 \\ &= (I_p - I)\tau\omega^2 \cos(\omega t + \beta_0), \\ I\ddot{\theta}_y - I_p\omega\dot{\theta}_x + c_4\dot{\theta}_y + \gamma_0y + \delta_0\theta_y + \Phi_4 \\ &= (I_p - I)\tau\omega^2 \sin(\omega t + \beta_0), \end{aligned} \right\} \dots\dots\dots(1)$$

where x, y are displacements of the rotating body in x, y directions; θ_x, θ_y are components of

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