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Studies on the Characteristics of Axial-Flow Pumps*

(Part 4, Comparison between Theory and Experimental Results)

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In this paper, the values of some coefficients, which have been introduced in the theoretical analysis about the axial-flow pump performance shown in part 2 of this study, are determined by using the velocity distributions at the inlet and outlet from impeller blades obtained in part 3. These coefficients can be shown in comparatively simple forms, and an application of the theoretical solution to the actual problem becomes possible. In order to prove the accuracy of the theoretical solution, the flows before and behind the impeller blades and the theoretical head-flow characteristics of an axial-flow pump with impeller blades designed by the modified aerofoil theory are calculated by using these coefficients. The results coincide considerably well with the experimental ones.

1. Introduction

In the present report, some coefficients which have been introduced in the theoretical solution in part 2⁽¹⁾ of this study are decided by using the experimental results shown in part 3⁽²⁾. Further, the effects of the number of blades, setting angles and profil of blades on these coefficients are investigated. The determination of the values of these coefficients makes the theoretical solution applicable to an actual problem. To demonstrate the accuracy of theoretical solution, the velocity distributions in the pump casing and theoretical heads from the design point to shut-off point were calculated and found in good agreement with experimental ones, except in the reverse flow domains on both sides of impeller blades, where the loss of head affects the flows seriously.

2. Determination of coefficients

2.1 Coefficient of slip angle

As described in part 2, the flow in the impeller must be solved by regarding it as one dimensional flow along the impeller blades, then the coefficients of slip angle $1+p_1$ have been introduced⁽¹⁾. It has never been decided experimentally up to date, because the flow in an axial flow pump is usually treated as a flow through a cascade of blades. The

values of $1+p_1$ for an axial flow pump are decided only by the location and not changed by flow rates as shown already in part 2 of this study, therefore it is possible to use $1+p_1$ decided at the design point. Accordingly it may be determined about the blades designed by using the aerofoil theory⁽³⁾. That is, since an aerofoil, which has a desired theoretical head H_{th} under a given flow rate at each radius, can be obtained, $H_{th\infty}$, which is theoretical head for the infinite number of blades, can be obtained, too; $1+p_1 = H_{th\infty}/H_{th}$, therefore, will be given easily.

Here is a problem whether the angle between the tangential direction to the mean camber line at the tail end and the peripheral direction, or the angle given by the direction of a zero lift angle should be chosen for the exit angle of blades. According to the study of Wislicenus⁽⁴⁾, if the coefficient of lift is given by $C_A = 2\pi K \sin \delta$ (K : coefficient of interference, δ : attack angle) and drag is neglected, $1+p_1$ is given by the following equation

$$1+p_1 = \frac{1}{2} \left(\frac{t}{l} \frac{2}{\pi K \sin \beta_0} + 1 \right) \dots\dots\dots (1)$$

where, β_0 is the angle between the direction which gives a zero lift angle and the peripheral direction; and t/l gives the pitch-chord ratio. It is shown that if an exit angle is measured from the direction of zero lift angle, $1+p_1$ can be represented by using the aerofoil characteristics. As described previously, further, $1+p_1$ decided by such a method is

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independent of the flow rate. After all, it may be recognized that $1+p_1$ based on the zero lift angle is more proper. It will be shown in the following chapter that the flows on the delivery and suction sides of impeller blades calculated by using such a coefficient have a good agreement with those obtained from the experiments.

In order to know the radial flow at radius r_1 in the impeller, the effects of slip angle were taken into account in 3.2 of part 2, because the number of blades was finite. Since the flow in the impeller has a conspicuous radial velocity component, it may be considered as the flow in the channel put between two adjacent blades. Accordingly, $1+p_1$ given by the relation⁽⁵⁾ derived by Pfeleiderer seems more appropriate than that shown by Eq. (1). Then the value given by Eq. (17) in part 2 was applied.

2.2 Critical capacity

As illustrated in part 1⁽⁶⁾, the flow which begins a reverse flow on the suction side of impeller blades is very important to explain the impeller performance at low flow rates and has been defined as the critical discharge q_c . It can be determined experimentally and is shown in Fig. 1, where, φ_{dmc} and $\varphi_{dm\eta \max}$ represent the non-dimensional values of mean axial velocity components on the delivery side at q_c and the max. efficiency point. The graph shows that its ratio is little affected by the number of blades and setting angle, which takes a nearly constant value 0.6 if the hub ratio is about 0.5. It, further, shows that the effect of the impeller blade-tips clearance on this value is comparatively small, too.

It is noteworthy that the capacity at the max. efficiency point becomes larger than what gives the min. drag-lift ratio as explained in part 3, if the setting angle of blades is increased and the guide vanes are fixed. Provided that the guide vanes

and delivery pipe are so arranged as to present the max. efficiency at the optimum attack angle, for a larger setting angle, then the capacity at the max. efficiency point will become larger than that above described. For such a case, accordingly, $\varphi_{dmc}/\varphi_{dm\eta \max}$ becomes fairly small and there exists a wide axial-flow domain. After all, since an aerofoil is apt to stall in the axial flow domain for the impeller of large setting angle, there is often an unstable zone in the pump performance curve.

Capacities which begin a reverse flow on the delivery side of impeller blades are obtained from the experiments and shown in Fig. 1, simultaneously. These capacities are a little larger than those which begin a backward flow on the suction side, usually, and it seems that this reverse flow on the delivery side makes the radial flow in the impeller increase, as a result, causing a backward flow on the suction side.

2.3 Radial flow at R_1

For the sake of convenience to find the impeller performance at low flow rates, the flow in the impeller has been investigated by dividing it into two different domains: one is affected seriously by the casing wall but another is little affected by it. The boundary between these two domains has been defined by a cylindrical surface with radius R_1 ($=r_1/r_2$, r_2 : the radius of the casing wall). As explained in part 2, it is given by the radial point where a reverse flow begins in front of the impeller in the centrifugal flow domain and the relation between φ_{dm} ($=v_{dm}/u_t$, v_{dm} : mean axial velocity on the delivery side, u_t : peripheral velocity of the impeller blade-tips) and R_1 has been discussed to some extent (see Fig. 1 in part 2).

The values of R_1 were decided from the experiments for various types of impellers and graphed in Fig. 2. The ordinate is all given by the

Fig. 1 Capacities which begin reverse flows on both sides of impeller blades

capacity ratio to the max. efficiency point. Every measured point lines up along a straight line given by connecting two points on the graph; $R_1=1$ at the design capacity (which contains the leakage flow) and $R_1=0.8$ at $\varphi_{dm}=0$. It means that R_1 is affected seriously by the casing wall because the width of jet flow domain is narrow, and is little affected by the state of impeller blades.

The radial flow capacity in the impeller can be decided by measuring the capacities of fluid which flowed out on both sides of the impeller, in the jet flow domain, if the radial flows on both sides of the impeller are neglected. From it, the non-dimensional mean radial velocity $y_{R_{1m}} (=c_{r_{1m}}/u_t, c_{r_{1m}}: \text{mean radial velocity at } r_1)$ is obtained, and graphed in Figs. 3 and 4. They show that the effects of the number of blades and aerofoil are comparatively small and $y_{R_{1m}}$ decreases a little with increasing β_t , but its effect is little except in the case of a very small β_t . Accordingly, since the radial flow capacity at R_1 and the fluid velocity after the impeller in the jet flow domain augment with

increasing width of blades measured in the axial direction, the theoretical head and brake horsepower increase rapidly at low flow rates. All the figures show that a decreasing flow brings an increasing $y_{R_{1m}}$, but its increasing ratio decreases rather near the shut off point. This fact is dependent on the decreasing of the effects of centrifugal force, because r_1 becomes small there as explained already in the part 1.

The complexity of the flow on R_1 made it necessary to introduce a correcting factor ξ (see Eq. (16) of part 2). In order to find the value of this coefficient, $y_{R_{1m}}$ obtained from the experiments and the theoretical equation given by putting $\xi=1$ must be compared. For this purpose, it becomes necessary to know the flows before and behind the impeller. Since the flow on the delivery side of impeller blades is too complex even at the b.e.p. to analyse, when β_t is changed, which case is excepted and the calculation is performed only on the cases where the number of blades is varied. As the theoretical head of thin blades at the design

Thin blades
 $\beta_t = 14.7^\circ$
 $\circ \dots z=6$
 $\times \dots z=4$
 $\bullet \dots z=3$
 $\triangle \dots z=2$
 $\blacktriangle \dots z=3 (\epsilon^*=0.047)$
 $\otimes \dots z=4$ (with the parallel annular part on suction side of impeller)

(a)

Gött. blades
 $z=4$
 $\circ \dots \beta_t = 27.5^\circ$
 $\times \dots = 20^\circ$
 $\bullet \dots = 14.7^\circ$
 $\triangle \dots = 8^\circ$
 $\beta_t = 14.7^\circ$
 $\nabla \dots z=6$
 $+\dots z=3$
 $\blacktriangledown \dots = 2$

(b)

* $\epsilon = e/B$, e : width of blade-tips clearance
 B : width of pump annulus

Fig. 2 Values of $R_1(\epsilon=0.017)$

point could not be determined theoretically for the sake of speciality of its design procedure, it was decided by using the velocity distributions measured. In this case, the value of $1+p_1$, which was based on the direction of zero lift angle, was treated as being constant over all radii (mean value at $R=0.8\sim0.9$ was applied), for convenience sake. For the impeller with 4 Gött. blades, which corresponds to the design state, theoretical head and the value of $1+p_1$ are found by the design calculation at each radius. But in other cases, theoretical heads given by the experimental results are used.

The full line in Fig. 3 shows the mean radial velocity at various flow rates, for various types of impellers, which are derived from Eqs. (6)~(16) in part 2 by putting $\xi=1$. The graphs reveal that the calculated results have the same tendency as those obtained from the experiments. For the impellers with $z=3, 4$ Gött. blades and $z=3$ thin blades, the calculated values well coincided

Thin blades Gött. blades
[all $\varepsilon=0.017$ except (e)]

Fig. 3 Mean radial velocity at R_1 (effects of the number of blades and blade-tips clearance)

with the experimental ones. In general, for an impeller with a number of blades or a low pitch-chord ratio, the calculated values become larger than those obtained from the experiments, but for the other cases, the figure shows a reverse tendency. On the other hand, the impeller blade-tips clearance little affects the radial flow measured at R_1 , but its theoretical value for the wider clearance decreases somewhat since $1+p_1$ increases. But its difference can be neglected for the clearance used usually.

Since a difference between the theoretical and experimental values is nearly constant for various capacities, as shown in Fig. 3, it becomes possible to coincide the former with the latter by adding a constant value without distinction of the capacity. Its difference ($\Delta y_{R1,m}$) is given by deducing the experimental value from the theoretical one, and is shown in Fig. 5. The figure expresses that it is governed not by an aerofoil but only by the

Fig. 4 Mean radial velocity at R_1 (effect of the setting angle of blades)

pitch-chord ratio at the mean effective radius. Accordingly, the following relation will be given as the correcting factor for the theoretical solution.

$$\xi = 1 - \frac{\Delta y_{Rt,1,m}}{(y_{Rt,1,\xi=1})_m} \quad (2)$$

where, $(y_{Rt,1,\xi=1})_m$ is a mean radial velocity at R_1 determined theoretically by putting $\xi=1$. Values in Fig. 5 is available for cases where the pitch-chord ratio is about 0.5.

For the next problem, ξ in the axial flow domain must be decided. In this domain, a difference of capacities flowing on the delivery and suction sides of impeller blades in the jet flow domain gives the radial flow capacity at R_1 , but since R_1 is nearly equal to one, its difference becomes small and the effects of the impeller blade-tips clearance on the flow increase seriously. It, therefore, becomes difficult to determine experimentally the radial flow at R_1 . Since there is no abrupt change in the flow at the outlet at q_e , as described in the previous paper, the radial flow at R_1 must be suddenly reduced there. This phenomenon depends on the increasing of the stagnation pressure in the impeller, because the width of the jet flow domain becomes narrow. Further, as there exists a fluid having entered from the suction side in the jet flow domain, the radial flow entering from the fundamental flow domain must be changed into an axial flow at once in this domain. It, accordingly, is necessary to take into consideration this effect for deciding the value of ξ .

Following approximate methods were applied to know the magnitude of the stagnation pressure. That is, a two-dimensional flow along the orthogonal walls is treated as a fundamental model (see Fig. 6). Then, a pressure rise Δp produced by the stagnation pressure is shown as

$$\Delta p/\gamma = \epsilon_1 \cdot c^2/2g \quad (3)$$

where, c is the velocity unaffected by the stagnation pressure. The coefficient ϵ_1 can be derived easily for a two-dimensional potential flow and has been obtained analytically by Matsumoto⁽⁷⁾ as a function

Fig. 5 Correcting factor for the mean radial velocity at R_1

of the width of flow b on a section far removed from the vertical wall and the distance l from the corner. If the velocity c is reduced to c' by the stagnation pressure, then

$$\Delta p/\gamma = c^2/2g - c'^2/2g = \epsilon_1 \cdot c^2/2g$$

$$c'/c = \sqrt{1 - \epsilon_1} \quad (4)$$

By using ϵ_1 obtained by Matsumoto, the values of c'/c are plotted as a function of l/b in Fig. 6, which shows that these relations are given by a nearly linear line.

The flow in the impeller is far different from such two-dimensional potential flow but it is limited to near the casing wall and is regarded as an axially symmetrical flow. Therefore, a decreasing ratio of the radial flow at R_1 by the stagnation pressure may be assumed to be proportional to the distance from the casing wall. In general, the radial velocity in the axial flow domain is so small that its effect on the pump performance is comparatively small and then such an approximation is allowed.

As described previously, there being no abrupt change in the flow at the outlet from impeller blades at q_e , the mean radial velocity at R_1 in the axial flow domain is shown at q_e as follows

$$(y_{Rt,1,m})_a = (y_{Rt,1,m})_c - (\varphi_{sj,m} + \varphi_s) \frac{1 - R_1^2}{2\lambda \cdot R_1 L} \quad (5)$$

where, $(y_{Rt,1,m})_c$, $\varphi_{sj,m}$ represent the mean radial velocity at R_1 and mean axial velocity (positive) on the suction side in the jet flow domain at q_e , respectively, in the centrifugal flow domain; φ_s denotes the axial velocity component in front of the impeller at q_e in the axial flow domain. They are all represented as non-dimensional ones. Further, $L = h_1/r_2$ (h_1 ; the width of the mean camber line measured in the axial direction at r_1), λ shows a reducing ratio of the effective area on the cylindrical surface of radius r_1 in the impeller by the existence of blade-sections.

Fig. 6 Deceleration state by the stagnation pressure of flow along orthogonal walls

pump characteristics decreases considerably too, then such an approximation is permitted. In the following chapter, it will be corroborated by numerical calculation.

3. A consideration on the flow in the jet flow domain

The most important basis to the fundamental conception about impeller performance was to explain the experimental fact that the angle between the relative discharge velocity and the peripheral direction becomes larger than the discharge angle of impeller, as described in part 2. For this purpose, it was discovered that in the jet flow domain, the effect of the casing wall on the flow is more important than the effect of the existence of blades; the fluid, accordingly, does not flow along blades. But is it possible to neglect the effect of blades as a boundary condition for the impeller blades with a small pitch-chord ratio? On this problem, some discussions were made about the impeller with six thin blades, whose pitch-chord ratio t/l was the minimum of all the impellers experimented ($t/l=0.86$ at mean effective radius). Calculations were made for $\varphi_{dm}=0.053$ (it corresponds to the flow in the centrifugal flow domain) by using the full line (experimented value) in Fig. 9 (a) as φ_a -distribution. By substituting

Fig. 9 A consideration about the flow on the delivery side of impeller blades in the jet flow domain

it into Eqs. (25)~(27) of part 2, the distribution of $\chi_d (=c_{ud}/u_t, c_{ud}$; peripheral velocity component) is derived. The value measured is used for χ_d at R_1 . The result calculated is shown in Fig. 9 (b) by a full line. It is known obviously that the value calculated is smaller than the one experimented.

As the reason for this, it is pointed out that χ_d over the inner radii, accordingly χ_d at R_1 become small in consequence of the reverse flow on the delivery side. But this result shows that the angle between the relative and peripheral velocity directions in the jet flow domain becomes larger than that derived theoretically by neglecting the existence of blades as a boundary condition. It, further, shows that it is allowable to calculate the flow in the jet flow domain, in disregard of the existence of blades, even for the impeller blades with such a small pitch-chord ratio.

4. Numerical example

To corroborate the accuracy of the analytical method for deciding the theoretical characteristics of an axial flow pump, obtained from the present study, the velocity distributions before and behind the impeller and the impeller characteristics were calculated for an impeller with 4 Gött. blades, whose setting angle is 14.7° at impeller tips (see part 3). At the design point, non-dimensional mean axial velocity on the discharge side of the impeller $\varphi_{dm0}=0.209$ and theoretical head $\psi_{th0} [=H_{th0}/(u_t^2/g)]=0.153$. The procedure is begun by determining the coefficient of slip angle according to the method described above. Then the calculation

Fig. 10 Comparison of calculated results with experimental ones I ("discharge-theoretical head" characteristics)

of each equation in part 2 was carried out. The critical capacities, R_1 and κ were decided by using the values shown in the present paper. On the radial flow at R_1 , $\xi=1$ is assumed, although its theoretical mean radial velocity is a little larger than that measured (see Fig. 3).

The relation of the theoretical head-capacity characteristics derived from calculations is shown by a full line in Fig. 10. The chain line on the figure corresponds to the case, in which the effect of the casing wall on the radial flow is neglected, that is, no jet flow domain. Referring to the figure, this $\varphi_{dm}-\psi_{thm}$ characteristics curve is nearly equal to the linear line through the design point, and

there is a serious difference in the centrifugal flow domain.

Theoretical head determined experimentally is given by the following relation, provided that the effects of the casing and hub walls near the impeller can be neglected.

$$\psi_{thm} = \frac{\int_{R_{md}}^1 2\pi R \chi_d \cdot \varphi_d \cdot R_t dR - \int_{R_b}^{R_{ms}} 2\pi R \chi_s \cdot \varphi_s \cdot R_t dR}{\pi(1-R_b^2) \cdot \varphi_{dm}} \quad \dots\dots\dots (7)$$

where, $R_t = r/r_t$ (r_t : radius of blade-tips), $R_b = r_b/r_2$ (r_b : radius of hub). R_{md} and R_{ms} denote the most outside and inside radii of the circulating flows on the delivery and suction sides of impeller blades,

(a) $\varphi_{dm}=0.209$

(b) $\varphi_{dm}=0.100$

(c) $\varphi_{dm}=0.058$

(d) $\varphi_{dm}=0.014$

Fig. 11 Comparison of calculated results with experimental ones II (velocity distribution on the delivery side of impeller blades)

respectively; if there is no circulating flow, they correspond to the radii of hub and casing walls. Subscripts *s* and *d* refer to the inlet and outlet conditions of impeller blades. The relation of discharge-theoretical head derived from Eq. (7) by using the measured velocity distribution, is also shown by a broken line in Fig. 10. Referring to the figure, it is known that the values calculated are found in fairly good agreement with those obtained from the experiment and that the analytical method shown in the present paper is reasonable. A little difference in the axial flow domain is owing to the existence of the blade-tips clearance.

For some delivery capacities, the velocity distributions calculated on the delivery side are compared with those experimented in Fig. 11. In the axial flow domain, there is a little difference at large radii because of the existence of blade-tips clearance, but its tendency is confirmed. As explained in part 2, for reduced φ_{dm} the flow calculated in the reverse flow domain is different from the one measured, because it is affected seriously by the guide vanes. Especially, its effect on χ_d is remarkable. But these differences in the circulating flow domain can be neglected for deciding the theoretical head, since it is only dependent on the delivered flow near the casing wall. From the figure it is found that these two flows are in good agreement in the jet flow domain. On the other hand, as known clearly from the comparison of (c) and (d) in Fig. 11, φ_d in the jet flow domain rather decreases near the shut-off point.

The head distributions on the delivery side are compared in Fig. 12. In the figure, a full line shows the theoretical value and symbol $\bigcirc$ denotes the value obtained from the following equation by using the experimental result

$$\psi_{th} = \chi_d \cdot R_t \dots\dots\dots (8)$$

As there is hardly any pre-rotation, Eq. (8) shows the distribution of theoretical head for the delivery flow. But for the circulating flow it does not indicate the head, then it becomes too small there. Symbol $\bullet$ expresses the non-dimensional total head $\psi_{t2} [=E_2/(u_t^2/g)]$ on the delivery side of impeller blades. Since the total head on the suction side is zero, it corresponds to the actual head given by the impeller; ψ_{th} on the outside being larger than the value calculated near the design point is owing to the existence of blade-tips clearance. In the centrifugal flow domain, the calculated values coincide with ψ_{th} near the casing wall.

A comparison of the velocity distributions on the suction side is given in Fig. 13. In the centrifugal flow domain, φ_s -distributions are not confirmed in the positive flow domain because $\varphi_s = \text{const.}$ is assumed in the theoretical analysis, but in the reverse flow domain they are confirmed. But for the peripheral velocity component χ_s , the theoretical value is far different from the experimental one, since the pre-rotation is neglected and $\chi_s = 0$ is assumed at R_1 , and the head loss is not considered at all. In fact, as a large energy is necessary to send backward a large quantity of fluids on the suction side, there must be a large difference of

Fig. 12 Comparison of calculated results with experimental ones III (head-distribution on the delivery side)

(a) $\varphi_{dm}=0.209$ (b) $\varphi_{dm}=0.058$ Fig. 13 Comparison of calculated results with experimental ones N (velocity distribution on the suction side of impeller blades) z' : distance measured in axial direction from the inlet of impellerFig. 14 Radial velocity distributions at R_1 in impeller blades

moment of momentum between the fluid particles in the positive and the negative flow domains. But the increasing of energy loss reduces the energy of reverse flow, accordingly, the reverse flow measured has a little energy in spite of the large χ_s (see Fig. 5 of part 1). In the theoretical calculation, this energy loss being neglected, the fluid need not receive a large energy to flow backward and becomes very small. This disagreement has no relation for determining the delivery head but is very important to know the brake-horse power characteristics at low flow rates.

The radial velocity distributions at R_1 are calculated and shown in Fig. 14 for various flow rates. Since $\varphi_{dm}=0.136$ corresponds to the axial flow domain, its radial velocity becomes very small.

5. Conclusions

Some coefficients which have been introduced in the analysis of the axial-flow pump characteristics in part 2 are determined and represented in a comparatively simple relation by using the experimental results given in part 3. It, further, was corroborated by numerical calculations on an actual example that theoretical results already obtained are found in good agreement with the experimental ones.

The method, which decides the theoretical head of an axial-flow pump at partial capacities, can be derived by the studies up to the present paper. On the other hand, it will be also possible to represent the pump characteristics in the axial flow domain by applying the aerofoil theory. Moreover, to find the relations between the capacity and actual head or brake horse-power, it is necessary to discuss the hydraulic losses in the pump casing. For these problems, some investigations will be carried out in the following paper.

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