The Prediction of Fatigue Fracture under Combined Stresses at Stress Concentrations

By George Sines**

On unnotched specimens, fatigue failure criterion was proposed under a combination of alternating and static stresses. The fatigue test data were then examined to obtain a general fatigue failure criterion, especially on notched specimen with stress concentrations. It is pointed out that after the stresses have been found for the critical regions, a criterion which uses the applied static stresses and alternating stress must be used to predict whether they are safe.

Introduction
Fatigue test data will be examined to obtain a general fatigue failure criterion and then this criterion and other considerations will be used to analyze failure at stress concentrations.

The local nature of the phenomenon of fatigue failure makes it necessary to carefully analyze the stress distributions in a machine part or structure because a fatigue crack can start in one small region of high alternating stress and propagate to...
cause complete failure regardless of how strong the other portions may be. After the stresses have been found for the critical regions, a criterion which uses the applied static stresses and alternating stress must be used to predict whether they are safe. The parameters of the criterion must be able to be evaluated from simple tests on laboratory specimens.

**Stress description**

In a stressed body it is always possible to find three orthogonal directions which are the normals to three planes on which no shear stresses act. The normal stresses on these three planes and the three orthogonal directions are a convenient way to describe the stress at a point. These are called the principal stresses and will be designated by $P_1$, $P_2$, $P_3$ and when ordered $P_1 \geq P_2 \geq P_3$.

If the principal stresses fluctuate in a regular sinusoidal manner with time, they can be described by an alternating stress amplitude superimposed upon a static stress as in Fig. 1. In the following presentation the principal stresses will be either exactly in or out of phase and all the principal stresses will vary with the same frequency. For other phase differences refer to the work of Nishihara and Kawamoto(1).

We most often will be concerned with stresses at the surface of a part, because cracks most often start there. Cracks usually start at the surface because:

1) Many parts have bending or twisting moments applied to them, thus causing the highest stresses to be at the surface.

2) Surface stresses may be further increased by the presence of stress concentrations such as scratches, tool marks, and notches.

3) There is good reason to believe that the grains at the surface are inherently weaker in fatigue because on one side their deformation is not confined by neighboring grains(2).

**Combined alternating stresses**

First, fatigue tests under alternating stress, without any static stress, will be examined. A convenient way to present the results is to plot the amplitudes of the alternating principal stresses which caused fracture after a certain number of repetitions. In Fig.2, the biaxial test data of Sawert(5) is plotted. In the lower quadrant, $\sigma_1$ is the greatest principal stress, the intermediate one is zero, and the least is the negative $\sigma_1$. In the upper quadrant, the least principal stress is zero. Combined bending and torsion data appear in the lower quadrant. For pure torsion (shear) the principal stresses are equal in magnitude and opposite in sign and this stress state is represented by points along the line 0-2. As more bending is applied in the 2-direction, the stress state is represented by points closer to the $\sigma_2$ axis.

It must be noticed that the principal stresses alternate each cycle, going from a given tensile stress to an equal compressive stress. In the lower quadrant, the principal stresses are out of phase; that is, at the time one stress reaches its greatest tensile value, the other principal stress reaches its greatest compressive value. In the upper quadrant the stresses are in phase— they reach their greatest tensile value at the same time and in the opposite half cycle they also reach their greatest compressive value together.

The test data are plotted in Fig. 2 for a life of
10^6 cycles for different combinations of alternating stresses for the particular metal. The curve connecting these tests data separates the \( \sigma_1 - \sigma_2 \) plane into two regions. Every combination of stress represented by a point within the region can be applied safely 10^6 times without failure; all points outside this region will cause failure at a number of cycles. Sawert's data and Gough's data were taken very early and are typical of later results. Except for the curve 1-3 for a cast iron the curves are close to the ellipse. Most fatigue data for metals fall close to the ellipse except that for cast iron or annealed hypereutectoid steel whose behavior is discussed in Appendix I.

The criterion proposed by Sawert was that when

\[
1/3 \left( (p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_3 - p_1)^2 \right)^{1/2} \geq \tau_{\text{ref}} \text{ crit}
\]

failure occurred. Where the \( p \)'s are the amplitudes of the alternating principal stresses and \( \tau_{\text{ref}} \text{ crit} \) is an experimental parameter for a particular material for a given cyclic life. (A similar criterion is used to predict yielding). The expression above has been shown to average the effect of shear stresses on many differently oriented slip planes.

### Simple combinations of alternating and static stresses

Next the effect of static stress on the permissible amplitude of alternating stress will be found and inserted into the fatigue criterion. Experiments on the following simple combinations will be examined:

- Static tension on alternating axial stress, Fig. 3
- Static compression on alternating axial stress, Fig. 3
- Static torsion on alternating torsion, Fig. 4
- Static torsion on alternating bending, Fig. 5
- Static tension on alternating torsion, Fig. 6
- Static bending on alternating torsion, Fig. 6
- Static compression on alternating torsion, Fig. 6

The data for Fig. 3 on the effect of static compression on alternating axial stress were critically selected; if all investigations were included, the general trend would be horizontal. It is difficult to avoid uncontrolled bending when applying compression and the presence of bending gives false results. The tests presented here were chosen because the testing methods used ensured true axial loading.

The author knows of no tests on the effect of static compression on alternating torsion except those presented in Appendix II. The author urges other investigators to conduct similar tests to see if these results are reproducible.

The results of the combinations of static stress and alternating stress tests are summarized by Fig.

### Table: Effect of Mean Stress on Yield Strength

<table>
<thead>
<tr>
<th>Material</th>
<th>Mean stress ( \sigma_m ) (psi)</th>
<th>Yield strength (psi)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024-T(245-T) Alum.</td>
<td>( +26000 )</td>
<td>48 000</td>
<td>Newmark and Coworkers</td>
</tr>
<tr>
<td>0.41 C steel</td>
<td>( +35000 )</td>
<td>55 000</td>
<td>Nishihara and Sakurai</td>
</tr>
<tr>
<td>0.65 C steel</td>
<td>( +38000 )</td>
<td>57 000</td>
<td></td>
</tr>
<tr>
<td>0.44 C steel</td>
<td>( +36000 )</td>
<td>57 000</td>
<td></td>
</tr>
<tr>
<td>Duralumin</td>
<td>( +17000 )</td>
<td>32 000</td>
<td>Nishihara and Koijima</td>
</tr>
<tr>
<td>Mild steel</td>
<td>( +26000 )</td>
<td>38 000</td>
<td>Roß and Elchinger</td>
</tr>
<tr>
<td>Mild steel</td>
<td>( +37000 )</td>
<td>47 000</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 3** Effect of mean axial stress on axial fatigue

7. Only the linear portions of the curves are presented where the combined stress did not exceed the yield strength. When excessive yielding occurs, the curves drop rapidly from the linear relationship shown.

Examination of the center column of Fig. 7, entitled the static stress on planes of maximum alternation of shear stress, shows that when the sum \( N_1 + N_2 \) is positive, as it is for cases 1 and 5, an increase in the static stress reduces the permissible alternation of stress. When the sum is negative, however, as in case 2 and for the data in the Appendix II, the permissible alternation is increased. And when the sum is zero regardless of the applied static stress, as in cases 3 and 4, the static stress had no effect. Thus there appears to be a simple correlation between the alternation of stress and the sum of the static normal stresses acting on the planes of maximum alternation of shear.

Since the sum of the orthogonal normal stresses is independent of their directions, it really was not necessary to have examined the planes of greatest alternation of shear, but the correlation could have been seen for any two orthogonal planes.

### Criterion for fatigue failure with superimposed static stresses

From the fatigue data which has been presented, a criterion can be proposed which includes the effect of different combinations of alternating stress with
Fig. 4 Effect of static shear stress on shear fatigue

Fig. 5 Effect of static shear on permissible amplitude of alternating bending stress

(a) Oono

(b) Lea and Budgeon

Table: Torsional yield strength

<table>
<thead>
<tr>
<th>Material</th>
<th>Fatigue strength (psi)</th>
<th>Est. shear yield (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 0.078 C steel</td>
<td>34 000</td>
<td>34 000</td>
</tr>
<tr>
<td>+ 0.251 C steel</td>
<td>34 000</td>
<td>54 000</td>
</tr>
<tr>
<td>+ 0.304 C steel</td>
<td>32 000</td>
<td>40 000</td>
</tr>
</tbody>
</table>

Material | Fatigue strength (psi) | Est. shear yield (psi) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 0.32 C steel</td>
<td>37 500</td>
<td>36 000</td>
</tr>
<tr>
<td>× 0.35 Cr- Ni steel</td>
<td>44 500</td>
<td>38 000</td>
</tr>
<tr>
<td>× 0.14 C steel</td>
<td>36 500</td>
<td>22 000</td>
</tr>
</tbody>
</table>
static stress. It is the simple statement that the permissible alternation of the octahedral-shear stress is a linear function of the sum of the orthogonal normal static stresses. It is mathematically expressed

$$1 \frac{1}{3}(p_1 - p_2)^2 + (p_3 - p_2)^2 + (p_3 - p_1)^2)^{1/2}$$

then failure will occur, where $p_1$, $p_2$, and $p_3$ are the amplitudes of the alternating principal stresses and $S_x$, $S_y$, and $S_z$ are the orthogonal static stresses. The $A$ is a constant for the material, proportional to the reversed fatigue strength, and $\alpha$ gives the variation of the permissible range of stress with static stress. Both $A$ and $\alpha$ are given for the desired cyclic lifetime.

### Determination of the constants for the criterion

The values for $A$ and $\alpha$ which describe the fatigue properties for a material can be determined from any two fatigue curves whose static stresses are appreciably different. Two curves which are often convenient for their determination are the reversed axial test and the zero-tension fluctuating stress test. To illustrate the process, use a reversed axial test for which $p_2$, $p_3$, $S_x$, $S_y$, and $S_z$ all equal zero. The criterion reduces to $(\sqrt{2/3}p_1 = A$. For the zero-tension test, $p_1', p_2', S_x'$ and $S_z'$ are zero and $S_y' = p_1'$, and the criterion reduces to $(\sqrt{2/3}p_1' = A - \alpha p_1'$. Solving the two above equations $\alpha = A/p_1' - \sqrt{2/3} = (\sqrt{2/3}(f_1f_2') - \sqrt{2/3}$ where $f_1'$ is the amplitude of the fluctuating stress which causes failure at the same lifetime as the reversed stress $f_1$.

### The graphical presentation of the criterion

For a free surface, the criterion is graphically presented in Fig. 8. At a free surface one principal stress is zero, thus permitting it to be plotted as a two-dimensional function of the other two principal stresses. It appears as a series of "concentric" ellipses, the size of the ellipse depending on the sum of the static normal stresses. The more positive (tensile) the sum of the static stress, the smaller the ellipse; conversely the more negative the sum, the larger the ellipse.

If the static stresses are fixed, the permissible combinations of alternating stress amplitudes can be chosen. The ellipse corresponding to the sum of
the static stress is selected, and any combination of
alternating stresses within the area it encloses
is safe; conversely, all combinations on the outside
will cause premature failure.

If the criterion is to be applied to the interior
of a body where all of the three principal stresses
are different from zero, the graphical representation
becomes a three-dimensional plot with the third
principal stress orthogonal to the other two. The
failure surface in this space is a cylindrical surface
with generators having the directional cosines
\((1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})\) intersecting the planes
of the axes in the ellipses shown in the two-dimensional
representation\(^{12}\).

The validity of the criterion when extrapolated
to three dimensions has never been proved. It is
probably made conservative by the inherent surface
weakness when applied to an interior region, but
if it is applied to a surface region which has
tractions acting on it, the fretting from any
mechanical contact or the action of a fluid would
probably make its predictions not conservative.

Factors affecting fatigue strength

Residual stress

Residual stresses may be induced in a part
during shaping, forming, heat treatment, or surface
work\(^{15}\). It makes no difference whether the
static stresses are residual ones caused by some
internal misfit between adjacent regions or whether
they are from applied external loads. The residual
stresses \(R_x', R_y', R_z'\), should be treated just as other
static stresses by adding them to the other static
stresses in the criterion in the following manner—

\[
1/3 \left( (p_1-p_0)^2 + (p_2-p_0)^2 + (p_3-p_0)^2 \right)^{1/2} 
\geq A - \alpha (S_x + S_y + S_z + R_x' + R_y' + R_z')
\]

The orthogonal axes, \(x', y', z'\) for the residual
stresses need not be in the same direction as those
for the static normal stresses. It can be seen that
compressive residual stresses, which are negative,
will permit greater alternation of stress for the same
cyclic life; conversely, tensile residual stresses
reduce the permissible alternation of stress.

Stress concentrations

No matter how safe a structure may be when
only a static load is applied it may be very unsafe
under repeated loads. Machine parts made of
commonly used metals have their fatigue strength
reduced much more by notches than their static
strength is reduced. A fatigue crack can start
wherever the amplitude of alternating stress is
excessively small and propagate to cause failure.
The critical regions must be checked carefully.
Critical regions are those which undergo
high alternation of stress or have a high static
tensile stress superimposed upon an alternating stress.
After those regions have been identified, the local
stresses must be carefully determined and then
inserted into the criterion to predict whether or not
failure will occur.

Notches and holes intensify the stress, i.e.,
the greatest principal stress \(P_1\) at the notch will be
higher than the average greatest principal stress in
the region. The stress state can be described by
\(k_2\) and \(k_3\) where the \(k\)'s are defined by \(P_\text{II}=k_2P_1\)
and \(P_\text{III}=k_3P_1\). The stress state will usually be
changed by the stress concentration. The exact
state of stress as well as the intensity of stress must
be known at the notch.

Notch sensitivity

The theoretical stress concentration factor for
a notch is always greater than the strength reduction
which it causes under alternating stress.
Several factors make this so:

One factor is the "size effect" which describes
the phenomenon that the fatigue strength of a given
volume of material under high stress may be lower
than that of a smaller volume of the same material
under high stress. The reason for this may be that
the probability of stressing a worse microscopic
weakness in the metal is greater when a larger
volume is under high stress. The volume of metal
highly stressed at a notch is usually much less than
that in a standard fatigue specimen so that the
material at the notch appears stronger and the
reduction in fatigue strength is less than that
predicted by the stress concentration factor. Of
course, this would not be true for very large parts
with very big notches where the highly stressed
volume might be greater than a standard fatigue
specimen.

Fig. 8 The plotted stress criterion
Another factor is that the individual grains, or some other similar unit, tend to average the stress which appears across them and thereby lower the peak theoretical stress. A reversible movement of dislocations around the grain boundaries could average the alternating stress. Neuber(18) looks at it in another way: The grains are considered to have an inherent stress concentration associated with them. The inherent stress concentration of the grains superimposed upon a geometrical one of about the same size does not add to it, but partially cancels it.

An example of the effect of these factors(17) can be seen in Fig. 9 where a test specimen with a theoretical stress concentration factor of $K_t=2.5$ only reduced the fatigue strength by a factor of $K_f=1.67$. If the total effect of the stress concentration factor had been the curve for the notched sample would have been $1/K_f=0.4$ that of the unnotched fatigue strength $f$. However, the strength reduction factor was only $K_f=1.65$ so the notched fatigue strength was $f/K_f=0.6$; thereby showing that the material appeared to be

$$\frac{f/K_f-f/K_t}{f/K_t} \times 100 = 50\%$$

stronger than the material of the unnotched specimen.

The notch sensitivity index proposed by Peterson(19), $q = (K_f-1)/(K_t-1)$ varies from $q=0$, when the notch has no effect, to $q=1$, when the strength reduction factor is equal to the theoretical stress concentration factor. Graphs for $q$ versus the sharpness of the notch are given in the literature(19). For a given material, the index $q$ is smaller for notches with small radii at their roots. These graphs can be converted to an apparent increase in strength of the notch material by solving the expression for $q$ to get $K_f$ and substituting it in the above equation.

$$\frac{-q(K_f-1)+K_t-1}{q(K_t-1)+1} \times 100 = P\%$$

increase in fatigue strength.

**Other effects**

Some processes, such as shot peening, surface rolling, and case hardening may improve the fatigue strength of the metal as well as induce residual stress. Other processes may weaken the surface metal, for example, decarburization of the surface of a steel part may occur during heat treatment and considerably reduce the fatigue strength. Many factors such as corrosion during cyclic stressing, high temperature, and a rough surface finish can greatly reduce the fatigue strength.

Of course, fatigue fracture is not the only mode of failure-brittle fracture from impact loads, excessive plastic deformation, wear, and corrosion are equally dangerous.

**An example of necessity of consideration of local stresses**

The following illustration shows the necessity of careful consideration of local stresses. In Fig. 4 it was seen that the permissible alternation of torque on a circular shaft was not influenced by the steady torque unless the yield strength of the material was exceeded. Now let us consider the case where design requires a small transverse hole in the shaft. The superficial reaction would be that the permissible alternation of torque would still be independent of the static torque and that it would only be necessary to reduce the alternating torque to compensate for the strength reduction of the hole. Because of the local nature of the fatigue failure, that reaction would be very much in error.

Neuber(14) has calculated the stress distribution around the hole when it is small compared to the shaft. In Fig. 10, the points of high alternation of stress are at A, B, and C. At points A, which are a short distance from the hole, the alternating stress is pure shear, while at points B and C it is an alternating uniaxial stress. We will examine these regions for several conditions of loading:

(a) For an alternating torque with no static torque, detailed stress analysis and use of the
criterion will show that points B and C are equally critical while A is slightly less so.

(b) For a static torque superimposed upon the alternating one and for the case where the peak sum does not cause yielding, the critical points are easily identified. For a static torque as applied in Fig. 10, the unfavorable static tensile stresses will be at points B, thus making them the most critical.

(c) If a large static torque sufficient to cause yielding at points A, B and C is applied initially, then the metal at C will be plastically shortened so that when the torque is removed it will be under residual tensile stress. At points B the metal will be plastically extended thus causing residual compressive stresses, and at A plastic distortion will result in residual shear stresses. Now if only an alternating torque is applied, points C will be most critical because of the presence of the unfavorable residual tensile stress. It is difficult to calculate the magnitude of the residual stress but it can be measured by various techniques(22). If this is not feasible, then a rough conservative estimate can be made by assuming that it is equal to the yield strength of the metal.

(d) When residual stresses are induced by a static torque as in paragraph (c), and then a static torque of the same sign but of less magnitude is applied along with an alternating stress, it is not readily apparent whether points B or C are most critical. Only very detailed study can determine at which point the sum of the residual stress and applied stress is most tensile.

(e) When residual compressive stresses are induced at all points by some process like shot peening and then the bar is subjected to an alternating torque, the residual stresses may sometimes be reduced by the loading cycle. If the compressive residual stress added to the peak compressive stress from the load exceeds the compressive yield strength, then this will reduce the magnitude for the beneficial residual compressive stress at both B and C and possibly at A. A rough estimate of the resulting residual compressive stress can be made by setting the sum of it and the concentrated applied stress equal to the yield strength and solving for \( \sigma \).

However, there is some indication that under repeated stress the yield strength may be lower than the statically determined value.

Many more combinations of residual stress and applied stress can occur for this simple specimen and they cannot all be discussed here. It should be remembered that the peak value of the applied stress may change the residual stress and that the sum of the actual residual stress and static stress at the critical points must be inserted into the criterion. Consideration of the possible apparent increase in fatigue strength discussed under "Notch Sensitivity" may increase the permissible alternation of stress.

**Discussion**

It must be remembered that only the linear portions of the curves showing the effect of static stress on fatigue were used so the criterion is not applicable for cases where appreciable yielding occurs. Also, materials which contain internal cracks or flake-like inclusions are excluded but are discussed in Appendix I. Within these restrictions, the attempt was made to devise a criterion as general, i.e., would cover the widest possible combinations of alternating and static stresses, and would still be convenient to use. In order to have a convenient criterion, the number of parameters used to describe the fatigue properties of the metal was limited to two. Other criteria(22),(23) containing only two parameters but limited to fewer combinations of stress or criteria containing more parameters are potentially more accurate over the range of their applicability.

Any criterion should be checked against known results before it is used in order to see if it correctly predicts the well-established behavior showing the beneficial effect of residual compression on alternating axial stress shown in Fig. 3 and the lack of effect of static torsion on torsion fatigue shown in Fig. 4. The effect of static tension on torsion fatigue is less well-established, because data of Nishihara and Kawamoto(22) disagree with that of Gough and of Hohenemser and Prager presented in Fig. 6. This difference does not necessarily disagree with the criterion but may just indicate that \( \alpha \) is almost zero for some metals.

Many criteria for fatigue from combined stresses are reviewed by J. Marin in his book, "Engineering Materials"(24). In these criteria the full strength of the material is not recognized because they lead to the prediction that the static compressive stress is as deleterious as static tensile stress, while data show that static compressive stress can increase the permissible alternation of stress. The criteria also suggest that static torsion would reduce the permissible range of alternating torsion, while data show that it has no effect unless the yield strength is exceeded.

The design procedure presented here are not in general use, but the author hopes they will serve as a useful guide where the utmost strength of the material must be utilized. Any design which differs greatly from standard practice should be carefully fatigue-tested before it is adopted for service.
Analyses such as these try to avoid "blind safety factors," which often are multiplicative and are so often extremely wasteful. The aim is to make the parts as uniformly strong as possible. If this is done, whatever "safety factor" is applied is a true measure of safety and may be selected according to the danger to personnel, property, or of delays incurred if failure should occur in service. The safety factor should not be used to compensate for ignorance of stress distribution, material properties, or the behavior of materials under combinations of stresses, although some allowance may be needed for the lack of information concerning the actual service loads.

Appendix I. The anomalous behavior of cast iron and hypereutectoid steel

The data for cast iron presented in Fig. 2 and other data on cast iron and annealed hypereutectoid steel fall nearer to a vertical line than to the ellipse of the criterion. Both of these materials show flakes of graphite dispersed throughout the metal. Graphite has a modulus of elasticity much lower than that of the surrounding iron crystals. Therefore, the flakes of graphite carry very little stress in tension. They act like holes in the iron structure. The effect of these flakes can be seen by considering a simplified model of three holes oriented at different angles as shown in Fig. 11.

When \( \sigma_1 \) is applied, the regions \( A_1 \) and \( A_2 \) at the edge of the one flake will be much more highly stressed than any other region because of the elastic-stress concentration caused by the hole. Regions \( B_1 \) and \( B_2 \) will also be more highly stressed than the average, but still considerably less than \( A_1 \) and \( A_2 \). The regions \( C_1 \) and \( C_2 \) around the edge of the flake longitudinal to the applied stress will be stressed even less than the average for the bulk of the material. The regions most highly stressed by the transverse stress \( \sigma_2 \) are \( C_1 \) and \( C_2 \), while regions \( A_1 \) and \( A_2 \) are stressed very little. The detailed analysis of cracks oriented at different angles under several combinations of stress shows that the sum of the effects caused by the transverse and longitudinal stresses together at cracks oriented at angles between that of \( A_1-A_2 \) and \( C_1-C_2 \) is less than that caused by them individually at \( A_1 \) and \( A_2 \) or \( C_1 \) and \( C_2 \) except for the range of combinations of applied stresses where the transverse stress approaches a value equal to the longitudinal one. In that case the combined effect is only slightly greater at one orientation than the isolated individual effects are at other orientations. Thus the greatest damage from the two stresses is localized at different points, which causes their effect to be independent. Complete independence of the effect of \( \sigma_2 \) and the effect of \( \sigma_1 \) would cause the data to fall on a vertical line in the lower quadrant of Fig. 2, but for cast iron there must be some slight interaction because it deviates slightly from the vertical.

Appendix II. A fatigue test under alternating torsion with static compression

Material

The aluminum alloy 6061-T6 was chosen for this test because its ratio of yield strength to fatigue strength is high; therefore, the yielding from the combined stress, which would complicate the analysis, was avoided. The specimens were cut from 1 by 4 inch rolled plate.

Specimen

The specimen is shown in Fig. 12. The test sections are the two cylindrical surfaces separated by a short square section in the center. The specimen was grasped at the ends and the alternating torque applied to the center. The device which was used to apply the static compressive stress required a longitudinal hole through the specimen. The final lathe cut on the test surfaces was less than 0.005 inch and made with a sharp tool. The last polishing was in the longitudinal direction with Minnesota Mining and Manufacturing silicon carbide No. 600 grit Wetordry paper.
Apparatus

Static compressing device The assembled device is shown in Fig.13. The stress rod passes through the Belleville spring assembly and the hollow specimen which are compressed by tightening the hexagonal nut on the draw screw. The danger inherent in applying an axial compressive load to a specimen is that eccentricity of the load will cause large undesired bending stresses. The central rod arrangement and very precise machining avoid bending stresses. The device was calibrated by using strain gages on a dummy specimen.

Fatigue testing machine A photograph of the complete machine appears in Fig.15 and a schematic drawing in Fig.14. A variable throw crank attached to a 1750 r.p.m. motor applies an alternating deflection to the torque arm. The torque arm is flexible so that small cracks or a slight movement of the specimen in its grips does not change appreciably the torque applied. The desired alternating stress was obtained by adjusting the crank throw to reproduce deflections of the torque arm determined by hanging weights on the torque arm at the connecting rod pin with the connecting rod disconnected.

Test results

The fatigue curves for alternating torsion with and without applied compressive stress are presented in Fig.16. The specimens failed from a very fine longitudinal crack forming in the test section. A crack length of 1/4 inch was sufficient to stop the testing machine to terminate the test.

Discussion of results

The formation of the longitudinal crack was delayed by the longitudinal compressive stress, although it had no component on the plane of the crack. This lends support to the criterion which states that it is the sum of the orthogonal components of the static normal stresses which is effective.

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Residual Stresses Produced by Plastic Tension in Notched Plate Specimens and Fatigue Strength*

By Shuji Taira** and Yasunori Murakami***

Plate specimens of a medium carbon steel (0.48% C) with sharp V notches on both edges were stretched so that the nominal tensile stress at the minimum cross section would have the values of 0.8σ and σ, respectively. As the result of this plastic tension, residual compressive stresses were introduced around the notch root. These residual stresses were measured by the X-ray diffraction method, and their changes due to alternating stressing in reversed bending were investigated. In the fatigue tests, the specimens with residual stresses were found to have higher fatigue limits and also prolonged fatigue lives over those without residual stresses, and these improvements were larger in proportion to the values of residual stresses. The growth of fatigue cracks was also investigated, and it was clarified that residual compressive stresses have an effect of suppressing the propagation of fatigue cracks. Applying the theory of threshold stress to spread the fatigue cracks proposed by T. Isibasi, and based on the change in residual stresses and on the crack growth curves, the contribution of residual stresses to the fatigue strength was calculated.

1. Introduction

Many machine parts have more or less stress raisers such as holes, notches and fillets, and the fatigue strength of such parts is expressed usually with the fatigue notch factor. As to the fatigue fracture of notched pieces, propagation of fatigue cracks initiated at the base of the notch must be taken into consideration. Numbers of studies have been made on the propagation of fatigue cracks, and a number of analytical as well as experimental formulas are reported. As T. Isibasi suggested, the residual compressive stresses existing at the notch root might increase the maximum alternating stress sustained by the notched member. Concerning this subject, D. Rosenthal and G. Sines(1) reported that the fatigue strength of U notched aluminum plates was improved by the residual compressive stresses at the notch root, and on the contrary, lowered by the residual tensile stresses. Similar experiments were made on round bar specimens, and the same results were obtained(2). In these cases, residual stresses were introduced by plastic tension or compression.

The fatigue strength of notched members can be improved by other treatments, for example, S. Takeuchi and T. Honma obtained the increase in fatigue limit of 73% by shot peening(3), and H. Nakamura reported a rise in fatigue limit to 4.7 times by induction hardening(4). The remarkable improvement in fatigue limit in these cases might be attributed largely to the favorable effect of