Investigation on Dynamic Creep and Rupture of a Low Carbon Steel*

By Shuji Taira** and Ryoichi KoteraZawa***

Dynamic creep and rupture tests were carried out with annealed specimens of a 0.15 percent carbon steel under the combined static and alternating stress with stress ratios (alternating stress/mean stress) of more than unity. The test temperature was 450°C. The results of the present study together with those obtained in the previous study for the stress ratios less than unity were analysed in the light of the authors' theory, which aims to relate the strength under dynamic stress to the static creep strength.

The conclusion obtained is summarized as follows:
1. The forms of the dynamic creep curves are similar to those of the static creep curves even for the case of the stress ratio of more than unity.
2. The prediction of dynamic creep rupture strength from the static creep rupture data together with the fatigue data under reversed stress is established over the entire range of stress ratios from zero (static stress) to infinity (reversed stress). This is based on the idea that the damages of material under varying stress are composed both of the creep damage previously discussed and the fatigue damage caused by the dislocation oscillation under alternating stress.
3. The equivalent static stress, which was defined previously to predict the dynamic creep strength from the static creep data in the stage of transient creep, is applicable to the case of creep including the steady state component in addition to the transient component and also to the case of rupture.
4. The influence of the dislocation oscillation due to alternating stress on the dynamic creep was also discussed.

1. Introduction

Recent trend towards the use of materials for engineering application operating at high temperatures and heavy loads makes it necessary to consider various sorts of properties of materials including dynamic creep and fatigue. In many cases, the engineering purpose urges the necessity of estimation of the strengths from the data of simpler tests such as the static creep tests\(^{(1)}\)\(^{(2)}\)\(^{(3)}\). Concerning this problem, the authors have proposed a method of analysis, which aims to predict the dynamic creep and the rupture strength from the informations on static creep and rupture tests, and have proved the applicability of the analysis\(^{(4)}\)\(^{(5)}\). However, this work was limited to the case of a dynamic stress acting in the tension range only, and the problem including the compression range, where the stress ratio \(A\) (alternating stress \(\sigma_a\)/mean stress \(\sigma_m\)) is larger than unity, has not been treated. In this report, analysis is extended to a wider application, covering the whole range of stress ratios from zero (static test) to infinity (reversed stress fatigue test) and comparison is made with the experimental results on a low carbon steel.

With respect to the dynamic creep, the analysis in the earlier work was limited within the range of transient stage of creep owing to the difficulty of analysis. In this study, a discussion is made on the extension of the analysis to the steady state stage, including the transient stage.

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** Professor, Faculty of Engineering, Kyoto University.
*** Assistant Professor, Faculty of Engineering, Kyoto University, Sakyo-ku, Kyoto.
2. Testing apparatus, test specimen and test material

The testing machine used in this study was the dynamic creep and fatigue testing machine as reported previously\(^{(6)}\). In this machine, combined alternating load and static load are applied to a specimen by the combination of rotating eccentric masses and coil springs, and both components are controlled independently.

The specimen is heated in an electric furnace fixed in the testing machine and the temperature is controlled within \( \pm 1^\circ \)C by an automatic temperature controller. The amount of creep deformation is measured by dial gauges with 1/100 mm scale.

The test material was a 0.15 percent carbon steel. Its chemical composition, condition of heat treatment and mechanical properties at room temperature are given in Table 1. The test specimen used was 6 mm in diameter and its shape is shown in Fig. 1.

3. Test results

The tests were carried out under the condition given in Table 2, in which the stress ratios of 2.0 and \( \infty \) (reversed stress fatigue test) are the test condition for the present study and the stress ratios less than unity are the one for the previous investigation\(^{(6)}\).

Fig. 2 shows the creep curves for the dynamic stress of \( A=2.0 \). It is known from the figure that the creep curve for this stress condition is similar to the static creep curve, with the exception that the third stage creep does not appear and the fracture is likely to occur abruptly at the end of the second stage of creep. It might be considered that the third stage creep begins with the development of microscopic cracks or voids in the material\(^{(6)}\). These defects act as stress raisers in the material and may be regarded as a sort of internal notches. As to the effect of notches on the strength of material, a weakening effect is observed under a dynamic stress with large stress ratio, while a strengthening effect appears under that with small stress ratio\(^{(6)}\). Thus, the internal notches generated at the end of the second stage of creep weaken the material and lead to a rapid fracture in the case of a large stress ratio.

Fig. 3 is the diagram of maximum stress versus the time required for fracture. It is known from this figure that the cases of \( A=0.23 \) and 0.70 give the longest life for a definite maximum stress. This tendency is similar to the case of an aluminum alloy 24S-T4 at 500°F given by Lazan et al\(^{(8)}\), and also to the case of a 13 chromium steel at 450°C reported in the previous paper of the authors\(^{(11)}\). It is to be noted that the life under the dynamic stress of \( A=2.0 \) is shorter than that under the static stress which is equal in magnitude to the maximum stress of the dynamic stress. In the case of \( A=\infty \), the value of maximum stress

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Table 1 Chemical composition, condition of heat treatment and mechanical properties at room temperature

<table>
<thead>
<tr>
<th>Composition</th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cu</th>
<th>Fe</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>0.15</td>
<td>0.26</td>
<td>0.43</td>
<td>0.013</td>
<td>0.007</td>
<td>0.09</td>
<td>bal</td>
</tr>
</tbody>
</table>

Heated after machining at 900°C for one hour in vacuum with subsequent cooling in furnace.

| Yield point | 23.2 kg/mm² |
| Tensile strength | 39.4 * |
| Elongation | 34.6 % |
| Reduction of area | 68.5 * |

Table 2 Test conditions

| Temperature | 450°C |
| Frequency of alternating stress | 1 800 c.p.m. |
| Stress ratio \( A \) (Alternating stress/mean stress) | 0º (static stress) |
| | 0.23º |
| | 0.70º |
| | 2.0 |
| | \( \infty \) (reversed stress) |
| Duration of tests | up to 100 hours |

* stress ratio for the tests in the previous study.
is only 67 percent of the maximum stress for $A = 0$ giving the same life.

Fig. 4 is a stress-range diagram represented as the combination of mean stress and alternating stress which cause fracture or 3, 5 and 8 percent creep strains in 100 hours. It is drawn by using the results described above together with those reported previously. In general, the safe stress-range is the area bounded by both limiting lines for allowable strain and for fracture. In the figure, however, the limiting lines for allowable strains lie below the fracture line as far as the stress ratios are less than 2.0, but in the range of stress ratios greater than 2.0 there would be the cases where the strength criterion is defined by fracture, especially in the range of stress ratios close to infinity.

4. Analysis of dynamic creep rupture

The analysis presented by the authors\(^\text{(5)-(3)}\) showed a good agreement with the experimental results in the case of stress ratios less than unity, but in the case of stress ratios larger than unity, appreciable discrepancy was noted, that is, the experimental strength was much smaller than the analytical one. In the previous report\(^\text{(5)}\), the authors attributed this discrepancy to the difference in the type of fracture between the case of $A<1$ and that of $A>1$. To predict the strength for such a case, an analysis will be made as follows.

To carry out the analysis, it is necessary to examine more closely the difference in the type of fracture. Under a dynamic stress with stress ratio larger than unity, the tensile and compressive stresses are imposed alternately, while in the case of stress ratio less than unity varying tensile stress only is applied.

Under a static stress, dislocations piled up locally around some obstacles are assumed to move slowly and steadily in one direction by the climb process or some other mechanism, while, under a dynamic stress, back and forth oscillation between obstacles is caused by the alternating component of the dynamic stress\(^\text{(3)(33)(44)}\). It may be supposed that the oscillation of dislocations between the obstacles is much greater when the tensile and compressive stresses are applied alternately than when varying uni-directional stress is applied, and consequently under a dynamic stress with large stress ratio, much more plastic deformations are produced in the material than under a dynamic stress with small stress ratio. In fact, the total plastic strains produced in the material up to fracture amount to 1 000—10 000 percent under a reversed stress\(^\text{(14)}\), whereas it is only several percent or several tens of percent under a static stress. As has already been reported by the authors, the amount of plastic deformations produced in the material plays an important role in the fatigue fracture of material under an alternating stress. Consequently, the rate of increase of damage under such a dynamic stress condition may be regarded as being influenced by the plastic deformation due to the dislocation oscillation.

As stated above, the dislocation oscillation is caused by the alternating stress and may be regarded as being scarcely affected by the mean stress\(^\text{(17)}\). At any moment in the period of a dynamic stress cycle, there would occur a deformation which is essentially of the same amount as that occurring when the stress at that moment is applied statically, in addition to the deformation caused by the oscillation of dislocations. Therefore, the damage produced under a dynamic stress may also be divided into the following two components. One is the component which increases at the same rate as in the case where the stress at that moment is applied statically, and the other is the one caused by the dislocation oscillation and is determined by the alternating stress. The former has
already been discussed in the earlier paper(4) and, according to its nature, will be noted as a creep damage hereafter. The rate of increase of this damage is expressed as a function of stress, that is:
\[
d\phi/\text{d}t = a\sigma^{a_{\sigma}}
\]  
where \(a\), and \(a_{\sigma}\), are constants which are independent of either stress or time. The latter may be called a fatigue damage, since it is caused by alternation of stress, and it would be reasonable to assume that the difference of current stress to mean stress, \(\sigma - \sigma_{n}\), is effective to the creation of the fatigue damage \(\phi_{f}\). In the same manner as the case of the creep damage, we may write the rate of increase of fatigue damage as:
\[
d\phi_{f}/\text{d}t = a_{f}(\sigma - \sigma_{n})^{a_{f}}
\]
where \(a_{f}\) and \(a_{f}\) are constants. In this equation, the rate of increase of damage is defined as the derivative of the damage with respect to time, from the consideration that the fatigue at elevated temperature is a rate process(15)(16). As has been reported by the authors, various test results concerning the influence of frequency or other factors on the elevated temperature fatigue can be explained consistently from this viewpoint(16).

As to the criterion of fracture, the following two cases are considered as fundamental cases.

Case (1). Material fracture when the sum of the creep and the fatigue damage reaches a critical value \(\phi_{c}\):
\[
\phi_{c} = \phi_{c} + \phi_{f}
\]  
Case (2). Material fractures when either the creep or the fatigue damage reaches its own critical value:
\[
\phi_{c} = \phi_{c}, \quad \text{or} \quad \phi_{f} = \phi_{f}
\]

According to the condition of present experiment, the dynamic stress is expressed as a function of time \(t\) as:
\[
\sigma = \sigma_{n} + \sigma_{a} \sin \omega t = \sigma_{n}(1 + A \sin \omega t)
\]
where \(\sigma_{n}\): mean stress
\(\sigma_{a}\): alternating stress
\(\omega\): angular frequency of alternating stress
\(A\): stress ratio

Substituting this into Eqs. (1) and (2), and integrating with respect to time \(t\) from 0 to rupture time \(t_{r}\), we have:
\[
\phi_{c} = a_{\sigma_{n}}^{a_{\sigma}}t_{r}^{a_{\sigma}} + a_{f}^{a_{f}}t_{r}^{a_{f}}
\]
where
\[
I = \frac{1}{2\pi} \int_{0}^{2\pi} (1 + A \sin \omega t)^{a_{\sigma}}d(\omega t),
\]
\[
a_{f}^{a_{f}} = \frac{d_{f}}{2\pi} \int_{0}^{2\pi} (1 \sin \omega t)^{a_{f}}d(\omega t).
\]

In Eq. (6), the second term in the right hand side vanishes in the case of static creep rupture, while the first term becomes negligibly small as compared with the second term in the case of reversed stress fatigue. In these cases, therefore, the time to fracture is expressed as a power function of static stress or alternating stress, that is, a linear relation holds between the logarithm of static stress or alternating stress and the logarithm of time to fracture. This is demonstrated in the experimental results shown in Fig. 3, and it suggests the validity of the expression for the rate of increase of damage as a power function of stress.

On the other hand, if the static stress which causes rupture at the same time as the dynamic stress does, that is, the equivalent static stress for rupture, is denoted as \(\sigma^{'e}\), then the following relation must be satisfied
\[
\phi_{c} = a_{\sigma_{n}}^{a_{\sigma}}t_{r},
\]

By Eqs. (6) and (9), the relations between \(\sigma_{n}\), \(\sigma_{f}\), and \(\sigma^{'e}\) are obtained as:
\[
\sigma_{n} / \sigma^{'e} = (1 + A^{a_{f}}(a_{f} / a_{e})^{a_{f} / a_{e}})^{a_{\sigma}}
\]

The right hand side of these equations can be calculated, provided the constants \(\sigma_{n}, \sigma_{f}\), and \(\sigma^{'e}\) are known. These constants may be determined from the results of static [creep rupture and reversed stress fatigue tests, by the aid of the following equations.

\[
\left\{
\begin{array}{l}
\sigma_{e}^{'e} = a_{e} \log \sigma + \log t_{r} = \log \phi_{c} / a_{e}, \\
\sigma_{f}^{'e} = a_{f} \log \sigma + \log t_{r} = \log \phi_{c} / a_{f},
\end{array}
\right.
\]

in static creep rupture test,

\[
\left\{
\begin{array}{l}
\sigma_{e}^{'e} = a_{e} \log \sigma + \log t_{r} = \log \phi_{c} / a_{e}, \\
\sigma_{f}^{'e} = a_{f} \log \sigma + \log t_{r} = \log \phi_{c} / a_{f},
\end{array}
\right.
\]

in reversed stress fatigue test.

Consequently, the equivalent static stress \(\sigma^{'e}\) can be calculated for any combination of mean stress and alternating stress from the results of static creep rupture test together with those of reversed stress fatigue test.

Fig. 5 shows the comparison of the analytical results with experimental ones on the low carbon steel stated in the preceding section. In this figure, the stress-range diagram, an example of which was shown in Fig. 4, is exhibited in a non-dimensional expression by taking the ratios of mean stress and alternating stress to equivalent static stress. The circles indicate the experimental results.
and the full line expresses the analytical one, which was calculated by using the following values of constants obtained from the static creep rupture and reversed stress fatigue data. 

\[ \alpha_s/\alpha_r = 32, \quad \alpha'_s/\alpha'_r = 2.9 \times 10^6 \]  \hspace{1cm} (13)

A fairly good agreement is seen in this figure between the analytical and the experimental results over the whole range of stress ratios from zero to infinity. This indicates the possibility of prediction of the dynamic creep rupture or fatigue strength by this analytical method.

In case (2), that is, in the case where fracture occurs when either the creep or fatigue damage reaches a critical value, the equations corresponding to Eqs. (10), (11) in case (1) become

\[ \sigma_m/\sigma'_r = (\Gamma)^{-1/\alpha_s} \]

or

\[ \sigma_m/\sigma'_r = (A_s^{\alpha_s}a_s^{\alpha'_s}/a'_s^{\alpha'_s}^{1-\alpha_s})^{-1/\alpha_s} \]

and

\[ \sigma_s/\sigma'_r = A(\Gamma)^{-1/\alpha_s} \]

or

\[ \sigma_s/\sigma'_r = (a_s^{\alpha_s}a_s^{\alpha'_s}^{1-\alpha_s})^{-1/\alpha_s} = [\sigma_s/\sigma'_r]_{A=\infty}^{1/\alpha_s} \]  \hspace{1cm} (14)

The equivalent static stress is the larger one of the two \( \sigma'_r \) 's calculated from Eq. (14) or (15). To calculate \( \sigma'_r \) by using Eq. (14) or (15), we need to know \( \alpha_s \) and \( \sigma'_s/\sigma'_r \), which may be obtained from static creep rupture and reversed stress fatigue data.

In Fig. 5, the dotted line and the chain line show the limiting stress value calculated from Eq. (14) or (15), the former corresponding to the first equation in Eq. (14) or (15) and the latter to the second in Eq. (14) or (15). The limiting stress is indicated by the chain line in the range of stress ratios larger than that at the intersecting point of the dotted and chain lines, and it is illustrated by the dotted line in the range of smaller stress ratios. These two lines coincide with the solid line over the almost entire range of stress ratios but the neighbourhood of the intersecting point, and agree well also with the experimental results.

In such a way, in either case of those discussed above, the creep damage is influential in the case of dynamic stress with stress ratio less than about 1.7 and the fatigue damage has a vital influence in the case of larger stress ratio than about 1.7. In other words, the strength can be estimated from the static creep rupture data in the range of stress ratios less than about 1.7, and it can be predicted from the reversed stress fatigue data in the range of stress ratios larger than about 1.7. From such a standpoint, the fracture of material under a dynamic stress with small stress ratio may be called the dynamic creep rupture while that under a dynamic stress with large stress ratio may be the fatigue fracture.

Fig. 6 shows the comparison of the calculated results with the experimental ones of an aluminum alloy 24S-T4 at 500°F given by Lazan et al.\(^{(16)}\). The figure is drawn in the same manner as Fig. 5. Although a difference between the limiting stress indicated by the full line and those indicated by the chain and dotted lines is seen over the somewhat wider range of stress ratios, it is not a serious one and about the same discussion as the case of low carbon steel can be made also in this case.

In Fig. 5, there is a slight disagreement between the analytical curve and experimental points in the range of large stress ratios. This discrepancy is overcome by taking the reduction of cross-sectional area of specimen into considera-
As is seen in Fig. 2, appreciable creep strain is produced in the case of \( A = 2.0 \), and, therefore, the cross sectional area of the test specimen decreases with the progress of creep and correspondingly the applied stress is increased. In the analysis discussed above, for the creep damage, the effect of reduction in cross-section has already been taken into account as this damage was estimated from the static creep rupture data obtained under a constant load test, but, for the fatigue damage, it has not been done as this damage was estimated from the data of reversed stress fatigue test conducted under a constant stress amplitude.

In the case of the low carbon steel, the true stress amplitude may be regarded as 7.5 percent larger than the nominal value according to the results shown in Fig. 2. If this increase in stress amplitude is assumed as 7.5 percent, a point is obtained on the line of \( A = 2.0 \) in Fig. 5, and a straight line connecting this point and the point for \( A = \infty \) is drawn in a double-dot chain line as seen in the figure. A fairly good agreement is seen with experimental point for \( A = 2.0 \). To obtain the correct line by taking the reduction of cross-sectional area into account, the dynamic creep data in the required range of stress ratio is necessary. However, the error will be little for assuming that the allowable stress-range is determined by the double-dot chain line together with the dotted line in Fig. 5, and it will be of satisfactory accuracy and simplicity for practical purpose.

The increase in stress amplitude due to the reduction in cross-sectional area depends of course on the magnitude of creep deformation. In the case of aluminum alloy shown in Fig. 6, the creep deformation is less than one percent\(^{(10)}\) and its effect is negligible. Consequently, the solid line and chain line agree well with experimental points in the figure, suggesting the validity of the above consideration concerning the effect of reduction of cross-sectional area.

5. Analysis of dynamic creep

(1) An extension of the applicable range of the equivalent static stress

The analysis of dynamic creep presented previously by the authors\(^{(2)}\) was for the case of a creep in which the creep strain could be expressed by the transient or steady component only. However, another case is also expected in practice in which the creep strain is composed of the transient and steady components of the magnitude comparable with each other. A discussion will be made on such a case as follows.

In the analysis reported previously, the creep strain was expressed as a function of stress \( \sigma \) and time \( t \) approximately as follows:

\[
\varepsilon = a\sigma^{\alpha/\beta} t^{\alpha/\beta} \quad \text{(16)}
\]

where \( \sigma \), \( \alpha \) and \( \beta \) are constants independent either of stress or time, and \( 0 < \beta < 1 \) for transient creep, \( \beta = 1 \) for steady state creep. In this case, the mechanical equation of state of strain hardening type\(^{(20)}\) becomes

\[
d\varepsilon/dt = \beta t^{1/\beta} \sigma^{\alpha/\beta} e^{(1-\beta)/\beta} \quad \text{(17)}
\]

Integration of Eq. (17) with respect to time \( t \) for the case of varying stress given by Eq. (5) gives the dynamic creep strain as

\[
e = a\sigma^{\alpha/\beta} t^{\alpha/\beta} \quad \text{(18)}
\]

where

\[
\sigma_t = \sigma_0^{1/\beta} \quad \text{(19)}
\]

\[
I = \frac{1}{2\pi} \int_0^{2\pi} (1 + A \sin \omega t)^{\alpha/\beta} d(\omega t) \quad \text{(20)}
\]

Eq. (18) means that the dynamic creep curve under the mean stress \( \sigma_m \) and the alternating stress \( \sigma_a \) is the same as that under the static stress \( \sigma_t \) given by Eq. (19). Thus, \( \sigma_t \) may be called the equivalent static stress for dynamic creep. As seen from Eq. (20), \( I \) is determined from the stress ratio \( A \) and the ratio of stress index \( \alpha \) to time index \( \beta \). Consequently, provided \( \alpha/\beta \) is known from static creep data, the dynamic creep curve for any combination of mean and alternating stresses is obtained as the static creep curve under the equivalent static stress \( \sigma_t \) calculated from Eqs. (19) and (20).

In the case where the steady state component is included in the creep strain together with transient component, Eq. (16) gives a smaller value of strain than the experimental result in the steady state stage and an additional term expressing the steady state component is required to obtain a good agreement in this stage. Such an expression for creep strain necessitates a cumbersome numerical calculation to analyse the dynamic creep strain from the static creep data as seen in the appendix, and does not suit to the practical purpose. However, this difficulty is overcome by applying the equivalent static stress defined for the transient creep to the steady state stage.

It has already been shown in the previous paper that the creep curves under dynamic stress are similar in their shape to those under static stress (c.f. Figs. 7 and 10 in this report). Consequently, the static creep curve under the equivalent static stress defined in the transient stage is supposed to coincide with the curve under dynamic stress even in the steady state stage. An experimental example is shown in Fig. 7. The circles indicate
the results of the dynamic creep test for the low carbon steel and the dotted lines are the creep curves under the equivalent static stress for transient creep, which are interpolated from the results of the static creep test by the aid of the experimental equation shown in the appendix. Although a slight discrepancy is noted, agreement is satisfactory for practical purpose.

The full lines in the figure are the creep curves calculated numerically on the basis of the mechanical equation of state in the way described in the appendix. The curves obtained from the equivalent stress do not appreciably differ also from the curves obtained by numerical calculation.

In the discussion made above, only the creep strain up to the steady state stage was discussed and that of the accelerated stage was not treated. By comparison of Fig. 7 with Fig. 10, we notice that the dynamic creep curves are not so different in their shape from the static creep curves in the accelerated stage of creep also, and that the equivalent static stress for the dynamic creep would coincide with the equivalent static stress for the dynamic creep rupture defined in the preceding section. In fact, the index for determining the equivalent stress in this experiment is \( \alpha/\beta=31 \) for dynamic creep and \( \alpha_s=32 \) for dynamic creep rupture. These values are almost equal to each other and therefore the equivalent stresses under a dynamic stress also coincide with each other approximately. Fig. 8 is the diagram showing the relation between the static stress and the time which is required to produce definite strains and rupture, that is, the design data diagram, drawn by using the results of the static tests in this study. As reported previously(J), the indexes \( \alpha/\beta \) and \( \alpha_s \) are obtained as the slopes of straight lines in this diagram. As seen clearly in the diagram, these lines are parallel to each other, indicating the same value of \( \alpha/\beta \) and \( \alpha_s \). It is to be noted in the diagram that the line for \( \epsilon=8\% \), which is a strain in the steady state stage, is also parallel to the line for \( \epsilon=3\% \) or \( 5\% \), also being a strain in the transient stage. This shows the applicability of the above method of obtaining \( \alpha/\beta \) in the transient stage of creep to the steady state stage.

Summarizing the above discussions, a single value of the equivalent static stress corresponding to a dynamic stress with any combination of mean and alternating stresses can be applied to the deformation in all stages of creep, whether it is transient, steady state or accelerated stage, and also to rupture, provided the lines in the design data diagram may be regarded to be parallel to each other as in the case of this experiment. The equivalent static stress is calculated by using the index \( \alpha/\beta \) or \( \alpha_s \) which is determined as the slope of these lines. This is true for many other materials as seen in the design data diagrams given in several literatures(J). In such cases, therefore, dynamic creep and rupture strength can be estimated by considering that the dynamic creep behaviour including the whole process up to fracture is entirely the same as the behaviour under the equivalent static stress.

(2) Influence of dislocation oscillation due to alternating stress on the dynamic creep

In the case of the analysis of dynamic creep rupture, it was seen that the fatigue damage was generated in addition to the creep damage by the dislocation oscillation due to alternating stress, and accordingly the rupture strength was considerably less than the strength estimated from the information on static creep rupture tests. Since the dislocation oscillation exerts such an influence on the dynamic creep rupture, it would produce some effect on the dynamic creep also. An example is shown in Fig. 9. This figure is the stress-range diagram for 5 percent allowable strain in a non-dimensional expression, drawn in the same manner as Fig. 5. In this figure, the analytical line based
on the strain hardening theory (full line) agrees well with the experimental points in the range of stress ratios less than 0.70, but some discrepancy is noted at the stress ratio of 2.0. In the latter case, the estimated strength is on the unsafe side of the experimental one, and therefore it is necessary to conduct an analysis taking this effect into account.

In this study, the authors will conduct an analysis on the assumption that the creep rate under such a dynamic stress condition is multiplied by a factor \( \psi \), which is equal to the ratio of the total damage \( \phi_1 + \phi_2 \) to the creep damage \( \phi_1 \). According to the assumption, the creep rate may be written as

\[
d\varepsilon/dt = \psi \beta \varepsilon^{1/2} \sigma^{\alpha/2} \varepsilon^{-(1 - \beta)/3}.
\]

\[
\psi = (\phi_1 + \phi_2)/\phi_1 = 1 + \frac{\sigma_f}{\sigma_0} A_{\alpha} a_{\alpha} a_f - a_s.
\]

Analysis on this line gives the equivalent static stress \( \sigma_e^* \) as

\[
\sigma_e^* = \sigma_0 \psi^{1/\beta}.
\]

where \( \sigma_0 \) is the equivalent stress estimated on the basis of the mechanical equation of state.

In Fig. 9, the analytical curve obtained from Eqs. (21) and (22) is shown by a dotted line. If the reduction of cross-sectional area of specimen is taken into account in the same way as the case of fatigue, analytical curve becomes as indicated by a double-dot chain line. A good agreement is seen in the figure, showing the validity of the analysis.

6. Conclusion

Dynamic creep and rupture tests were carried out with a low carbon steel under dynamic stress with stress ratios more than unity. The results of the present study together with those obtained in the previous study for the stress ratios less than unity were analysed in the light of the authors' theory, which aims to relate the strength under dynamic stress to the static creep and rupture strength. The conclusion obtained is summarized as follows.

1. The forms of dynamic creep curves are similar to those of static creep curves even for the case of stress ratio of \( A = 2.0 \). In this case, however, the third stage of creep does not appear and the fracture occur at the end of the second stage.

2. The prediction of dynamic creep rupture strength from the static creep rupture data together with fatigue data under reversed stress established over the entire range of stress ratios from zero (static stress) to infinity (reversed stress). This is based on the idea that the damages of material under varying stress are composed of the creep damage and the fatigue damage caused by the dislocation oscillation due to the alternating stress.

3. According to the analysis stated in 2, the dynamic creep rupture strength under a dynamic stress with stress ratio less than unity can be estimated from the static creep rupture data, and the fatigue strength under a dynamic stress with stress ratio larger than about 2 may be predicted from the results of reversed stress fatigue test.

4. The equivalent static stress for dynamic creep, which is designated to predict the dynamic creep strength from static creep data in the stage of transient creep, is applicable to the steady state stage and also to rupture. In other words, the dynamic creep behaviour in all stages of creep up to rupture is the same as the static creep behaviour under the equivalent static stress determined for the transient stage.

5. The influence of the dislocation oscillation due to alternating stress on the dynamic creep was also discussed.

The experiments as well as the analyses presented here are concerned mainly with a low carbon steel. Consequently, some modification will be necessary for the application of the analyses to various metallic materials, taking the structural change due to precipitation or other factors into consideration. However, the authors believe that a method of analysis as the basis for such general cases is established by this study, and intend to investigate further on the wider application of the analyses.

Appendix

The experimental equation used in Section 5
\( \varepsilon = a_0 \sigma^{\alpha_0} + a_1 \sigma^{\alpha_1} t^{\beta_1} + a_2 \sigma^{\alpha_2} t \)  \hspace{1cm} (a)

where \( \varepsilon \) is strain in percent, \( \sigma \) is static stress in kg/mm\(^2\), and \( t \) is time in hours, and the values of constants are

\[
\begin{align*}
    a_0 &= 6.06 \times 10^{-4}, \quad a_1 = 2.39 \times 10^{-7}, \quad a_2 = 3.0 \times 10^{-29} \\
    \alpha_0 &= 2.60, \quad \alpha_1 = 4.88, \quad \alpha_2 = 19.25, \quad \beta_1 = 0.332
\end{align*}
\]

Fig. 10 shows the comparison of the calculated results from Eq. (a) with the experimental ones.

The numerical calculation based on the mechanical equation of state was also carried out by using this equation in the following way. Strain increment \( \Delta \varepsilon \) in one cycle of stress alternation is expressed as

\[
\Delta \varepsilon = \frac{1}{\omega} \int_{0}^{2\pi} (a_1 \beta_1 \sigma^{\alpha_1} t^{\beta_1} + a_2 \sigma^{\alpha_2}) d(\omega t) \quad \cdots \hspace{1cm} (b)
\]

where \( \tau \) is determined from the following equation,

\[
\varepsilon = a_0 \sigma^{\alpha_0} + a_1 \sigma^{\alpha_1} t^{\beta_1} + a_2 \sigma^{\alpha_2} t \quad \cdots \hspace{1cm} (c)
\]

In Eq. (c) is the average strain in one cycle. As the time \( \Delta t \) that elapsed during one cycle of stress is equal to \( 2\pi/\omega \), the average dynamic creep rate \( \dot{\varepsilon} \) in one cycle becomes

\[
\dot{\varepsilon} = \frac{\Delta \varepsilon}{\Delta t} = \frac{1}{2\pi} \int_{0}^{2\pi} (a_1 \beta_1 \sigma^{\alpha_1} t^{\beta_1} + a_2 \sigma^{\alpha_2}) d(\omega t) \quad \cdots \hspace{1cm} (d)
\]

By Eqs. (c) and (d), the relation between the strain \( \varepsilon \) and the strain rate \( \dot{\varepsilon} \) is determined and from this relation the strain is calculated as a function of time.

References

(2) M.J. Manjoine: ibid, p. 788.