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Self-Excited Rolling Motion of a Vehicle Suspended by Air Springs*

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In this paper the self-excited rolling motion occasionally seen in a vehicle suspended by air springs is treated. Analysis and discussion are made to see under what conditions this motion arises and how to design the control system of air springs so as to be able to prevent such motion. On an example of vehicles in which this motion has occurred, the amplitude and frequency of this motion are calculated.

1. Introduction

An air spring has been often used for a suspension system of vehicle to improve the vibrational characteristics of vehicle. Air springs are generally used together with height control systems, and thereby air springs with very small stiffness have practical utility. As the vehicles suspended by air springs with the height control valves constitute closed loop systems, there is a possibility of instability being caused. In fact, cases were rarely reported of self-excited rolling motions of comparatively low frequency having occurred in vehicles so equipped.

Investigations were made to see under what conditions this motion arises and how to design the control system of air springs in such manner as to be able to prevent it. On an example of vehicles in which this motion had occurred, the amplitude and frequency of this motion were calculated.

2. Nomenclature

x : displacement of air spring, (cm)
 C_1 : capacity of auxiliary tank, (cm³)
 C_2 : capacity of air spring, (cm³)
 C_{20} : capacity of air spring at equilibrium state, (cm³)
 P_1 : absolute pressure in auxiliary tank, (kg/cm²)
 P_{10} : absolute pressure in auxiliary tank at equilibrium state, (kg/cm²)
 p_1 : pressure variation from equilibrium state in auxiliary tank, ($P_1 = P_{10} + p_1$), (kg/cm²)

P_2, P_{20}, p_2 : the same quantities in air spring as those in auxiliary tank, ($P_2 = P_{20} + p_2$), (kg/cm²)
 W_{10} : total weight of air in auxiliary tank at equilibrium state, (kg)
 W_{20} : total weight of air in air spring at equilibrium state, (kg)
 q_1 : air flow from control valve to auxiliary tank, (kg/sec)
 q_2 : air flow from auxiliary tank to air spring, (kg/sec)
 n : polytropic index of air
 a : constant given by the relation $a = Pv^n$ (v : specific volume), (kg·cm)
 A : mean cross area of air spring, (cm²)
 A_e : effective bearing area of air spring, (cm²)
 R_I : flow resistance of control valve and piping system, (s/cm²)
 R_{II} : nonlinear flow resistance of orifice between air spring and auxiliary tank, (s²/kg·cm²)
 R_2 : equivalent resistance of orifice, (s/cm²)
 Y : degree of opening of valve, (non-dimensional)
 y : displacement of piston, (cm)
 l_2, l_3, l_4 : lengths of various parts of lever system shown in Fig. 1, (cm)
 k : spring constant of buffer spring, (kg/cm)
 F_0 : initial tension of buffer spring, (kg)
 c : viscous damping coefficient of dashpot, (kg·s/cm)
 y_0, y_1 : values of dead zone and saturation of control valve, (ref. Fig. 4), (cm)
 P_s : absolute pressure of source tank, (kg/cm²)
 P_0 : absolute pressure of atmosphere, (kg/cm²)
 I_G : moment of inertia of car body about longitudinal axis through its center of gravity, (kg·s²·cm)

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- I : moment of inertia of car body about axis of rotation center, ($\text{kg}\cdot\text{s}^2\cdot\text{cm}$)
 r : distance between rotation center and center of gravity of car body, (cm)
 l : half the lateral distance between centers of air springs on both sides, (cm)
 M : mass of car body, ($\text{kg}\cdot\text{s}^2/\text{cm}$)
 g : acceleration due to gravity
 θ : angle of inclination of car body
 K : spring constant of rubber part of air spring, (kg/cm)
 c' : sum of damping coefficients of rubber part of air spring and oil damper installed parallel to the spring, ($\text{kg}\cdot\text{s}/\text{cm}$)

3. Transfer characteristics and block diagram of the system

A simplified schematic diagram of a vehicle suspended by air springs is shown in Fig. 1. When heights of air springs change from the desired height as shown in Fig. 2, the control valves operate in such a way as to restore them.

(i) Transfer characteristics of air system

There exists the following relation for the air spring:

$$(P_{20} + p_2) \{ (C_{20} - Ax) / (W_{20} + \int q_2 dt) \}^n = a \quad (1)$$

Fig. 1 Schematic diagram of the system

Fig. 2 Schematic diagram of the system under action of control valve

If $P_{20} \gg |p_2|$, $C_{20} \gg A|x|$ and $W_{20} \gg \left| \int q_2 dt \right|$

then, neglecting small quantities in Eq. (1), we have

$$p_2 = \frac{n}{C_{20}^n} \left\{ a W_{20}^{n-1} \int q_2 dt + A P_{20} C_{20}^{n-1} x \right\} \\ = n P_{20} \left\{ \frac{Ax}{C_{20}} + \frac{1}{W_{20}} \int q_2 dt \right\} \quad (2)$$

For the case of auxiliary tank we have the following relation similarly to the case of air spring:

$$p_1 = \frac{n}{C_1^n} a W_{10}^{n-1} \left\{ \int q_1 dt - \int q_2 dt \right\} \\ = n P_{10} \frac{1}{W_{10}} \left\{ \int q_1 dt - \int q_2 dt \right\} \quad (3)$$

Between Y and q the following assumption may be made.

$$\left. \begin{aligned} q_1 &= \frac{P_s - P_1}{R_1} Y; & Y > 0 \\ q_2 &= \frac{P_1 - P_0}{R_1} Y; & Y < 0 \end{aligned} \right\} \quad (4)$$

If $|p_1| \ll P_{10}$, $(P_s - P_{10})$ and $(P_{10} - P_0)$ (5)

Eq. (4) may be written approximately as follows*:

$$\left. \begin{aligned} q_1 &= \frac{P_s - P_{10}}{R_1} Y; & Y > 0 \\ q_2 &= \frac{P_{10} - P_0}{R_1} Y; & Y < 0 \end{aligned} \right\} \quad (6)$$

When Eq. (5) holds, Eq. (4) or (6) may be used instead, even if the exact relations are such that

$$\left. \begin{aligned} q_1 &= \sqrt{\frac{P_s - P_1}{R}} Y; & Y > 0 \\ q_2 &= \sqrt{\frac{P_1 - P_0}{R}} Y; & Y < 0 \end{aligned} \right\} \quad (7)$$

for the linearized form of Eq. (7) is Eq. (6) in the case of Eq. (5). If P_0 and P_s are chosen to satisfy

$$P_s - P_{10} = P_{10} - P_0 \quad (8)$$

then the two equations in Eq. (6) may be represented by one equation as follows:

$$q_1 = \frac{P_s - P_0}{2R_1} Y \quad (9)$$

Since the system with this condition is considered better in its performance than that for $P_s - P_{10} \neq P_{10} - P_0$, only this case will be discussed hereafter.

q_2 is related with $(P_1 - P_2)$ as follows:

$$\pm q_2^2 = \frac{P_1 - P_2}{R_{II}}, \quad \pm: P_1 - P_2 \geq 0 \quad (10)$$

* In air spring system in practice, the value of P_0/P_1 is usually smaller than the value of the critical pressure ratio, 0.53, and P_1/P_s is nearly equal to or higher than 0.53. Therefore, the second equation of Eq. (6) is more appropriate than the second one of Eq. (4).

As $P_{10} = P_{20}$, this equation is precisely equivalent to

$$\pm q_2^2 = \frac{p_1 - p_2}{R_{II}}, \quad \pm: p_1 - p_2 \geq 0 \quad (11)$$

With the equivalent resistance R_2 this nonlinear equation may be linearized as follows:

$$q_2 = \frac{p_1 - p_2}{R_2} \quad (12)$$

R_2 will be discussed later.

(ii) Transfer characteristics of lever system

The lever system transmitting the vertical displacement of air spring to the valve piston is composed of two rods which have the common hinge as shown in Fig. 1 or 2. The initially strained spring named buffer spring restrains the relative rotation of these two rods. When the spring in a control valve is assumed so soft that it scarcely resists the movement of the piston, the transfer characteristic between x and y is obtained as shown in Fig. 3.

The transfer characteristic between y and Y may be approximated as shown in Fig. 4.

$$\begin{aligned} \text{and} \quad b_{50} &= IC_1 C_{20} l_3^2 / n^2, \quad b_{40} = b_{50} (l_4 - l_3)^2 / l_3^2 \\ b_{41} &= \frac{P_{20}}{n} \left(\frac{a}{P_{20}} \right)^{1/n} I (C_1 + C_{20}) l_3^2, \quad b_{42} = 4l^2 C_1 C_{20} l_3^2 / n^2 \\ b_{30} &= b_{41} (l_4 - l_3)^2 / l_3^2, \quad b_{31} = b_{42} (l_4 - l_3)^2 / l_3^2, \quad b_{32} = 4l^2 \frac{P_{20}}{n} \left(\frac{a}{P_{20}} \right)^{1/n} (C_1 + C_{20}) l_3^2 \\ b_{33} &= \{ 4l^2 A_e A P_{20} C_1 + C_1 C_{20} (4Kl^2 - Mgr) / n \} l_3^2 / n \\ b_{20} &= b_{32} (l_4 - l_3)^2 / l_3^2, \quad b_{21} = b_{33} (l_4 - l_3)^2 / l_3^2 \\ b_{22} &= \left\{ 4l^2 A_e A P_{20}^2 \left(\frac{a}{P_{20}} \right)^{1/n} + \frac{P_{20}}{n} \left(\frac{a}{P_{20}} \right)^{1/n} (C_1 + C_{20}) (4Kl^2 - Mgr) \right\} l_3^2 \\ b_{10} &= b_{22} (l_4 - l_3)^2 / l_3^2, \quad b_{00} = 4l^2 A_e P_{20}^2 \left(\frac{a}{P_{20}} \right)^{2/n} (P_s - P_0) \frac{l_3}{2l_2} (l_4 - l_3)^2 \end{aligned} \quad (16)$$

κ_1 and κ_2 in Eq. (15) will be discussed later.

5. Equivalent transfer function of nonlinear element

(i) Buffer spring

If the sinusoidal force $F_m \sin \omega t$ is applied on the buffer spring, the fundamental harmonic of the displacement of this spring becomes κ_1 times $F_m \sin \omega t$, where κ_1 is the quantity called equivalent gain and is the function of F_m as follows:

$$\begin{aligned} \kappa_1 &= 0; & F_m < F_0 \\ \kappa_1 &= \frac{2}{\pi k} \left\{ \cos^{-1} \frac{F_0}{F_m} - \frac{F_0}{F_m} \sin \left(\cos^{-1} \frac{F_0}{F_m} \right) \right\}; & F_m > F_0 \end{aligned} \quad (17)$$

F_m is related with y_m , the amplitude of y , as

$$F_m = \frac{l_3}{l_4 - l_3} c \omega y_m \quad (18)$$

(ii) Control valve

When $y_0 = 0$, the equivalent gain κ_2 of the element shown in Fig. 4 is obtained as follows:

$$\begin{aligned} \kappa_2 &= 1/y_1; & y_m < y_1 \\ \kappa_2 &= \frac{2}{y_1 \pi} \left\{ \sin^{-1} \frac{y_1}{y_m} + \frac{y_1}{y_m} \cos \left(\sin^{-1} \frac{y_1}{y_m} \right) \right\}; & y_m > y_1 \end{aligned} \quad (19)$$

(iii) Equation of motion of car body

The equation of rolling motion of the car body is as follows (ref. Fig. 5):

$$I\ddot{\theta} = Mgr\theta - 4A_e p_2 l - 4Kl^2 \theta - 4c'l^2 \dot{\theta} \quad (13)$$

(iv) Total block diagram

The total block diagram of this system is shown in Fig. 6, where the element indicated G_r is the simplified representation of the block shown in Fig. 3.

4. Characteristic equation

The characteristic equation of the system shown in Fig. 6 is expressed in the form

$$f(s) = a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 \quad (14)$$

where

$$\left. \begin{aligned} a_5 &= b_{50} c \kappa_1 R_2 \\ a_4 &= b_{40} R_2 + b_{41} c \kappa_1 + b_{42} c' c \kappa_1 R_2 \\ a_3 &= b_{30} + b_{31} c' R_2 + b_{32} c' c \kappa_1 + b_{33} c \kappa_1 R_2 \\ a_2 &= b_{20} c' + b_{21} R_2 + b_{22} c \kappa_1 \\ a_1 &= b_{10} \\ a_0 &= b_{00} \kappa_2 / R_1 \end{aligned} \right\} \quad (15)$$

Fig. 3 Transfer characteristic between displacement of valve piston and displacement of air spring

Fig. 4 Transfer characteristic between degree of the opening of valve and displacement of valve piston

Fig. 5 Mode of rolling motion of car body

Fig. 6 Total block diagram of the system

(iii) Orifice

When $q_2 = q_{2m} \sin \omega t$, the equivalent resistance R_2 is given by

$$R_2 = \frac{8}{3\pi} R_{II} q_{2m} \quad (20)$$

Expressing q_{2m} as a function of y_m and substituting this q_{2m} into the above equation of R_2 , we have

$$R_2 = \frac{\sqrt{2}}{\omega} B \sqrt{H-1}$$

where

$$\left. \begin{aligned} H &= \sqrt{1 + (NR_{II}^2 y_m^2 / B^4)} \\ N &= \frac{16}{9\pi^2} \omega^4 n^2 P_{10}^2 \left[\left\{ \frac{P_s - P_0}{2R_1} \cdot \frac{1}{C_1} \left(\frac{a}{P_{10}} \right)^{1/n} \frac{\kappa_2}{\omega} + \frac{Al_2 l_3 \kappa_1 c \omega}{(l_4 - l_3)^2 C_{20}} \right\}^2 + \left(\frac{Al_2}{C_{20} l_3} \right)^2 \right] \\ B &= \frac{n P_{10}}{2} \left(\frac{a}{P_{10}} \right)^{1/n} \left\{ \frac{1}{C_1} + \frac{1}{C_{20}} \right\} \end{aligned} \right\} \quad (21)$$

6. Discussion on stability

First, let us discuss the case of $\kappa_1 = 0$ and $R_2 = 0$ which corresponds to the case of small amplitude. In this case, the characteristic equation becomes

$$b_{30}s^3 + b_{20}c's^2 + b_{10}s + b_{00}\kappa_2/R_1 = 0 \quad (22)$$

By the Hurwitz's criterion the system is found to be stable under the present condition

$$c' > \frac{b_{30}b_{00}}{b_{20}b_{10}} \frac{\kappa_2}{R_1} \quad (23)$$

When $c' > 0$, if $y_0 \neq 0$ and the position of the valve piston y at equilibrium state is within the region $y_0 < y < -y_0$, the system is always stable unless a considerably large disturbance is introduced.

Practically, it is often the case that y at equilibrium state is equal to y_0 or to $-y_0$, and the equivalent gain κ_2 in the case of $y_0 \neq 0$ at a small amplitude does not become zero. Therefore, the existence of neutral zone in the $y-Y$ relation has no particular meaning for the stability of the system. Thus, for simplicity of discussion we put $y_0 = 0$ as in Eq. (19).

As κ_2 never exceeds $1/y_1$, the following inequality is the condition for stability in the case of $R_2 = 0$ and $\kappa_1 = 0$:

$$c' > \frac{b_{30}b_{00}}{b_{20}b_{10}R_1 y_1} \quad (24)$$

Using Eq. (16), we have

$$\frac{b_{30}b_{00}}{b_{20}b_{10}} = \frac{IA_s P_{20} \left(\frac{a}{P_{20}} \right)^{1/n} (P_s - P_{20}) \frac{l_3}{l_2}}{4l^2 A_s P_{20} A + (4Kl^2 - Mgr) \frac{C_1 + C_{20}}{n}} \quad (25)$$

As the second term of the denominator of this equation is usually negligible as compared to the first term, omitting this we have

$$\frac{b_{30}b_{00}}{b_{20}b_{10}} = \frac{I(P_s - P_{20})l_3}{4l^2 l_2 A} \left(\frac{a}{P_{20}} \right)^{1/n} \quad (26)$$

If the number of passengers increases, P_{20} increases because of $M_g = 4P_{20}A_s$. Therefore, the stability of the system in the case of small amplitude is not appreciably affected by the number of passengers. It is seen that r/l affects the stability.

In order to discuss the effect of R on the stability, we put

$$\kappa_1 = 0 \text{ and } c' = 0$$

We have the following condition for stability :

$$R_2 > \frac{b_{30}^2 b_{00}}{b_{10}(b_{21}b_{30} - b_{10}b_{40})} \frac{\kappa_2}{R_1} = \frac{IP_{20} \left(\frac{a}{P_{20}} \right)^{2/n} (C_1 + C_{20})^2 (P_s - P_{10}) \frac{l_3}{l_2}}{C_1^2 A \left\{ 4l^2 A_s A P_{20} + \frac{(C_1 + C_2)}{n} (4Kl^2 - Mgr) \right\}} \frac{\kappa_2}{R_1} \quad (27)$$

R_2 tends to increase the stability of the system.

Putting

$$R_2 = 0$$

in Eq. (14), we obtain the stability condition for the system as follows :

$$\frac{\kappa_2}{R_1} < \frac{b_{10}b_{20}c'}{b_{00}(b_{30} + b_{32}c'\kappa_1)} + \frac{b_{10}b_{32}b_{22}c'^2\kappa_1^2}{b_{00}(b_{30} + b_{32}c'\kappa_1)^2} \quad (28)$$

The right hand side of Eq. (28) takes the least value

$$\frac{b_{10}b_{20}c'}{b_{00}b_{30}} \left\{ 1 - \frac{b_{20}b_{32}c'^2}{4b_{22}b_{30}} \right\}$$

with the variation of $c\kappa_1$ at

$$c\kappa_1 = \frac{c'b_{20}b_{30}}{2b_{22}b_{30} - b_{20}b_{32}c'^2} \quad (29)$$

Then the safety condition for the stability in the case of $R_2 = 0$ is given as follows :

$$\frac{\kappa_2}{R_1} < \frac{b_{20}b_{10}c'}{b_{00}b_{30}} \left\{ 1 - \frac{b_{20}b_{32}c'^2}{4b_{22}b_{30}} \right\} \quad (30)$$

The value of the second term in the brackets of Eq. (30) for the practical system is usually negligible compared to unity. Comparing this term, we have Eq. (24).

The above discussions lead to the following conclusions :

(a) R_1 and c' have the effect of increasing the stability of the system.

(b) $c\kappa_1$ does not show any fixed tendency for the stability of the system. However, there exists a worst value of $c\kappa_1$, given by Eq. (29), for the stability.

(c) The smaller κ_2 is, the more stable is the system ; κ_2 affects the stability inversely as R_1 .

(d) R_2 has good effect on the stability of the

system.

(e) The larger l is, the more stable is the system.

(f) From the fact that R_2 becomes very small at small amplitude, it seems necessary that the system parameters are so chosen as to satisfy Eq. (30) in which κ_2 is replaced by $1/y_1$.

7. Example of numerical calculations of self-excited rolling motion

It is well known that the amplitude and frequency of self-excited oscillation are determined by solving the equation formed by substitution of $j\omega$ into s of the characteristic equation. That is, the amplitude and frequency of self-excited rolling motion are determined by solving the following equations simultaneously :

$$\begin{cases} \Re\{f(j\omega)\} = 0 \\ \Im\{f(j\omega)\} = 0 \end{cases} \quad (31)$$

Let us consider the problem with a given set of various values as follows :

$$\left. \begin{aligned} C_1 &= 3.4 \times 10^4, \quad C_{20} = 3.0 \times 10^4, \quad A_s = A = 1770 \\ P_{10} &= P_{20} = 3.78, \quad P_0 = 1, \quad P_s = 6.56 \\ l_1 &= 13, \quad l_2 = 15, \quad l_3 = 3, \quad l_4 = 5 \\ M_g &= 19660, \quad I_G = 2.88 \times 10^3, \quad r = 100 \\ l &= 103.5, \quad I = 2.035 \times 10^5, \quad n = 1 \\ a &= 8.43 \times 10^5, \quad K = 49.6 \end{aligned} \right\} \quad (32)$$

$$\left. \begin{aligned} y_1 &= 0.25, \quad k = 10, \quad R_1 = 60 \\ R_{II} &= 640, \quad F_0 = 10 \end{aligned} \right\} \quad (33)$$

$$\left. \begin{aligned} c &= 44 - 2\omega; \quad \omega < 22 \\ c &= 0; \quad \omega > 22 \end{aligned} \right\} \quad (34)$$

$$\left. \begin{aligned} c' &= 5; \quad \omega < 1 \\ c' &= 5/\omega; \quad \omega > 1 \end{aligned} \right\} \quad (35)$$

From Eq. (30) we have

$$\left. \begin{aligned} b_{50} &= 1.868 \times 10^{15}, & b_{40} &= 3.303 \times 10^{14} \\ b_{41} &= 9.881 \times 10^{16}, & b_{42} &= 3.934 \times 10^{14} \\ b_{30} &= 4.392 \times 10^{16}, & b_{31} &= 1.748 \times 10^{14} \\ b_{32} &= 2.081 \times 10^{16}, & b_{33} &= 1.567 \times 10^{17} \\ b_{20} &= 9.247 \times 10^{15}, & b_{21} &= 6.964 \times 10^{16} \\ b_{22} &= 3.928 \times 10^{18}, & b_{10} &= 1.746 \times 10^{18} \\ b_{00} &= 1.199 \times 10^{20} \end{aligned} \right\} \dots\dots (36)$$

Seeking numerically the pairs of ω and y_m satisfying the first equation of Eq. (31) we have the curve representing $R_e\{f(j\omega)\}=0$ shown in Fig. 7. Similarly, we have the curve for $I_m\{f(j\omega)\}=0$ of Fig. 7. The intersection of these two curves

$$y_m = 0.08 \text{ cm}, \quad \omega = 2.51 \text{ rad/sec}$$

gives the amplitude and frequency of the rolling motion. It seems that these values are of the same order as the ones observed in practical cases. If $y \neq 0$, there may exist an unstable limit cycle of

Fig. 7 Loci of points satisfying Eq. (31) on y_m - ω plane

smaller amplitude together with the stable limit cycle obtained above.

Discussion

J. ACHIWA: (1) In this paper it is mentioned that the expression (7) in the special case is included in the general Eq. (4) by giving R_1 a suitable value. But I consider this is not right unless R is the function of P .

(2) In the chapter "Discussion on stability" the effect of c' and R_1 on stability in the extremely small amplitude is described in the special case of $\kappa_1=0$ and $R_2 \neq 0$ and the effect of R_2 in the case of $\kappa_1=0$ and $c'=0$. But this explanation in such a special case is not practical and we require the very solution in the case of $c' \neq 0$. The reason is that in the general traveling vehicle motion the resistance of orifice R_2 , damping factor of the shock absorber etc. exist together, not alone. Similarly the explanation on the influence of κ_1 is given in the case of $R_2=0$, but we need a solution to the general case $R_2 \neq 0$. Even if the calculation becomes complicated, we prefer a general solution to the special one.

(3) The expression (34), some leveling valve damping property, means that the damping factor c decreases with an increase in its angular frequency ω , but it is incomprehensible for us, because this phenomenon is quite contrary to our ordinary experience. Likewise the expression (35), i.e. the damping property of rubber, is hard to be understood, for we know the experimental fact that the rubber damping factor c' generally increases with an increase in ω . The example is calculated on the assumption that no free travel is given at the leveling valve arm end, but this

assumption seems irrational for the example given. Because one of the important purposes of the leveling valve with a dashpot is to by-pass the high frequency relative motions between the body and wheels by giving a free travel at the arm end. Therefore we need the solution assuming such free travel for enabling application of the results to any actual case.

T. TAKADA: (1) The damping coefficients c and c' , which are shown in the Eq. (34) and (35), are of nonlinear characteristics of special type. What type of damper can show such damping characteristics?

(2) How is the time-lag of the control valve, which the author used in the example of calculation?

(3) On actual cars with air spring, self-excited rolling motions can occasionally be seen without any action of control valve in the case of passengers getting into or out of cars. It is considered that such motions are due to the extreme softness of air spring compared with that of metal spring. I would like to know the suitable method of preventing the said motion.

Author's closure

The authors wish to thank Mr. Achiwa and Mr. Takada for their discussion.

Mr. Achiwa made a statement in his first paragraph concerning the relation between R_1 and R in Eqs. (4) and (7). The dimension of R , which is not illustrated in the section of nomenclature,

is different from that of R_1 .

The solution of the stability in the general case that Mr. Achiwa would like to see can not be made in the form from which the influence of each parameter on stability can be seen. Of course, the particular discussion of stability of the numerical system having a few parameters may be made. But this is not the intent of this paper.

One of the questions raised by Mr. Achiwa and Mr. Takada pertains to the appropriateness of Eq. (34). Eq. (34) has been derived from an experiment on the frequency response of the system shown in Fig. 3. In this experiment the input amplitude x_m has been kept constant. Analysing the data, we knew that the value of c varied with frequency. Then, we set c as Eq. (34). However, if this is unreasonable from the viewpoint of the principle of action of the dashpot, we must seek any other cause of the variation of c . Since y_m varies with frequency when x_m is held constant, even if c has the amplitude dependency instead of the frequency dependency, the frequency response of the system shown in Fig. 3 behaves as if c varies with frequency. Analysing the data under the assumption that c has the amplitude dependency, we have

$$c = 420 y_m$$

Calculating the self excited rolling motion using this value instead of Eq. (34), we have

$$y_m = 0.084 \text{ cm}, \quad \omega = 2.54 \text{ rad/sec}, \quad x_m = 0.48 \text{ cm}$$

The second comment made by both Mr. Achiwa and Mr. Takada is concerning Eq. (35). A loss of rubber, an area of a hysteresis loop of load-deflection diagram of rubber, increases with frequency at first, then reaches a constant at certain frequency. Since c' is given by a loss divided by the frequency ω , c' behaves as Eq. (35) with the frequency ω . But there will be open to discussion about the value of the constants in Eq. (35).

Mr. Achiwa made a comment in his last paragraph concerning the numerical calculation executed under the assumption $y_0 = 0$. If we calculate the self excited rolling motion under the condition $y_0 \neq 0$, we have an unstable limit cycle together with a stable limit cycle. Since the stable limit cycle under $y_0 \neq 0$ seems to be not so far different from that under $y_0 = 0$ if y_0 is not so large, we have calculated about the case of $y_0 = 0$.

The question raised by Mr. Takada in his second paragraph is concerned with the time-lag of the control valve used in the example. It ranges from 0.5 to 2 sec.

The last statement made by Mr. Takada concerning the rolling motion without any action of control valve occasionally observed in practice is very interesting. However, this motion seems to be the free oscillation of the carriage-spring system, because the control valve does not operate. It seems to be sufficient to increase c' in order to prevent this motion.