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Fatigue Tests of Metals at Ultrasonic Frequency*

(2nd Report, For Materials Having High Endurance Limits)

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In the ultrasonic fatigue machine previously reported, it was necessary to adopt the test pieces contracted in the mid portion in order to induce the higher stress there, and a graphical method was required for the estimation of the value of the stress induced. In this paper, the analysis of vibration of the contracted test piece at resonance condition is developed and the result is confirmed with experiments. By this, the stress estimation can be done without the graphical complexity, if an oscillatory amplitude at the top of the test piece is measured. As an application of this analysis, $S-N$ characteristics for bearing steel and copper are obtained.

1. Introduction

The fatigue tester described in the 1st Report⁽¹⁾ has been often adopted, for the rapidity of testing, as a method of study in metal physics. But, if it is intended for a routine test, the following points must be noted :

(1) As an alternating stress is induced by the free vibration of the test piece as shown again in Fig. 1, the stress cannot be raised up to any large value. The range of stress over about 30 kg/mm^2 is beyond the capacity of this tester when the sample (mild steel) takes a uniformly cylindrical form. For tests on materials having high endurance limits, the sample of the tapering form that is contracted at the mid portion must be applied. In this case, it is not so difficult to induce stresses to the extent of about 100 kg/mm^2 .

(2) The second point to be noted is about the method of evaluating the stress induced. The untapered sample make a cos-distribution along its longitude for the oscillatory amplitude and therefore a sin-distribution for the stress. Then, the maximum stress induced at the middle can be theoretically derived from the amplitude at the top of the test piece. For the tapered one as shown in the previous paper⁽¹⁾, however, the stress had to be derived graphically from the distribution of the amplitude. This necessitated measurements of the oscillatory amplitude at many points along the longitude of the sample. And the scattering of the experimental plots on the $S-N$ curve seemed to be

partially due to the inaccuracy of the graphical method (see Figs. 10 and 11 in the 1st Report).

In this paper, the analysis of vibration of the tapered sample at resonant condition is developed in order to avoid the in accuracy in and the complexity of this graphical procedure. By this, the stress estimation can be done as easily as in the case of the untapered one. As an application of this method, fatigue tests on bearing steel and copper have been done.

Fig. 1 The vibration system

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Fig. 2 The form of the sample

2. Fundamental vibration of the tapered sample

The test piece adopted here has such a shape as a uniform cylinder at definite intervals from both ends and therefrom a taper contracted exponentially, the middle being minimum, as shown in Fig. 2.

A differential equation for the plane elastic wave induced in such a rod is given by

$$\frac{\partial^2 \xi}{\partial x^2} + \frac{1}{A} \frac{\partial A}{\partial x} \frac{\partial \xi}{\partial x} = \frac{1}{c^2} \frac{\partial^2 \xi}{\partial t^2} \quad (1)$$

where ξ is the oscillatory amplitude, A the cross sectional area, c the propagation velocity, t the time. By putting $\xi = \xi_0(x)e^{i\omega t}$ (ω the angular frequency), Eq. (1) becomes

$$\frac{d^2 \xi_0}{dx^2} + \frac{1}{A} \frac{dA}{dx} \frac{d\xi_0}{dx} + \frac{\omega^2}{c^2} \xi_0 = 0 \quad (2)$$

For the sample shown in Fig. 2, $A = A_0$ in intervals $x = 0 \sim a$ and $(a+2b) \sim (2a+2b)$ and $A = A_0 e^{-\gamma x}$ and $A = A_0 e^{-\gamma(a+2b-x)}$ in intervals $x = a \sim (a+b)$ and $(a+b) \sim (a+2b)$, respectively. It being symmetrical with respect to the middle, it is sufficient to consider only the left half. Thus, we have

$$x = 0 \sim a; \quad \frac{d^2 \xi_0}{dx^2} + \frac{\omega^2}{c^2} \xi_0 = 0 \quad (3)$$

$$x = a \sim (a+b); \quad \frac{d^2 \xi_0}{dx^2} - \gamma \frac{d\xi_0}{dx} + \frac{\omega^2}{c^2} \xi_0 = 0 \quad (4)$$

$$L_1 \cos \left\{ \frac{\omega}{c_1} (a+b) \right\} + L_2 \frac{\sin \left\{ \frac{\omega}{c_1} (a+b) \right\}}{\sinh \left\{ \frac{\omega}{c_1} (a+b) \right\}} = 0 \quad (13)$$

$$L_1 \cos \left(\frac{\omega}{c_1} a \right) + L_2 \frac{\sin \left(\frac{\omega}{c_1} a \right)}{\sinh \left(\frac{\omega}{c_1} a \right)} = \xi_{00} e^{-\gamma a/2} \cos \left(\frac{\omega}{c} a \right) \quad (14)$$

$$L_1 \left\{ \frac{\gamma}{2} \cos \left(\frac{\omega}{c_1} a \right) \mp \left(\frac{\omega}{c_1} \right) \frac{\sin \left(\frac{\omega}{c_1} a \right)}{\sinh \left(\frac{\omega}{c_1} a \right)} \right\} + L_2 \left\{ \frac{\gamma}{2} \frac{\sin \left(\frac{\omega}{c_1} a \right)}{\sinh \left(\frac{\omega}{c_1} a \right)} + \left(\frac{\omega}{c_1} \right) \frac{\cos \left(\frac{\omega}{c_1} a \right)}{\cosh \left(\frac{\omega}{c_1} a \right)} \right\} = -\xi_{00} \left(\frac{\omega}{c} \right) e^{-\gamma a/2} \sin \left(\frac{\omega}{c} a \right) \quad (15)$$

Elimination of L_1 and L_2 from Eqs. (13)~(15) gives the frequency equation

$$\begin{vmatrix} \left\{ \frac{\gamma}{2} \cos \left(\frac{\omega}{c_1} a \right) \mp \left(\frac{\omega}{c_1} \right) \frac{\sin \left(\frac{\omega}{c_1} a \right)}{\sinh \left(\frac{\omega}{c_1} a \right)} \right\}, & \left\{ \frac{\gamma}{2} \frac{\sin \left(\frac{\omega}{c_1} a \right)}{\sinh \left(\frac{\omega}{c_1} a \right)} + \left(\frac{\omega}{c_1} \right) \frac{\cos \left(\frac{\omega}{c_1} a \right)}{\cosh \left(\frac{\omega}{c_1} a \right)} \right\}, & -\xi_{00} e^{-\gamma a/2} \left(\frac{\omega}{c} \right) \sin \left(\frac{\omega}{c} a \right) \\ \cos \left(\frac{\omega}{c_1} a \right), & \frac{\sin \left(\frac{\omega}{c_1} a \right)}{\sinh \left(\frac{\omega}{c_1} a \right)}, & \xi_{00} e^{-\gamma a/2} \cos \left(\frac{\omega}{c} a \right) \\ \cos \left\{ \frac{\omega}{c_1} (a+b) \right\}, & \frac{\sin \left\{ \frac{\omega}{c_1} (a+b) \right\}}{\sinh \left\{ \frac{\omega}{c_1} (a+b) \right\}}, & 0 \end{vmatrix} = 0$$

which is simplified as

$$\left(\frac{\omega}{c} \right) \sin \left(\frac{\omega}{c} a \right) \frac{\sin \left(\frac{\omega}{c_1} b \right)}{\sinh \left(\frac{\omega}{c_1} b \right)} - \left(\frac{\omega}{c_1} \right) \cos \left(\frac{\omega}{c} a \right) \frac{\cos \left(\frac{\omega}{c_1} b \right)}{\cosh \left(\frac{\omega}{c_1} b \right)} + \frac{\gamma}{2} \cos \left(\frac{\omega}{c} a \right) \frac{\sin \left(\frac{\omega}{c_1} b \right)}{\sinh \left(\frac{\omega}{c_1} b \right)} = 0 \quad (16)$$

The solutions of Eqs. (3) and (4) are easily obtainable, that is,

$$x = 0 \sim a;$$

$$\xi_0(x) = K_1 \cos \left(\frac{\omega}{c} x \right) + K_2 \sin \left(\frac{\omega}{c} x \right) \quad (5)$$

$$x = a \sim (a+b);$$

$$\xi_0(x) = e^{(\gamma/2)x} \left[L_1 \frac{\cos \left(\frac{\omega}{c_1} x \right)}{\cosh \left(\frac{\omega}{c_1} x \right)} + L_2 \frac{\sin \left(\frac{\omega}{c_1} x \right)}{\sinh \left(\frac{\omega}{c_1} x \right)} \right] \quad (6)$$

$$\text{where } c_1 = c \left\{ \pm \left(1 - \frac{\gamma^2 c^2}{4\omega^2} \right) \right\}^{-1/2} \quad (7)$$

the + sign or - sign being taken so as to make c_1 real. In the above equations, trigonometric terms and hyperbolic terms correspond to the + sign and - sign respectively, and K_1 , K_2 , L_1 and L_2 are constants to be determined from the mode of vibration and boundary conditions. Now, we are treating the fundamental mode at resonance when the test piece is made to vibrate with the oscillatory amplitude ξ_{00} at its end. Then, we have

$$|\xi_0(x)|_{x=0} = \xi_{00} \quad (8)$$

$$\left| \frac{d\xi_0(x)}{dx} \right|_{x=0} = 0 \quad (9)$$

$$|\xi_0(x)|_{x=a+b} = 0 \quad (10)$$

Continuities of both the displacement and the stress at $x=a$ require that

$$|\xi_l(x)|_{x=a} = |\xi_r(x)|_{x=a} \quad (11)$$

$$\left| \frac{d\xi_l(x)}{dx} \right|_{x=a} = \left| \frac{d\xi_r(x)}{dx} \right|_{x=a} \quad (12)$$

where subscripts l and r mean both sides of the cross section at $x=a$, the left side and the right side.

The above relation yields $K_1 = \xi_{00}$, $K_2 = 0$ and

This equation, however, may be used for calculation of the total length of the test piece instead of the resonant frequency, this being done by assuming the length, $2b$, of the tapered part. In this case, it would be more handy if Eq. (16) were re-written as

$$\tan\left(\frac{\omega}{c}a\right) = \frac{\left\{\left(\frac{\omega}{c_1}\right)\cos\left(\frac{\omega}{c_1}b\right) - \frac{\gamma}{2}\sinh\left(\frac{\omega}{c_1}b\right)\right\}}{\left(\frac{\omega}{c}\right)\sinh\left(\frac{\omega}{c_1}b\right)} \quad (17)$$

From Eqs. (13) and (14), the constants not known yet are also derived,

$$L_1 = \xi_{00} e^{-\gamma a/2} \frac{\cos\left(\frac{\omega}{c}a\right) \sin\left\{\frac{\omega}{c_1}(a+b)\right\}}{\sinh\left(\frac{\omega}{c_1}b\right)} \quad (18)$$

$$L_2 = -\xi_{00} e^{-\gamma a/2} \frac{\cos\left(\frac{\omega}{c}a\right) \cosh\left\{\frac{\omega}{c_1}(a+b)\right\}}{\sinh\left(\frac{\omega}{c_1}b\right)} \quad (19)$$

By substituting Eqs. (18) and (19) into Eq. (6), the oscillatory amplitude becomes

$$\left. \begin{aligned} x=a \sim (a+b); \quad \xi_0(x) &= \xi_{00} e^{\gamma(x-a)/2} \cos\left(\frac{\omega}{c}a\right) \frac{\sin\left\{\frac{\omega}{c_1}(a+b-x)\right\}}{\sinh\left(\frac{\omega}{c_1}b\right)}, \\ \text{and } x=0 \sim a; \quad \xi_0(x) &= \xi_{00} \cos\left(\frac{\omega}{c}x\right) \end{aligned} \right\} \quad (20)$$

The maximum stress induced in the middle section is

$$\sigma_{\max} = E \left| \frac{d\xi_0(x)}{dx} \right|_{x=a+b} = -\xi_{00} E e^{\gamma b/2} \frac{\left(\frac{\omega}{c_1}\right) \cos\left(\frac{\omega}{c}a\right)}{\sinh\left(\frac{\omega}{c_1}b\right)} \quad (21)$$

Thus, the maximum stress can be theoretically derived from the value of the amplitude ξ_{00} at the top of the test piece. For the untapered test piece, the maximum stress, $\sigma_{\max}'$, is simply obtainable, that is,

$$\sigma_{\max}' = -\xi_{00} E \left(\frac{\omega}{c}\right) \quad (22)$$

Assuming that these two test pieces have an equal amplitude at their tops, the ratio of these stresses is

$$M = \frac{\sigma_{\max}}{\sigma_{\max}'} = \frac{c}{c_1} \frac{\cos\left(\frac{\omega}{c}a\right)}{\sinh\left(\frac{\omega}{c_1}b\right)} e^{\gamma b/2} \quad (23)$$

Fig. 3 The magnification factor of the stress induced

M may be regarded as a magnification factor of the induced stress which results from tapering the test piece. Fig. 3 denotes the relation between M and the ratio, $d_{\min}/d_{\max}$, which is derived from Eq. (23) for $2b=60$ mm and $\omega=2\pi \times 18.3 \times 10^3$ rad/sec by using $E=2.10 \times 10^6$ kg/cm² and $c=5.14 \times 10^5$ cm/sec, the value measured from the untapered sample.

Fig. 4 The photograph of samples

3. Experimental

The analysis developed above is based on the assumption that the displacement is uniform in each cross section and takes place in only the direction of the axis. Then, four kinds of samples as shown in Fig. 4 were applied to test the next two points, the length of the test piece and the displacement distribution. (1) In the first place, the lengths of the test pieces were decided so as to fit for the resonant frequency of the tester, 18.3 kc/sec, by a trial and cutting method. And these were compared with theoretical lengths which derived from Eq. (17). The result is tabulated in Table 1, a satisfactory agreement being seen between two. (2) Secondly, to test whether Eq. (20) represents the real ξ -distribution or not, the values of ξ were measured at many points along the longitude with a microscope ($\times 200$), when ξ_{00} was kept at 14×200^{-1} mm. Marks $\times$ and $\bullet$ in Fig. 5 are the experimental plots, while the curves are theoretical. The curve No. 1, for the untapered sample was given by $\xi_0(x) = \xi_{00} \times \cos[(\omega/c)x]$, while No. 2 and No. 3 were obtained from Eq. (20) by taking the sin-terms and No. 4 by taking the sinh-terms. It should be noticed that the value in the parenthesis of Eq. (7) is negative

Table 1 The length of samples

No. of samples	The length of samples mm		The length of the tapered part mm	$d \text{ min}/d \text{ max}$	r
	Theoretical	Experimental			
No. 1		140.3	0	1	0
No. 2	128.0	125.0	60	0.8	0.148 76
No. 3	110.5	107.0	60	0.6	0.340 55
No. 4	87.1	86.0	60	0.4	0.610 86

Fig. 5 Distribution of the oscillatory amplitude

for the No. 4 sample. From these experiments, the result of this analysis seems to be applied with enough accuracy.

4. Application

As an example of application, a fatigue test was done on the so-called bearing steel (Cr 1.4 %, C 1.05 %, Si 0.25 %, Mn 0.4 %, Fe 96 %). The fatigue limit appeared at round 10^8 reversals and was 40 kg/mm² (see Fig. 6). This number of reversals is appreciably higher than that obtained from the ordinary measurement, while the value of stress is lower than that expected for this material, (this being a contrast to Neppiras's for brass⁽²⁾). And also the slope of the S-N curve is considerably sharp. Enough cooling water was made to flow down on the full surface of the test piece. Under such condition, though no appearance of the temperature rise was observed, the appreciable heating-up might take place at micro-local-regions in the sample. If so and if micro-cracks resulted from such atomic processes as multiplication and intersection of dislocation, these would be annealed out by this heating-up. Thus, with increasing the stress-amplitude, the number of the stress-cycle at fatigue failure would increase and the slope of the S-N curve would be steep. Although one run

Fig. 6 The S-N curve (bearing steel)

Fig. 7 The S-N curve (copper)

finished within 2 hours or so, erosive action by cooling water might make a little contribution to the slope of the curve. To make such speculations, however, more experimental evidences will be need.

As an example of the untapered sample, a fatigue test was done on copper (99.9 %) annealed at 500°C for 0.5 hour (see Fig. 7). In this case, only small scatters were also seen about experimental plots. The breaking stress at 10^8 reversals was about 8 kg/mm², nearly the value expected from the ordinary measurement.

5. Conclusion

In this paper, the analysis is developed for the fundamental vibration at resonance of the tapered

sample and the result is confirmed with experiments. The estimation of the stress induced can be done in the same manner as in the case of the untapered one. This makes tests on materials having high endurance limits possible without any graphical complexity. As an example of application, a fatigue test on the so-called bearing-steel was carried out with small scatters of the experimental plots for such an ultrasonic vibration method.

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Photoelastic Study on Flat Bars with a Double Notch on One Edge Subjected to Pure Bending*

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Flat bar specimens of Epoxy Resin with a double notch on one edge were subjected to pure bending. The double notch consisted of a primary circular or 60°-V notch and a secondary U notch formed at the base of the primary notch. The effects of the root radius and the depth of the primary notch and also that of the depth of the secondary notch on the stress concentration factor were investigated. The stress concentration factor showed a nearly similar change to that of a single notch when the depth of the secondary notch was varied. This was also true in the case of the variation in the depth of the primary notch. As to the effect of the root radius of the primary notch, the stress concentration factor made a reverse change to that of a single notch. In all cases examined, the stress concentration factors of a double notch showed smaller values than the product of the independent stress concentration factors of a primary and a secondary notch.

1. Introduction

The authors carried out a photoelastic study on flat bars with V notches on both edges in pure bending, the effects of the root radius, the included angle and the notch depth on the stress concentration factor being examined⁽¹⁾. In the present study, a double notch which consisted of a primary circular or 60°-V notch and a secondary U notch formed

at the base of the primary notch, was taken as a stress raiser. No study on the stress concentration factor in such a double notch as above-mentioned seems to have been reported, except the analytical work on a circular hole with a small semi-circular notch or a sharp slit⁽²⁾. The above work shows that the stress concentration factor of the over-all configuration is equal to the product of the independent stress concentration factors of the circular hole and the secondary stress raiser. Besides, according to the results of fatigue tests using round bar specimens with a sharp secondary V notch at the base of a primary notch, the fatigue notch factor

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