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Some Studies on the Rotating Bending Fatigue of Large Mild Steel Specimens*

(Part 3, Fatigue Strength and Crack Propagation
in Specimens with Shoulder Fillet)

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The authors have made two kinds of fatigue test on shoulder fillet specimens (one is 135 mm dia. at the testing part and 15, 25 and 40 mm in fillet radius, the other is 14 mm dia. having proportional sizes to the large one), for the purpose of obtaining fatigue notch factors on shoulder fillet by comparing their fatigue limits with those of 125 ϕ mm and 10 ϕ mm plain cylindrical specimens described in Part 1. On the other hand, the stress concentration factors have been determined on fillets of the large specimens by taking into account the effect of the ratio of gauge length to fillet radius.

Furthermore, the authors have studied the relationship between the stress and the propagating speed of fatigue crack by analysing shelly patterns on fracture surfaces of shoulder fillet specimens.

It has been disclosed from these tests that;

(1) The correcting factors are 1.06, 1.018 and 1.006 where the above ratios are 0.60, 0.36 and 0.225, respectively. Stress concentration factors given by R. E. Peterson are equal to the author's ones without correction.

(2) Fatigue notch factors of large specimens are nearly equal to those of smaller ones, and coincide with the corrected concentration factors of equivalent stress based upon Mises' criterion.

(3) Fatigue crack decreases remarkably its propagating speed as the stress concentration factor on the fillet part becomes larger.

Introduction

The authors previously reported on the results of fatigue tests with large plain cylindrical specimens in Part 1⁽¹⁾, and subsequently on the crack propagation in Part 2⁽²⁾. In this paper, the authors describe the fatigue strength and the crack propagation in large specimens with shoulder fillet. Rotating bending fatigue tests in shoulder fillet specimens have been

carried out by many researchers; for example, E. Lehr⁽³⁾, R. E. Peterson⁽⁴⁾ and O. J. Horger⁽⁵⁾ tested large specimens of 30~50 mm shaft diameters.

The authors tested shoulder fillet specimens of 135 mm diameter and also small specimens (about 1/10 of the large ones in size), in order to find out the size effect on the fatigue strength of test specimens taking into account the stress concentration.

Since the material of the specimens was the same as that of the specimens used for the previous tests, the fatigue notch factors were easily obtained. It should be emphasized that one of the most

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important problems in machine design is to establish a general relationship between the fatigue notch factor and the stress concentration factor. In this respect, the stress concentration factors of shoulder fillet specimens were directly determined by using wire strain gauges.

Furthermore, the propagating speed of fatigue crack was investigated in the way as described in Part 1⁽¹⁾.

Test material

The specimens were made of 0.2% carbon steel (designated as BII in Part 1⁽¹⁾) which was refined in a 6-ton basic electric furnace and was forged into 175 mm diameter bars. Table 1 shows the chemical compositions, heat treatment and mechanical properties of the material.

Testing machines and specimens

The large specimens were tested in a 7.5 t·m rotating bending machine (cantilever type, 1250 r.p.m.) and small ones in a 10 kg·m machine (cantilever type, 6000 r.p.m.).

Fig. 1 (a) illustrates the large specimen, of which the larger part is 190 ϕ mm and the smaller part 133 ϕ mm in diameter, respectively. The smaller part was tapered in such a way as to equalize the bending stress along the shaft. Fillet radii of the specimens were 40, 25 and 15 mm, respectively, and the fillet parts were finished by emery paper until their surface roughness became about $2\mu H_{\max}$.

Table 1 Mechanical properties, chemical compositions and heat treatment of test materials

Material	Mechanical properties				Chemical composition %					Heat treatment
	Yielding point kg/mm ²	Tensile strength kg/mm ²	Elongation %	Reduction of area %	C	Si	Mn	P	S	
B II	30.0~33.5	49.8~52.0	31.8~37.5	52.0~62.7	0.22	0.28	0.59	0.031	0.015	850~860°C 5 hr F.C.

Fig. 1 (a) Large specimen

Fig. 1 (b) Small specimen

Fig. 1 (b) shows a typical example of the smaller specimen of which the larger diameter is 20 ϕ mm and the smaller one is 14 ϕ mm, respectively. These specimens were taken from the force fitted part of a larger specimen, the original position of each specimen in the cross section of the larger one being illustrated by the corresponding number as indicated in this figure. The fillet radii of the smaller specimens were 8.0, 4.3, 2.0 and 1.1 mm, respectively and their surfaces were finished by emery paper to $0.8\mu H_{\max}$ roughness.

Stress concentration factors for the fillet part

Stress measurements at the fillet part in both longitudinal and circumferential directions were carried out by using cross rosette strain gauges (9 mm gauge length) which were mounted on the fillet part of the larger specimen.

Fig. 2 illustrates a result of the strain measurement for the case of 15 mm fillet radius, where x represents the distance from fillet root to center of the strain gauge divided by the fillet radius.

Although the principal stresses in the fillet part may be directly computed from these values of strain, it must be considered that the measured value of strain give a mean value along the gauge length and the true maximum strain may be higher than the measured one.

When $\bar{\epsilon}(x)$ and $\epsilon(x)$ represent the true strain and the measured strain respectively, the relation

Fig. 2 Strains on fillet part

between $\bar{\epsilon}(x)$ and $\epsilon(x)$ may be given by the following equation;

$$\epsilon(x) = \frac{\int_{x-K}^{x+K} \bar{\epsilon}(x) dx}{2K} \dots\dots\dots (1)$$

where,

$$K = l/2\rho$$

l : gauge length

ρ : radius of curvature at fillet

It is necessary to approximate the distribution of measured strain $\epsilon(x)$ for computation of the true strain.

The authors attempted to approximate $\bar{\epsilon}(x)$ by a cubic Eq. (2), and then substituting Eq. (2) into Eq. (1), $\epsilon(x)$ can be given by Eq. (3).

$$\bar{\epsilon}(x) = Ax^3 + Bx^2 + Cx + D \dots\dots\dots (2)$$

$$\epsilon(x) = Ax^3 + Bx^2 + (C + AK^2)x + D + BK^2/3 \dots\dots (3)$$

Consequently, the function $\epsilon(x)$ is obtained by determining the coefficients in Eq. (3) which approximates the distribution of the measured strain shown in Fig. 2. These coefficients may be obtained by solving the following simultaneous equations, Eq. (4), which are obtained by substituting strains measured ϵ_i at the corresponding points x_i in Eq. (3).

$$\epsilon_i = Ax_i^3 + Bx_i^2 + (C + AK^2)x_i + D + BK^2/3 \dots\dots (4)$$

where, $i = 1, 2, 3$ and 4.

Fig. 3 Stress distributions on fillet part

Table 2 The relation between stress concentration factor and ρ/d

Fillet radius ρ mm	ρ/d	$l/\rho = 2K$	Stress concentration factor $\alpha_{K1} = \sigma_1/\bar{\sigma}_1$			Stress concentration factor $\alpha_{K0} = \sigma_0/\bar{\sigma}_1$		
			Corrected value	Measured value	Peterson	Corrected value	Measured value	Peterson
15	0.111	0.60	1.71	1.62	1.61	1.57	1.48	1.48
25	0.185	0.36	1.46	1.43	1.44	1.36	1.33	1.33
40	0.296	0.225	1.31	1.27	1.31	1.20	1.19	1.22
100	0.74	0.09	—	1.11	—	—	1.08	—

The values of ϵ_i and x_i are to be determined by trial and error method so as to approximate most suitably the measured strain distribution as shown in Fig. 2.

In Fig. 2, the dotted line denotes the function $\epsilon(x)$ (measured strain) and the full line shows $\bar{\epsilon}(x)$ as corrected according to Eq. (2). Since the variation of circumferential strain distribution ϵ_y along x is so small that the correction for ϵ_y is not necessary.

The stress distributions at the fillet based upon the corrected value and the measured one are shown in Fig. 3, where, σ_1 and σ_2 denote the corrected values of principal stresses in longitudinal and circumferential directions respectively, and σ_0 denotes the corrected equivalent stress based on Mises' criterion. On the other hand, σ_1' , σ_2' and σ_0' denote the measured values of principal stresses and the equivalent stress respectively.

In the same way, the stress distributions at fillet of 25 and 40 mm radius were obtained.

The stress concentration factors at fillet part thus obtained are summarized in Table 2 and illustrated in Fig. 4, where, α_{K1} , α_{K0} and α_{K1}' , α_{K0}' denote the concentration factors of σ_1 , σ_0 and σ_1' , σ_0' respectively. Also shown are the stress concentration factors for 100 mm radius fillet which were determined by the tests in plain cylindrical specimens.

The stress concentration factors α_{K1}' and α_{K0}' as measured by the authors coincide with those in Peterson's chart⁽⁶⁾ (K_t and K_t'), however, as the ratio of gauge length to fillet radius increases, the corrected values become larger than Peterson's values⁽⁶⁾. That is, α_{K1}/α_{K1}' and α_{K0}/α_{K0}' are equal to 1.058, 1.018 and 1.006 when the ratios of gauge length to fillet radius are equal to 0.6, 0.36 and 0.225 respectively.

Fatigue test results

Large specimen

The fatigue tests on large specimens were carried out by using four specimens of 25 mm fillet radius, three having 40 mm fillet radius, five having 15 mm fillet radius, respectively.

Fig. 5 shows the S-N curves for the large

specimens, being compared with those of large plain cylindrical specimens. The fatigue limits and fatigue notch factors based on the fatigue limits of large plain cylindrical specimen are tabulated in Table 3.

It should be noted that the fatigue notch factors β_K are nearly equal to the stress concentration factors α_{K0} based on Mises' criterion and about 0.9 of α_{K1} based on the maximum principal stress criterion.

The fracture surfaces of large specimens have two kinds of appearances: one has many origins of cracks as shown in Fig. 6 (a), and the other has a single origin as shown in Fig. 6 (b). The former

was observed when the ratio of testing stress σ to fatigue limit σ_w exceeded 1.15, and the latter was observed in the range from 1.0 to 1.10. In these figures many shelly patterns were observed on fractured surfaces, which were created artificially by adjusting the applied load on the specimen in order to obtain the crack propagating speed.

Small specimen

Fatigue tests on small specimens were performed by using 31 specimens and the results are illustrated in Fig. 7. The fatigue notch factors which are based on the results of 10 mm diameter plain cylindrical specimen shown in Fig. 8, are obtained and the results are summarized in Table 4.

It can be noted that β_K are nearly equal to α_{K0} which are determined from Fig. 4.

Fractured surfaces of the small specimens also have many crack origins when the stress ratio σ/σ_w exceeds 1.2, and single one in the range from 1.0

(d: dia. of shaft)
Fig. 4 α_K and P/d

Fig. 5 S-N curves of large specimen

Fillet radius ρ mm	Fatigue limit σ_w kg/mm ²	Fatigue notch factor β_K	Stress concentration factor	
			$\alpha_{K1} = \sigma_1/\bar{\sigma}_1$	$\alpha_{K0} = \sigma_0/\bar{\sigma}_1$
15	14.5(15.2)	1.61(1.54)	1.71	1.57
25	16.9	1.38	1.46	1.36
40	19.6	1.19	1.31	1.20
100	23.4	1.00	1.07	1.04

B15 (25 Rmm)
(a)

B16 (40 Rmm)
(b)

Fig. 6 Fracture surface

to 1.1. However, in the range from 1.1 to 1.2, the number of origins increases as the stress concentration factor becomes larger.

Relationship between the stress concentration factor and the fatigue notch factor

It has been considered very important to find out a general relation between the stress concentration factor and the fatigue notch factor taking into

Fig. 7 S-N curves of small specimen

Fig. 8 α_K and β_K

Table 4 Fatigue notch factor of small specimen

Fillet radius ρ mm	Fatigue limit σ_w kg/mm ²	Fatigue notch factor β_K	Stress concentration factor			
			$\alpha_{K1} = \sigma_1 / \bar{\sigma}_1$	$\alpha_{K0} = \sigma_0 / \bar{\sigma}_1$	Peterson's K_t	Peterson's K_t'
1.1	15.6	1.76	1.92	1.75	1.76	1.60
2.0	18.4	1.49	1.58	1.45	1.52	1.39
4.3	22.3	1.23	1.30	1.20	1.28	1.20
8.0	23.3	1.18	1.16	1.14	—	—

Table 5 Sequences of load change on B16 specimen (40 R)

Order of shelly line	0	1	2	3	4	5	6	7	8	9	10	11	12
Stress σ_n kg/mm ²	20.6	10.15	13.35	19.75	12.28	17.62	14.42	11.21	15.49	9.08	12.28	14.42	16.54
Reversal number $\Delta N \times 10^{-3}$	1428.6	23.1	19.7	10.8	72.6	20.2	39.0	108.9	15.5	20.3	29.2	20.0	10.5

consideration the size effect of specimen.

Using the results tabulated in Tables 3 and 4, the relationship between the stress concentration factors α_{K1} and α_{K0} and the fatigue notch factor β_K are illustrated in Fig. 8. It can be clearly recognized from this figure that the fatigue notch factor β_K is nearly equal to α_{K0} (stress concentration factor for the equivalent stress based on Mises' criterion) and $0.9 \alpha_{K1}$ (concentration factor for the principal stress) irrespective to the size of specimen, when the stress concentration factor is smaller than 2.0.

Propagation of fatigue crack

Propagating speed of fatigue crack in large shoulder fillet specimens can be obtained by analysing shelly patterns on the fracture surface as shown in Fig. 6 (b). Fig. 9 illustrates a sketch of shelly patterns. Stresses σ_n in the parallel part of the specimen and reversal numbers ΔN which correspond to the number of shelly line are given in Table 5.

Stress gradient in surface layer of the shoulder fillet

In the case of a plain cylindrical specimen,

Fig. 9 Sketch of shelly pattern

stresses along the edge line of the fracture surface could be computed with some simple presumptions, because the stress gradient in surface layer is linear. In the case of a shoulder fillet specimen, however, stresses along the edge line are hard to compute because of the unknown stress gradient in the fillet part.

The authors attempted, at first, to find out an approximate method for obtaining the stress gradient. That is, the gradient may be drawn in such a way as shown in Fig. 10, and this stress gradient is given by a function $f(\eta)$ in the longitudinal section of fillet part.

The function $f(\eta)$ must satisfy the following Eqs. (5) and (6) according to the theory on the bending of beam.

$$\int \sigma \eta dA = \int f(\eta) \cdot \eta dA = M \quad (5)$$

$$f(\eta) = \alpha_K \cdot \sigma_n = \frac{\alpha_K \cdot \eta_1 \cdot M}{I_0} \quad (6)$$

where,

dA : elemental area at a distance η from neutral axis

M : bending moment acting in the beam

η_1 : distance from the neutral axis to the surface of parallel part

α_K : stress concentration factor on the fillet part

σ_n : stress in the parallel part

I_0 : moment of inertia in the cross section of beam

Furthermore, the function $f(\eta)$ will satisfy the following condition.

When η is positive,

(a) $f(\eta)$ is symmetrical with respect to the point O in Fig. 10.

(b) $df(\eta)/d\eta > 0$

(c) $d^2f(\eta)/d\eta^2 > 0$

A simple example satisfying the above conditions may be given by Eq. (7).

$$f(\eta) = C_1 \eta^{2n+1} + C_2 \eta^{2m+1} \quad (7)$$

where, n and m are positive integers and m is larger than n .

Substituting Eq. (7) into Eq. (5) and Eq.

Fig. 10 Stress gradient on fillet part

(6) the constants C_1 and C_2 will be determined, and then Eq. (7) can be deduced to the following Eq. (8).

$$f(\eta) = \frac{M \eta^{2n+1}}{I_0 \eta_1^{2n}} \left\{ \frac{I_0 \eta_1^{2m} - \alpha_K I_{2(m+1)}}{\eta_1^{2(m-n)} I_{2(n+1)} - I_{2(m+1)}} + \frac{\alpha_K I_{2(n+1)} - I_0 \eta_1^{2n}}{\eta_1^{2(m-n)} I_{2(n+1)} - I_{2(m+1)}} \eta^{2(m-n)} \right\} \quad (8)$$

Where, $I_{2(n+1)}$ and $I_{2(m+1)}$ mean $2(n+1)$ th and $2(m+1)$ th order moment of inertia on the cross section.

The relation between α_K and m, n is given by substituting Eq. (8) in the above three conditions.

$$\frac{I_0 \eta_1^{2n}}{I_{2(n+1)}} < \alpha_K < \frac{I_0 \eta_1^{2m}}{I_{2(m+1)}} \quad (9)$$

It is true that many pairs of n and m may satisfy Eq. (9) for a definite value of α_K . However, when the stress concentration factors are smaller than 2.0, the function $f(\eta)$ can be assumed by taking n and m equal to 0 and 1, respectively. Then, the stress σ is given by Eq. (10).

$$\sigma = \frac{M \eta}{I_2} \frac{I_2 \eta_1^2 - \alpha_K I_4 + \alpha_K I_2 \eta^2 - I_2 \eta^2}{I_2 \eta_1^2 - I_4} \quad (10)$$

The stress on the edge line of fracture surface

The stress on the edge line of fracture surface may be computed by the same method as described in Part 2⁽²⁾. In this case, however, it is necessary to make such an assumption that the stresses can be expressed by Eq. (10) regardless of any change in the position of the neutral axis in the fractured cross section. This assumption might become inadequate when the shape of the effective area in the cross section bearing the bending stress becomes unsymmetric.

Therefore, the above-mentioned approximate method is useful only when the area of fractured part is comparatively small and the factor α_K remains small.

The maximum tensile stress $(\sigma_b)_{\max}$ at an arbitrary point "b" on the edge line may be given by computing the second and fourth order moment of inertia of non-fractured part of the cross section as long as the assumption is valid.

$$\nu_b = \frac{(\sigma_b)_{\max}}{\sigma_n} = \frac{1}{r} \left\{ \frac{y \cos \alpha}{i_{2z}} \frac{I_{2z} r^2 - \alpha_K I_{4z} + (\alpha_K - 1) I_{2z} y^2}{I_{2z} r^2 - I_{4z}} + \frac{z \sin \alpha}{i_{2y}} \frac{I_{2y} r^2 - \alpha_K I_{4y} + (\alpha_K - 1) I_{2y} z^2}{I_{2y} r^2 - I_{4y}} \right\} \quad (11)$$

where,

$$\tan \alpha = \frac{z}{y} \frac{I_{2z}}{I_{2y}} \frac{I_{2y} r^2 - \alpha_K I_{4y} + (\alpha_K - 1) I_{2z} z^2}{I_{2z} r^2 - \alpha_K I_{4z} + (\alpha_K - 1) I_{2z} y^2}$$

ν_b : stress magnification factor at point "b" due to fracture

r : radius of shaft

I_{2y}, I_{4y} and I_{2z}, I_{4z} : second and fourth order moment of inertia of non-fractured part of the cross section about the principal axis O_GZ shown

Fig. 11 Section of fractured part

in Fig. 11.

Relation between crack propagating speed and the stress on the edge line of fracture surface

The crack propagating speed is given by dividing the distance of shelly lines shown in Fig. 9 by the corresponding value of the reversal number ΔN tabulated in Table 5, the speed being expressed in microns per cycle (μ/c). On the other hand, the stress on the edge line is approximately given by $\nu_b \sigma_n$, σ_n being the nominal stress tabulated in Table 5.

Figs. 12 (a), (b), (c) and (d) indicate the relationships between the crack speed v_c and the stress $\nu_b \sigma_n$, which were obtained from measurements on the fracture surfaces of the specimens of 100, 40,

Fig. 12 v_c and $\nu_b \sigma_n$

Table 6 Relation between stress gradient and A, B

Specimen	P.C. 125 ϕ mm	S.F. 125 ϕ mm ($\rho=100$ mm)	S.F. 135 ϕ mm ($\rho=40$ mm)	S.F. 135 ϕ mm ($\rho=25$ mm)	S.F. 135 ϕ mm ($\rho=15$ mm)
α_K	1.0	1.04	1.19	1.36	1.50
ϕ_r kg/mm ² /mm	0.411	0.450	0.596	0.785	0.826
A	400	400	650	900	1300
B	17	16.5	25	33	44

Note: P.C., plain cylindrical specimen; S.F., shoulder fillet specimen

Fig. 13 A, B and stress gradient ϕ_r

25 and 15 mm fillet radii.

When computing ν_b in Eq. (11), α_{K0} values given in Table 3 were used as the stress concentration factors, taking into consideration the fact that the actual crack path deviated a little from the maximum stress point in the fillet.

As can be seen from Fig. 12, the crack speeds v_c are approximately represented by Eq. (12 a) or Eq. (12 b).

$$v_c = \frac{\nu_b \sigma_n - \sigma_0}{B}, \quad 1.5 \sigma_0 \leq \nu_b \sigma_n < \sigma_s \quad (12 a)$$

$$v_c = \frac{(\nu_b \sigma_n - \sigma_0')^2}{A}, \quad 1.5 \sigma_0 \leq \nu_b \sigma_n \quad (12 b)$$

where,

σ_s : yield stress of the tested material

σ_0 : minimum stress for the crack propagation

σ_0' : material constant

A, B : constants depending on the stress gradient namely the stress concentration factor

Table 6 shows the values of σ_0, σ_0', A and B which are obtained from Fig. 12 and also values of stress gradient ϕ_r computed by Eq. (13).

$$\phi_r = \left(\frac{d\sigma}{dn} \right)_{\eta=r} = \frac{\sigma_n}{r} (5\alpha_K - 4) \quad (13)$$

Fig. 13 illustrates the relationship between the stress gradient ϕ_r and the constants A and B as represented approximately by the following equations:

$$v_c = \frac{\nu_b \sigma_n - \sigma_0}{8 + 50 \phi_r^2}, \quad 1.5 \sigma_0 \leq \nu_b \sigma_n < \sigma_s \quad (14 a)$$

$$v_c = \frac{(\nu_b \sigma_n - \sigma_0')^2}{150 + 1500 \phi_r^2}, \quad 1.5 \sigma_0 \leq \nu_b \sigma_n \quad (14 b)$$

Conclusion

The following is a summary of the results as presented in this paper.

(1) The stress concentration factors on shoulder fillet specimens as obtained directly from the experiment were nearly equal to those reported by Peterson⁽⁶⁾. However, if the effect of the finite gauge length on the measured values of strains is taken into consideration, then the corrected stress concentration factors are determined as 1.06, 1.018 and 1.006 times K_t and K_t' given by Peterson when the ratios of gauge length to fillet radius are equal to 0.06, 0.36 and 0.225, respectively.

(2) When the stress concentration factor is smaller than 2.0, the fatigue notch factors are nearly equal to the stress concentration factors of equivalent stress based on the Mises' criterion, irrespective of the size of specimen.

(3) The number of the crack origins on the fillet part depends upon the ratio of the testing stress to the fatigue limit of the material. When the ratio exceeds 1.2, multi-cracks were originated in the fillet, while a single crack was observed when the ratio was smaller than 1.1.

(4) Propagating speed of fatigue crack on a large shoulder fillet specimen decreases remarkably, when the stress concentration factor is large. It is given in terms of the stress gradient ϕ_r in the fillet part by the following equations.

$$v_c = \frac{\nu_b \sigma_n - \sigma_0}{8 + 50 \phi_r^2}$$

$$v_c = \frac{(\nu_b \sigma_n - \sigma_0')^2}{150 + 1500 \phi_r^2}$$

References

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Discussion

N. OKAZAKI: (1) Is it not unreasonable to equate the re-fatiguing tests of the materials B 18, B 15 and B 23, which were not fractured in the S-N curve of large specimen tests in Fig. 5 and so were machined about 10 mm in the larger part for re-testing, with the fatigue test of the original materials? The re-test might be called a re-fatiguing test of a specimen that has undergone a material change in a fatigue test.

(2) You admit that the conventional theory of fatigue failure being caused according to the shearing strain energy theory has been well endorsed, while you presume that the stress gradient has considerable bearing on fatigue failure. Explanation of the relation between these two considerations will be appreciated.

(3) The expressions "low", "intermediate" and "high" in low overstress, intermediate overstress and high overstress seem to be more appropriate as scientific terms. What is your opinion?

H. OUCHIDA: (1) Much time and money were spent for the present study and the results there of are deemed valuable, indeed. Seeing that in the 1st Report, there was a scattering of experimental points due to presence of cracks in the tested specimens, in the present experiment, sound materials ought to have been used for notched specimens. If so, the result would have been more reliable.

(2) A small specimen taken from the core of a large specimen to determine the size effect must have contained a defect near its surface. Therefore it would be unreasonable to compare it with a large specimen free from any defect in the surface layer for fatigue strength or notch factor. Perhaps it is due to the defect that in Fig. 8 the value of β_K for the small specimen turns out greater than that for the large one.

In Perterson's and my tests, the value of β_K is greater for large specimens than that for small specimens, which is also expected from the size effect.

(3) In the conclusion (3) it is stated that the number of cracks in the fillet depends on the ratio of the tested stress to the fatigue limit, and is independent of the stress concentration factor, since the fatigue failure depends on the stressed condition of the area near the surface, it would be reasonable to think that these two factors are both involved thereby.

It is desirable that the quantitative relation between crack initiation and σ/σ_w is further confirmed as to larger values of α_K .

H. NAKAMURA: (1) You expressed B with the form $(8+50\phi_r^2)$, but the relation between α_K and B is linear as shown in append-Fig. 1. Therefore, let me suggest that you check the form " $a+b\alpha_K$ " as the expression of B .

(2) The form of B is very important, therefore I hope that you will continue these fatigue tests with test pieces of various sizes and decide the form of B more exactly.

(3) According to your research, σ_0 , the limit stress at which the crack spreads or not, is 11.5 kg/mm². But, σ_0 , which I experienced last year^(a), is about 7 kg/mm², and σ_0 , which Mr. Ouchida^(b) measured on the V-notched specimen, is about 6 kg/mm².

I suppose that the difference in the values of σ_0 , is due to the method of fatigue test. That is, when you tested by stress σ , sometimes you stopped fatigue test and repeated with a slightly lower stress in order to measure the crack depth, I fear that by these operations a coaxing and a compressive residual stress are generated and the condition of crack's tip changes.

Append-Fig. 1

Author's closure

(1) The fillet parts of the specimens B 18, B 15 and B 23 were removed by machining 10 mm in length towards the larger part, and these specimens were tested again in high stress levels, with the intention of reduce the effect of under-stressing.

Therefore, even if the effect may exist, it will be confined to the fatigue strength of high stress level.

(2) It is known that a mild steel subjected to biaxial reversal stress such as the fillet part of shoulder specimen is fractured when the equivalent stress based upon the Mises' criterion exceeds a

(a) Nakamura, Amakasu, and other: *Trans. Japan Soc. Mech. Engrs.*, Vol. 26, No. 161 (1960), p. 91.

(b) H. Ouchida: *Jour. Japan Soc. Mech. Engrs.*, Vol. 62, No. 484 (1959), p. 722.

definite value for the material. While, it is also recognized as the chief cause of the size effect that the fatigue strength is related to the stress gradient in the subsurface of the specimen.

Considering the above, it seems that the fatigue notch factors, representing the ratio of fatigue limit of the shoulder fillet specimen divided by that of plain cylindrical one having the same gradient, would be independent of the specimen size.

The equivalent diameter corresponding to the fillet part may be computed by Eq. (13), for instance, the large shoulder fillet specimens having 15, 25, 40 and 100 mm in fillet radius are equivalent in stress gradient to the plain cylindrical ones having 38.5, 48.3, 70 and 104 mm in diameter, respectively.

It was shown by Mr. Ouchida^(b) and Mr. Peterson^(c) that the fatigue limit of the specimen among these diameters varied scarcely.

Therefore, it may be presumed that the fatigue notch factor β_K based on the fatigue limit of 125 mm plain cylindrical specimen as shown in Table 3 coincides with that mentioned above. Accordingly, it is also expected that the factor β_K is equal to the stress concentration factor α_{K0} of the equivalent stress based on the Mises' criterion. For the small shoulder fillet specimens, the facts mentioned above may be also presumed.

Furthermore, it should be noted that the relation between β_K and α_K depends upon the fatigue limit of equivalent specimens having the same stress gradient as that of fillet part.

When the fatigue limit of equivalent specimen is larger than that of a plain cylindrical one having the same diameter as the shoulder fillet one, it may be observed that β_K is smaller than α_K , as shown by Mr. Ouchida^(b) and Mr. Peterson^(c).

(3) The expression given by Mr. Cummings may be adequate, in your opinion, when many sorts of materials are tested under a wide range of stress concentration factors.

(1) Although the test material includes some flaws with fine cracks, these are scarce, one or two are only observed at the center part of cross section

of a 125 ϕ mm specimen tested by hydrochloric acid etching.

Therefore, it is presumed that the results in Fig. 7 are fully reliable except those of specimens, 11B2 and 6B2, which were taken from the center part.

(2) The authors should like to reply the same as (2) to Mr. Okazaki.

(3) The descriptions in (3) of the conclusion represents a general expression of the results of the author's experiment. It has been mentioned in the fatigue test result of small specimens that crack origins increase in numbers as the stress ratio become larger between 1.1 and 1.2.

(1) Although the constant B in Eq. (12a) may be in proportion to α_K for a definite size of specimen, it is presumed in Part 1⁽¹⁾ that the propagating speed descends to a very low level when the size of specimen becomes small. Therefore, the authors have attempted to represent the constant as a function of the stress gradient, intending to involve both effects of stress concentration factor and the size of specimen. On the other hand, if B were approximated by the linear function of Φ_r in Fig. 13, it would be unreasonable that the crack speed of an extremely large specimen, for instance, the specimen subjected to a tension and compression reversal load, would become infinite. Such discrepancy may be avoided by approximating B as a quadratic function of Φ_r .

(2) The authors are now carrying out the test of the crack speed with various sizes of specimens, the results of which will be presented by a subsequent paper.

(3) It is presumed that the minimum value of the stress which keeps up the propagation of crack is to be defined with the plain cylindrical specimen having many fine cracks of which the depths are negligible comparing to the specimen size. Accordingly, σ_0 is to be computed with the nonfractured part of the specimen when the depth of crack is not negligible.

Since σ_0 is obtained by the operation lowering the stress from high level in which the crack is propagating, it seems that the coxing and strain-hardening effects which may be created at the tip of crack will be hardly affected.

(c) R.E. Peterson and A.M. Wahl: *Jour. Appl. Mech.*, Vol. 3 (1936), p. A-15.